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A ranking method based on data envelopment analysis for classification the insurers risk in Saman insurance company

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Abstract
Insurance industry is one of the most important factors for the economic development of the countries. For example, insurance industry can be important for the stability of financial systems mainly because they are large investors in financial markets, because there are growing links between insurers and banks and because insurers are safeguarding the financial stability of households and firms by insuring their risks. This paper focuses on the efficiency evaluation of the insurance industry. For this purpose, we use the dataset of the car insurance policies of Saman Insurance Company during the years 2018-2019 and implements an extended cross efficiency method to rank the insured for prediction the risk of insurers in terms of existence of damage risk or absence of damage risk.

Keywords: Insurance industry, Data Envelopment Analysis, Cross efficiency evaluation, Ranking.

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1. Introduction

Data Envelopment Analysis (DEA), introduced by Charnes and further developed by Banker [1,2] is a non-parametric technique for assessing the efficiency of a set of Decision Making Units (DMUs) with multiple inputs and multiple outputs. It assigns an efficiency score between 0 and 1 to each unit. The larger the efficiency score, the better performance the unit under evaluation has. Traditional DEA models cannot discriminate among efficient DMUs since they all get the efficiency score equal to 1. In this regard, several ranking methods have been developed in DEA literature. For more studies about ranking methods in DEA, see [3-6].

For example, Salehi [7] proposed a method for ranking all DMUs by using strong and weak supporting hyperplanes. They evaluated the rail freight and the passenger transportation in some Asian countries. Akhlaghi [8] introduced a linear programming (LP) model to determine the most BCC efficient decision making unit. For more studies, see [9-11].

Traditional DEA models compute the efficiency of each DMU as a ratio of weighted outputs to inputs in which the optimal weights are obtained under some constraints. In this regard, n different sets of weights are determined after evaluating n DMUs. Sexton [12] proposed a method for ranking DMUs based on their cross-efficiency score. They evaluated each unit by the n obtained sets of weights and defined the cross-efficiency score of each unit as the average of the obtained efficiency scores. Doyle and Green [13] pointed out that the cross efficiency method could provide a unique ordering of the units. Anderson et al. [14] showed that this method could eliminate the unrealistic weights without incorporating weight restrictions. Because of these advantages, the cross-efficiency method has been extensively applied in various cases. For more studies, see Sexton [12], [15-19].

Despite the benefits of the cross efficiency method, there exist some factors that reduce the utility of this method. The main drawback is that the cross-efficiency scores may not be unique due to the presence of alternative optimal weights. As a result, many authors incorporated secondary goal models into cross-efficiency method for the weight selection. Several studies have been proposed for secondary goals, for instance, [13,16,17,20,21].

The original DEA models consider the situation that all aspects of the production process are desirable. However, this assumption can be violated due to the existence of undesirable outputs denoted by the "bad" aspect of the production process. In the other word, in many real world applications, the production process, in addition to the desirable outputs, produces the undesirable outputs. Many authors have focused on treating undesirable outputs, some of the most commonly cited works include: [15], [22-35].

On the other hand, the traditional DEA models select the most desirable input and output weights for each DMU when evaluating that unit. Therefore, the units achieve the maximum efficiency score by applying these desirable weights. But, the efficiency of different DMUs is determined by the different input and output weights. It seems that, comparison and ranking the units are impossible challenges. Hence, there is a need to impose the weight restrictions to reduce the flexibility of weights and to improve the discrimination of DEA models. The weight restrictions are incorporated as the additional constraints on the input and output weights in the DEA model to show the value judgements of the decision maker (DM). See [36-40] for more studies about the weight restrictions in DEA.

Insurance industry is an important segment of the economy in every country. A closer look at the economies of the developed countries shows that the insurance industry has a significant contribution in the economic development of these countries. Therefore, evaluating the performance of insurance companies and providing a method for improving their performances are as the most important issues in the economic development of countries. In general, there are two approaches to assess the efficiency of an insurance company, namely the production approach and the financial intermediary approach. The production approach confines the role of financial institutions to that of service providers to account holders. In the intermediation approach, financial institutions act to channel the funds between savers and investors.

Because of increased consumer awareness and expectations, evolving business models, new technologies with emerging risks, new waves of regulations, and an unprecedented level of sanctions, insurance companies must revise their risk strategies and invest heavily in compliance. Awareness of risks has risen dramatically, and many companies have already started the journey toward structured, business-driven, forward-looking risk management practices. But there is still significant work to be done. Risk management is in fact a significant concern of boards of directors and executive managers across the insurance industry. Therefore, the insurance companies need to access to the powerful risk analysis tools in order to manage the potential risks. DEA is one of the most important techniques to identify the risk resources.

Given the importance of the risk management, in particular in the insurance industry, this study focuses on the risk management of the insurance industry. For this purpose, we use the dataset of the car insurance policies of Saman Insurance Company during the years 2018-2019 and implements the cross efficiency method to rank the insured for prediction the risk of insurers (in terms of existence of damage risk or absence of damage risk).

The rest of this paper is organized as follows: section 2 reviews the cross efficiency method. In section 3, we present an extension for the cross efficiency method in the presence of undesirable outputs and the weight restrictions. A numerical example is provided in section 4. Section 5 concludes the paper.

2. Preliminaries

This section reviews some basic preliminaries, e.g. the cross-efficiency method, the undesirable outputs and the weight restrictions in DEA.

2.1. Cross efficiency evaluation

Consider a system of n DMUs, denoted by $DMU_j, j = 1, \dots, n,$ where each unit consumes m different inputs to generate s different outputs. The i^{th} input and r^{th} output for DMU_j are denoted by x_{ij} and y_{rj}, respectively, for $i = 1, \dots, m$ and $r = 1, \dots, s$. Charnes et al. (1978) proposed the CCR model to evaluate the efficiency score of units which are provided in both envelopment and multiplier forms in Table 1.

Table 1. The envelopment and multiplier forms of CCR model.

The envelopment form	The multiplier form
$\min \theta_o$ (1a) s.t. $\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1, \dots, m,$ $\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1, \dots, s,$ $\lambda_j \ge 0, \quad j = 1, \dots, n,$ $\sum_{j=1}^{n} \lambda_j = 1,$ θ_o is free.	$\max \sum_{r=1}^{s} u_r y_{ro} + u_0$ (1b) s.t. $\sum_{i=1}^{m} v_i x_{io} = 1,$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad j = 1, \dots, n,$ $u_r \ge 0, \quad r = 1, \dots, s,$ $v_i \ge 0, \quad i = 1, \dots, m.$

Let $\{v_{1o}^*, \dots, v_{mo,}^*\ u_{1o}^*, \dots, u_{so}^*, u_0^*\}$ be an optimal solution for model (1b) evaluating DMU_o. Sexton et al. (1986) defined the cross-efficiency score of DMU_j corresponding to DMU_o as follows:

$$E_{oj} = \frac{\sum_{r=1}^{s} u_{ro}^* y_{rj} + u_0}{\sum_{i=1}^{m} v_{io}^* x_{ij}} \quad o, j = 1, \dots, n. \quad (2)$$

They defined the average of $E_{oj}'s$ as the cross-efficiency score of DMU_j as follows:

$$E_j = \frac{1}{n} \sum_{o=1}^{n} E_{oj}. \quad (3)$$

2.2. undesirable outputs

Suppose that, we have a set of n DMUs, $DMU_j, j = 1, \dots, n,$ where each unit has three factors, the inputs, the desirable outputs and the undesirable outputs. Assume that, G and B are the sets of the desirable outputs and the undesirable outputs, respectively. Also, suppose that all the input and output values are nonnegative and atleast one of them is non zero. The unit $DMU_o = (x_o, y_o)$ is the unit under evaluation.

Seiford and Zhu [35] introduced the production possibility set under the variable returns to scale (VRS) assumption:

$$T = \{(x,y) | x \ge \sum_{j=1}^{n} \lambda_j x_j, y_r \le \sum_{j=1}^{n} \lambda_j y_j, r \in G, y_r \ge \sum_{j=1}^{n} \lambda_j y_j, r \in B, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, \forall j\} \quad (4)$$

They proposed the following model to evaluate the units in the presence of the undesirable outputs:

$$\begin{aligned} & \min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\ & s.t. \\ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, && \forall i, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = y_{ro}, && \forall r \in B, \quad (5) \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, && \forall r \in G, \\ & \sum_{j=1}^{n} \lambda_j = 1, \\ & \lambda_j \ge 0, && j = 1, \dots, n, \\ & s_i^- \ge 0, s_r^+ \ge 0, && \forall i, r. \end{aligned}$$

Where ε is the Non-Archimedean.

The dual of model (5) is as follows:

$$\begin{aligned} & \max \sum_{r \in G} u_r y_{ro} - \sum_{r \in B} u_r y_{ro} + u_0 \\ & \text{s.t.} \\ & \sum_{r \in G} u_r y_{rj} - \sum_{r \in B} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0, \quad j = 1, \dots, n, \quad (6) \\ & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & u_r \ge \varepsilon, && r \in B, \\ & u_r \ge \varepsilon, && r \in G, \\ & v_i \ge \varepsilon, && i = 1, \dots, m. \end{aligned}$$

3. Applying the cross efficiency evaluation for the car insurance policies

In this section, we develop the cross efficiency method for the insurance industry. For this purpose, we consider the car insurance policies. This data set has five inputs (the Number of years of car operation, the price, the driver gender, the driver age, the province of driver's residence) and two outputs (the number of years without damages, the damage ratio). The damage ratio is calculated as the ratio of the amount of damage cost to the amount of premium paid by the insurer which is considered as undesirable output. Therefore, we aim to develop the cross efficiency evaluation to the situation where there are the undesirable outputs. On the other hand, according to the type of inputs and outputs in the car insurance policies, the decision maker decides to determine the relative importance of inputs and outputs via the restrictions on the input and output weights.

We formulate the following model to determine the efficiency of units in the data set of the car insurance policies:

$Z_o^* = \max u_1 y_{1o} - u_2 y_{2o} + u_0$ (7.1)

s.t. (7,2)

$u_1 y_{1o} - u_2 y_{2o} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0,$ j=1,…,n (7,3)

$\sum_{i=1}^{m} v_i x_{io} = 1,$ (7,4)

$u_2 \geq 2u_1,$ (7,5)

$v_2 \geq v_1,$ (7,6)

$v_1 \geq 2v_4,$ (7,7)

$v_5 \geq v_4,$ (7,8)

$v_5 \geq 2v_3,$ (7,9)

$u_1 \geq \varepsilon,$ (7,10)

$u_2 \geq \varepsilon,$ (7,11)

$v_i \geq \varepsilon$ (7,12)

Model (7.1)- (7.12) is formulated by adding the weight restrictions (7.5)- (7.9) to model (6). These weight restrictions are selected according to the importance of each indicator from the decision maker's point of view. Model (7.1)- (7.12) determines the efficiency of DMUs in the data set of the car insurance policies.

Let $\{v_{1o}^*, \dots, v_{mo}^*, u_{1o}^*, u_{2o}^*, u_0^*\}$ is an optimal solution for model (7.1)- (7.12) evaluating DMU_o. We define the cross-efficiency score of DMU_j corresponding to DMU_o as follows:

$$E_{oj} = \frac{u_{1o}^* y_{1o} - u_{2o}^* y_{2o} + u_0^*}{\sum_{i=1}^{m} v_{io}^* x_{ij}} \quad o, j = 1, \dots, n. \quad (8)$$

Then, the average of E_{oj} is defined as the cross-efficiency score of DMU_j as follows:

$$E_j = \frac{1}{n} \sum_{o=1}^{n} E_{oj}\,. \quad (9)$$

Finally, the cross efficiency score of units can be used to rank the decision making units.

In the next section, we report the results of the proposed approach for the car insurance policies.

4. **Numerical example**

In this example, the results of applying our proposed approach to the data set which includes 201 insurers who have purchased insurance policies from Saman Insurance company during the years 2018-2019, are reported. Each insurer is considered as a decision making unit with five inputs (the Number of years of car operation (x_1), the price (x_2), the driver gender (x_3), the driver age (x_4), the province of driver's residence (x_5)) and two outputs (the number of years without damages (y_1), the damage ratio (y_2)). Table 1 shows the information of units. For the second input (x_2), an integer number is assigned to the price of each insured car, so that the smaller integer number is assigned to the more expensive car. For the third input (x_3), the number 1 is attributed to the female gender and the number 2 to the male gender. For the fourth input (x_4), the numbers 1, 2 and 3 are assigned to the insurers age, so that the smaller number is assigned to the older insurer. According to the manager’s view point, an integer number is assigned to each province of drivers’ residence. The damage ratio (y_2)

is calculated as the ratio of the amount of damage cost to the amount of premium paid by the insurer which is considered as undesirable output.

Table 2. The information of units.

Inputs and Outputs	Mean	Median	Mode	Variance	Minimum	Maximum
x_1	5.4975	4	1	18.2712	1	21
x_2	10.8905	13	13	6.5180	1	13
x_3	1.6716	2	2	0.2144	1	2
x_4	2.1045	2	2	0.5440	1	3
x_5	1.5572	1.5	1	0.6980	1	3
y_1	2.0498	2	0	4.3875	0	8
y_2	53.8632	47	0	19433.18	0	1272.95

Now, our proposed approach is Implemented for ranking the units in this example. We solve model (7) to determine the efficiency score of DMUs and the results are summarized in Table 3. The columns 2, 4, 6, 8, 10 and 12 show the efficiency score of units. Then, we use the equation (8) to construct the cross efficiency matrix. This matrix is a square matrix of order 201. Table (4) shows the median, the mode, the variance, the minimum and the maximum of the cross efficiency scores of each unit according to other DMUs. Finally, the cross efficiency score of each unit is determined by using the equation (9). Table (5) reports the cross efficiency score and the rank of each DMU.

Table 3. The efficiency score of units.

DMU	Z_o^*	DMU	Z_o^*	DMU	Z_o^*	DMU	Z_o^*	DMU	Z_o^*	DMU	Z_o^*
1	0.9969	37	0.9994	73	0.9936	109	0.6232	145	0.4993	181	1.0000
2	1.0000	38	0.8792	74	0.9988	110	0.9972	146	0.9905	182	0.9868
3	0.9992	39	0.9990	75	0.9988	111	0.9990	147	0.4911	183	0.9883
4	0.9976	40	1.0000	76	0.9990	112	0.9988	148	0.9948	184	0.9877
5	0.9993	41	0.9992	77	0.4147	113	0.9914	149	0.9982	185	0.9914
6	0.9999	42	0.9994	78	1.0000	114	0.9991	150	0.9954	186	0.9900
7	0.9903	43	0.9993	79	0.9978	115	0.9963	151	0.9972	187	0.9923
8	0.9894	44	0.9662	80	0.9935	116	0.9995	152	0.4912	188	1.0000
9	0.9896	45	0.9984	81	0.5907	117	0.9979	153	0.9917	189	0.9925
10	1.0000	46	0.9993	82	0.4999	118	0.9974	154	0.9915	190	0.9931
11	1.0000	47	1.0000	83	1.0000	119	0.9990	155	0.9985	191	0.4914
12	0.9902	48	0.9980	84	0.9952	120	0.9996	156	0.9921	192	0.9902
13	0.9917	49	0.9994	85	0.9961	121	0.9926	157	0.5050	193	0.4920

14	0.9913	50	0.9996	86	0.9971	122	1.0000	158	0.9991	194	0.9945
15	0.9894	51	0.9995	87	0.4955	123	1.0000	159	0.9958	195	1.0000
16	0.8348	52	0.9983	88	0.9973	124	0.9997	160	0.9987	196	0.9915
17	0.9908	53	0.9981	89	0.9984	125	0.9977	161	0.9887	197	0.9886
18	0.9889	54	0.9795	90	0.5907	126	0.9990	162	0.9959	198	0.9960
19	0.6537	55	1.0000	91	0.9988	127	0.9988	163	0.9890	199	0.4904
20	0.4963	56	0.9992	92	0.9962	128	0.6111	164	0.9888	200	0.4946
21	0.9989	57	0.9975	93	0.9890	129	0.9953	165	0.4967	201	0.9916
22	0.5358	58	0.7637	94	0.9976	130	0.9972	166	0.9947		
23	0.5646	59	0.9991	95	0.9899	131	0.9993	167	0.4903		
24	0.9979	60	0.9835	96	1.0000	132	1.0000	168	1.0000		
25	0.9953	61	1.0000	97	0.6299	133	0.9977	169	0.9905		
26	0.9995	62	0.9992	98	0.9980	134	0.9895	170	0.9850		
27	0.5963	63	1.0000	99	0.9987	135	0.9981	171	0.9956		
28	0.8251	64	1.0000	100	0.9942	136	0.9904	172	1.0000		
29	0.4929	65	0.9996	101	0.9987	137	0.9898	173	0.5777		
30	0.9982	66	0.9972	102	1.0000	138	0.9920	174	0.5069		
31	0.9994	67	0.9991	103	0.9990	139	0.9941	175	0.9922		
32	1.0000	68	0.9990	104	0.9982	140	0.9937	176	1.0000		
33	0.9982	69	0.9990	105	0.9995	141	0.9871	177	0.9827		
34	1.0000	70	0.5908	106	0.9935	142	1.0000	178	0.9912		
35	0.9990	71	0.4928	107	0.8706	143	0.9952	179	0.9868		
36	0.4986	72	0.9947	108	0.9994	144	0.9986	180	0.9983		

Table 4. The statistical information of the cross efficiency matrix.

DMU	Median	Mode	Var	Min	Max	DMU	Median	Mode	Var	Min	Max
1	0.500	0.500	0.027	0.077	0.500	37	0.714	0.643	0.019	0.091	0.831
2	0.500	0.500	0.076	0.188	1.000	38	0.749	0.998	8.214	0.077	0.999
3	0.563	0.614	0.030	0.050	0.868	39	0.767	0.789	0.029	0.100	0.922
4	0.750	0.542	0.041	0.077	0.796	40	1.000	1.000	0.036	0.083	1.000
5	0.375	0.333	0.002	0.077	0.437	41	0.375	0.333	0.002	0.091	0.488
6	0.873	0.714	0.021	0.100	0.914	42	0.750	0.731	0.026	0.091	0.863
7	0.718	0.831	0.056	0.050	0.941	43	0.889	0.668	0.028	0.120	0.932
8	0.773	0.668	0.056	0.077	0.963	44	0.428	0.333	0.291	0.010	0.802
9	0.812	0.903	0.296	0.077	0.915	45	0.703	0.753	0.037	0.111	0.936
10	1.000	1.000	0.039	0.017	1.000	46	0.650	0.692	0.028	0.083	0.901
11	1.000	1.000	0.055	0.019	1.000	47	1.000	1.000	0.011	0.125	1.000
12	0.645	0.843	0.053	0.077	0.927	48	0.674	0.843	0.046	0.016	0.906

13	0.753	0.796	0.029	0.143	0.968	49	0.556	0.691	0.027	0.050	0.921
14	0.413	0.619	0.039	0.111	0.671	50	0.429	0.333	0.006	0.111	0.611
15	0.333	0.333	0.006	0.083	0.917	51	0.375	0.333	0.002	0.083	0.450
16	0.333	0.333	0.015	0.050	0.982	52	0.704	0.661	0.032	0.077	0.901
17	0.737	0.699	0.055	0.023	0.851	53	0.750	0.593	0.034	0.077	0.813
18	0.283	0.341	0.060	0.077	0.479	54	0.333	0.333	0.081	0.075	0.955
19	0.375	0.333	0.002	0.083	0.444	55	0.767	1.000	0.030	0.100	1.000
20	0.500	0.500	0.019	0.062	0.884	56	0.750	0.608	0.038	0.077	0.814
21	0.813	0.842	0.032	0.071	0.873	57	0.750	0.548	0.041	0.077	0.796
22	0.375	0.333	0.002	0.077	0.430	58	0.375	0.333	0.004	0.086	0.571
23	0.375	0.333	0.002	0.077	0.434	59	0.500	0.500	0.003	0.077	0.556
24	0.750	0.420	0.040	0.038	0.692	60	0.333	0.333	0.059	0.028	0.991
25	0.593	0.632	0.098	0.009	0.914	61	0.920	1.000	0.018	0.200	1.000
26	0.768	0.814	0.030	0.125	0.953	62	0.750	0.664	0.028	0.083	0.817
27	0.375	0.333	0.003	0.042	0.420	63	0.793	0.701	0.024	0.111	0.843
28	0.500	0.500	0.009	0.005	0.599	64	1.000	1.000	0.031	0.077	1.000
29	0.333	0.333	0.004	0.083	0.375	65	0.750	0.763	0.034	0.050	0.824
30	0.815	0.751	0.036	0.083	0.894	66	0.750	0.692	0.043	0.077	0.801
31	0.731	0.472	0.025	0.091	0.562	67	0.375	0.333	0.002	0.077	0.460
32	1.000	1.000	0.023	0.091	1.000	68	0.750	0.663	0.037	0.077	0.802
33	0.755	0.673	0.034	0.125	0.946	69	0.782	0.703	0.029	0.077	0.818
34	1.000	1.000	0.025	0.125	1.000	70	0.500	0.500	0.006	0.050	0.917
35	0.561	0.312	0.035	0.038	0.414	71	0.333	0.333	0.011	0.035	0.931
36	0.333	0.333	0.005	0.025	0.892	72	0.750	0.776	0.096	0.004	0.904

Table 4. Continued

DMU	Median	Mode	Var	Min	Max	DMU	Median	Mode	Var	Min	Max
73	0.714	0.673	0.095	0.071	0.815	109	0.500	0.500	0.004	0.083	0.625
74	0.781	0.693	0.035	0.077	0.909	110	0.750	0.596	0.047	0.083	0.702
75	0.429	0.333	0.006	0.083	0.905	111	0.556	0.500	0.007	0.077	0.605
76	0.750	0.674	0.028	0.111	0.818	112	0.765	0.598	0.024	0.111	0.817
77	0.299	0.299	2.277	0.077	0.841	113	0.750	0.603	0.061	0.118	0.745
78	0.750	1.000	0.027	0.091	1.000	114	0.500	0.500	0.004	0.077	0.577
79	0.750	0.673	0.041	0.064	0.805	115	0.750	0.564	0.058	0.077	0.743
80	0.750	0.668	0.084	0.048	0.763	116	0.678	0.602	0.020	0.083	0.914
81	0.375	0.333	0.002	0.083	0.460	117	0.750	0.498	0.046	0.077	0.706
82	0.375	0.333	0.004	0.083	0.835	118	0.333	0.333	0.008	0.020	0.935
83	0.375	0.333	0.011	0.146	1.000	119	0.750	0.531	0.038	0.099	0.787
84	0.750	0.659	0.098	0.009	0.718	120	0.750	0.678	0.026	0.083	0.914

85	0.333	0.333	0.006	0.037	0.659	121	0.643	0.721	0.084	0.039	0.856
86	0.750	0.681	0.044	0.077	0.796	122	0.429	0.333	0.008	0.100	1.000
87	0.333	0.333	0.013	0.029	0.789	123	1.000	1.000	0.022	0.083	1.000
88	0.750	0.693	0.051	0.019	0.800	124	0.816	0.653	0.030	0.075	0.804
89	0.793	0.591	0.032	0.111	0.818	125	0.750	0.587	0.049	0.005	0.785
90	0.375	0.333	0.003	0.075	0.646	126	0.689	0.823	0.025	0.077	0.914
91	0.790	0.668	0.022	0.167	0.808	127	0.750	0.598	0.034	0.077	0.765
92	0.750	0.703	0.057	0.083	0.769	128	0.375	0.333	0.002	0.083	0.442
93	0.750	0.598	0.109	0.106	0.819	129	0.750	0.698	0.077	0.050	0.756
94	0.750	0.714	0.042	0.046	0.806	130	0.750	0.498	0.043	0.077	0.798
95	0.750	0.776	0.075	0.036	0.779	131	0.680	0.769	0.032	0.038	0.915
96	1.000	1.000	0.018	0.333	1.000	132	1.000	1.000	0.108	0.009	1.000
97	0.375	0.333	0.002	0.077	0.434	133	0.750	0.642	0.055	0.062	0.713
98	0.349	0.333	0.007	0.027	0.975	134	0.750	1.000	0.062	0.077	2.498
99	0.750	0.698	0.033	0.083	0.815	135	0.342	0.333	0.004	0.063	0.800
100	0.750	0.652	0.099	0.001	0.703	136	0.500	0.500	0.022	0.057	0.383
101	0.506	0.500	0.010	0.046	0.846	137	0.333	0.333	0.010	0.077	0.289
102	1.000	1.000	0.014	0.143	1.000	138	0.750	0.456	0.080	0.077	0.681
103	0.700	0.541	0.022	0.125	0.793	139	0.750	0.714	0.107	0.024	0.853
104	0.750	0.568	0.036	0.077	0.814	140	0.500	0.500	0.031	0.001	0.454
105	0.784	0.589	0.023	0.111	0.715	141	0.750	0.699	0.065	0.077	0.827
106	0.750	0.601	0.100	0.019	0.703	142	0.412	0.333	0.004	0.077	1.000
107	0.549	0.697	10.408	0.048	0.999	143	0.750	0.643	0.065	0.066	0.775
108	0.600	0.500	0.007	0.077	0.625	144	0.750	0.398	0.045	0.041	0.709

Table 4. Continued

DMU	Median	Mode	Var	Min	Max	DMU	Median	Mode	Var	Min	Max
145	0.333	0.333	0.011	0.009	0.770	181	1.000	1.000	0.021	0.035	1.000
146	0.500	0.500	0.026	0.006	0.466	182	0.750	0.624	0.031	0.077	0.789
147	0.300	0.300	0.022	0.111	0.345	183	0.769	0.731	0.057	0.033	0.812
148	0.500	0.500	0.026	0.007	0.441	184	0.750	0.578	0.036	0.050	0.767
149	0.750	0.567	0.051	0.032	0.798	185	0.750	0.704	0.058	0.077	0.801
150	0.703	0.698	0.100	0.014	0.814	186	0.750	0.698	0.043	0.083	0.714
151	0.750	0.614	0.060	0.026	0.765	187	0.750	0.476	0.074	0.022	0.723
152	0.333	0.333	0.016	0.091	0.331	188	0.750	1.000	0.098	0.009	1.000
153	0.750	0.598	0.082	0.070	0.756	189	0.214	0.498	0.078	0.091	0.392
154	0.750	0.614	0.078	0.077	0.788	190	0.500	0.500	0.034	0.012	0.965
155	0.333	0.333	0.005	0.000	0.429	191	0.333	0.333	0.005	0.077	0.780
156	0.750	0.245	0.081	0.032	0.312	192	0.350	0.350	0.061	0.000	0.321

157	0.333	0.333	0.010	0.064	0.756	193	0.333	0.333	0.005	0.026	0.788
158	0.500	0.500	0.005	0.167	0.929	194	0.333	0.333	0.020	0.008	0.918
159	0.750	0.667	0.067	0.012	0.756	195	1.000	1.000	0.103	0.013	1.000
160	0.682	0.598	0.034	0.111	0.905	196	0.554	0.593	0.086	0.077	0.693
161	0.750	0.603	0.069	0.010	0.714	197	0.750	0.565	0.055	0.077	0.683
162	0.750	0.667	0.077	0.007	0.765	198	0.750	0.402	0.069	0.032	0.265
163	0.750	0.702	0.058	0.083	0.818	199	0.338	0.333	0.007	0.083	0.800
164	0.653	0.714	0.073	0.077	0.913	200	0.500	0.500	0.021	0.015	0.546
165	0.500	0.500	0.016	0.001	0.995	201	0.356	0.343	0.067	0.077	0.975
166	0.750	0.620	0.074	0.009	0.717						
167	0.333	0.333	0.015	0.008	0.265						
168	0.750	1.000	0.049	0.100	1.000						
169	0.750	0.646	0.032	0.072	0.913						
170	0.695	0.713	0.024	0.111	0.921						
171	0.333	0.333	0.013	0.029	0.973						
172	0.750	1.000	0.074	0.037	1.000						
173	0.500	0.500	0.023	0.042	0.879						
174	0.333	0.333	0.014	0.000	0.745						
175	0.589	0.446	0.073	0.077	0.677						
176	0.750	1.000	0.057	0.083	1.000						
177	0.750	0.346	0.083	0.006	0.667						
178	0.750	0.513	0.056	0.167	0.689						
179	0.428	0.333	0.021	0.000	0.864						
180	0.346	0.346	0.042	0.091	0.914						

Table 5. The cross efficiency score and the rank of units

DMU	Z_o^*	Rank	DMU	Z_o^*	Rank	DMU	Z_o^*	Rank
1	0.463	(104)	37	0.942	(8)	73	0.878	(30)
2	0.513	(95)	38	0.854	(40)	74	0.929	(16)
3	0.941	(9)	39	0.840	(46)	75	0.389	(115)
4	0.815	(62)	40	0.930	(15)	76	0.838	(47)
5	0.356	(129)	41	0.359	(126)	77	0.315	(146)
6	0.958	(3)	42	0.832	(51)	78	0.844	(45)
7	0.896	(26)	43	0.858	(38)	79	0.815	(62)
8	0.898	(25)	44	0.445	(109)	80	0.777	(82)
9	0.941	(9)	45	0.921	(19)	81	0.356	(129)
10	0.930	(15)	46	0.828	(55)	82	0.362	(124)
11	0.900	(24)	47	0.968	(2)	83	0.376	(116)
12	0.905	(22)	48	0.913	(21)	84	0.756	(87)

13	0.929	(16)	49	0.945	(7)	85	0.330	(142)
14	0.915	(20)	50	0.392	(114)	86	0.805	(68)
15	0.353	(132)	51	0.360	(125)	87	0.322	(145)
16	0.334	(141)	52	0.926	(17)	88	0.800	(70)
17	0.901	(23)	53	0.816	(61)	89	0.833	(50)
18	0.894	(27)	54	0.372	(119)	90	0.357	(128)
19	0.358	(127)	55	0.837	(48)	91	0.849	(42)
20	0.457	(107)	56	0.821	(58)	92	0.788	(76)
21	0.840	(46)	57	0.816	(61)	93	0.829	(54)
22	0.356	(129)	58	0.359	(126)	94	0.816	(61)
23	0.349	(136)	59	0.489	(99)	95	0.787	(77)
24	0.818	(59)	60	0.373	(118)	96	0.893	(28)
25	0.861	(36)	61	0.864	(34)	97	0.355	(130)
26	0.837	(48)	62	0.827	(56)	98	0.352	(133)
27	0.351	(134)	63	0.852	(41)	99	0.831	(52)
28	0.467	(103)	64	0.937	(12)	100	0.753	(89)
29	0.330	(142)	65	0.833	(50)	101	0.534	(92)
30	0.824	(57)	66	0.808	(65)	102	0.968	(2)
31	0.830	(53)	67	0.354	(131)	103	0.844	(45)
32	0.952	(6)	68	0.821	(58)	104	0.824	(57)
33	0.832	(51)	69	0.933	(14)	105	0.840	(46)
34	0.954	(4)	70	0.494	(96)	106	0.751	(90)
35	0.930	(15)	71	0.339	(139)	107	0.832	(51)
36	0.338	(140)	72	0.758	(86)	108	0.538	(91)

Table 5. Continued

DMU	Z_o^*	Rank	DMU	Z_o^*	Rank	DMU	Z_o^*	Rank
109	0.494	(96)	145	0.323	(144)	181	0.835	(49)
110	0.803	(69)	146	0.533	(93)	182	0.815	(62)
111	0.532	(94)	147	0.410	(112)	183	0.831	(52)
112	0.837	(48)	148	0.458	(106)	184	0.787	(77)
113	0.845	(44)	149	0.799	(71)	185	0.807	(66)
114	0.490	(98)	150	0.870	(32)	186	0.771	(85)
115	0.789	(75)	151	0.792	(74)	187	0.754	(88)
116	0.941	(9)	152	0.370	(120)	188	0.876	(31)
117	0.803	(69)	153	0.776	(83)	189	0.492	(97)
118	0.342	(138)	154	0.784	(78)	190	0.352	(133)
119	0.817	(60)	155	0.368	(121)	191	0.485	(102)
120	0.945	(7)	156	0.779	(81)	192	0.350	(135)

121	0.889	(29)	157	0.325	(143)	193	0.349	(136)
122	0.395	(113)	158	0.494	(96)	194	0.859	(37)
123	0.953	(5)	159	0.784	(78)	195	0.867	(33)
124	0.846	(43)	160	0.926	(17)	196	0.793	(73)
125	0.806	(67)	161	0.798	(72)	197	0.783	(79)
126	0.938	(11)	162	0.771	(85)	198	0.363	(123)
127	0.815	(62)	163	0.828	(55)	199	0.438	(110)
128	0.357	(128)	164	0.923	(18)	200	0.488	100
129	0.777	(82)	165	0.459	(105)	201	0.969	(1)
130	0.808	(65)	166	0.777	(82)			
131	0.935	(13)	167	0.365	(122)			
132	0.857	(39)	168	0.798	(72)			
133	0.792	(74)	169	0.940	(10)			
134	0.813	(63)	170	0.945	(7)			
135	0.346	(137)	171	0.360	(125)			
136	0.494	(96)	172	0.776	(83)			
137	0.353	(132)	173	0.452	(108)			
138	0.781	(80)	174	0.323	(144)			
139	0.863	(35)	175	0.878	(30)			
140	0.487	(101)	176	0.800	(70)			
141	0.835	(49)	177	0.773	(84)			
142	0.375	(117)	178	0.789	(75)			
143	0.779	(81)	179	0.417	(111)			
144	0.809	(64)	180	0.969	(1)			

Now, we determine the correlation coefficient between the obtained ranks and each input and output indices. For this purpose, each time, we ignore one of the input or output indicators in the evaluation and determine the rank of the decision making units using the other indicators. And then we determine the Spearman correlation coefficient between the different ranks obtained by the omission of each of the indicators. The results are reported in Table 6.

Table 6. The Spearman correlation coefficient

	The original rank	Ignoring x_1	Ignoring x_2	Ignoring x_3	Ignoring x_4	Ignoring x_5	Ignoring y_1	Ignoring y_2
The original rank	-	0.987	0.873	0.358	0.551	0.437	0.786	0.329
Ignoring x_1	-	-	0.881	0.369	0.623	0.676	0.800	0.518
Ignoring x_2	-	-	-	0.404	0.415	0.608	0.724	0.467

Ignoring x_3	-	-	-	-	0.528	0.325	0.709	0.359
Ignoring x_4	-	-	-	-	-	0.612	0.695	0.626
Ignoring x_5	-	-	-	-	-	-	0.574	0.448
Ignoring y_1	-	-	-	-	-	-	-	0.439
Ignoring y_2	-	-	-	-	-	-	-	-

As we see in Table 6, the lowest correlation coefficients are related to the elimination of the third input and the second output. In the other word, if we ignore the third input and the second output in the evaluation, then the obtained ranks by the extended cross efficiency evolution can be changed significantly.

5. conclusion

This study focused on the efficiency evaluation of the insurance industry. For this purpose, we used the dataset of the car insurance policies of Saman Insurance Company during the years 2018-2019 and implemented an extended cross efficiency method to rank the insured for prediction the risk of insurers in terms of existence of damage risk or absence of damage risk. Also, the correlation coefficient between the obtained ranks and each input and output indices was determined to specify to analyze the role of each indicator in the obtained rank for DMUs. This study can be used in the future policies of the insurance company. For example, the insurance companies can use the results of this paper to adjust the premiums received from different insurers and increase the satisfaction for insurers and their profitability by creating a rating system based on the insurers 'risk. A possible extension of this research would be to deal with negative data for further research.

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