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Increasing Discrimination Efficiency in Data Envelopment Analysis with Imprecise Input and Output

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Abstract
One way to increase the discrimination ability in data envelopment analysis (DEA) is to use the pessimistic view in the performance evaluation. A traditional and usual approach to move from the optimistic to the pessimistic view is to expand the production possibility set. By expanding the production possibility set, the distance between each unit can be increased from the efficiency frontier, and then a smaller number of units are located on the boundary. On the other hand, in practical applications, we are confronted with imprecise inputs and outputs. Expressions of inputs and outputs as imprecise data can give us an opportunity to use it in order to increase the efficiency discrimination. Our view of the ambiguity in the data focus on fuzzy relation. We introduce a fuzzy monotonicity assumption and construct a fuzzy production possibility set (FPPS) with varying degrees of feasibility. Using the tolerance approach a nonsymmetric fuzzy linear programming model and subsequently a parametric DEA model are constructed. By applying this model, it will be seen that, for a specific and small tolerance of constraints, The discrimination efficiency of the units increases. Finally, we propose a procedure for ranking of DMUs and employ it to rank iranian national universities.

Keywords: Imprecise data envelopment analysis, Fuzzy relation, production possibility set ranking.

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1. Introduction

Data envelopment analysis is a mathematical programming technique which is used to compute relative efficiency, rank, return to scale, benchmarks and other applications of Decision Making Units (DMUs). It was developed by Charnes et al.[4]. The conventional DEA methods require accurate measurement of both the inputs and outputs. But in real-world problems the observed data are sometimes imprecise. However, for the first time, imprecise data envelopment analysis (IDEA) was introduced by Cooper et al.[5] and various fuzzy methods were prepared for dealing with it. A literature review on fuzzy DEA models with imprecise data can be found in [17]. Recently, Hatami-Marbini et al.[8] provided a taxonomy and a review of the fuzzy DEA methods. For more details we refer readers to it. Most of the Researches often expressed this problem with bounded intervals, fuzzy numbers and Statistical data, but our view of the ambiguity in the data focuses on fuzzy relations that have paid less attention to. We want to use imprecise data to increase the efficiency discrimination.Increasing the power of efficiency discrimination is one of the fundamental issues in data envelopment analysis.

Shen et al.[11] used both the efficiency and the inefficiency frontiers to increase the discrimination capability of DEA models. When the number of inputs and outputs is high compared to the number of units, it is essentially important to increase the discrimination capability of models to split the units into efficient and inefficient ones (see [3]). Dyson et al. [7] propose that the number of units be at least $2 \times |I| \times |O|$, which attain a reliable degree of efficiency discrimination. (i) Using weight restrictions, (ii) incorporation of expert opinion and value judgements into models to obtain interactive models, (iii) providing models based on common weights set such as cross-efficiency, (iv) and using the super-efficiency method are approaches that are often used to enhance the discrimination ability of DEA models [1,2,6,14].

One way to increase the discrimination ability is to use the pessimistic view in the performance evaluation. A traditional and usual approach to move from the optimistic to the pessimistic view is to expand the production possibility set. By expanding the production possibility set, the distance between each unit can be increased from the efficiency frontier, and then a smaller number of units are located on the boundary. The most famous example of this is the comparison of performance scores under the CCR and BCC models. The corresponding production possibility sets of CCR and BCC models are represented by T_c and T_v respectively. We know that set T_v is a subset T_c and efficient units in the CCR model are efficient units of the BCC model, but the converse is not true.

In DEA, the production possibility set (PPS) is formed from observed input-output data based on some assumptions: convexity, monotonicity, inclusion of observations, constant returns to scale and minimum extrapolation. We believe that the monotonicity assumption should be replaced by a fuzzy monotonicity assumption. The basic reason for this idea is to envelop those points which are placed in the vicinity and outside the PPS, because of the impreciseness or vagueness of input-output data. We incorporate the fuzzy monotonicity assumption by fuzzy relations in PPS. Also we use the tolerance approach which was proposed by Sengupta [10] for the first time. At first, we obtain a non symmetric fuzzy LP model and then, a parametric LP model is constructed by applying the Verdegay's approach [15]. The proposed method and Sengupta-method both incorporate uncertainty into the DEA models by defining tolerance levels on constraint violations. Sengupta [10] introduced fuzziness in the objective function and the constraints of the conventional DEA model but did not provide an application roadmap of his proposed framework [12]. In this study, we propose a useful practical method to pick tolerance vectors. We apply the proposed method for a case study to rank national universities of Iran. It is shown that although almost all the DMUs were efficient by classic DEA model, they rank completely by the proposed method .

The rest of the paper is organized as follows: In section 2, we will have some basic ideas and definitions and section 3 contains the proposed model. Section 4 includes two examples and finally, the paper ends with a conclusion in section 5.

2. Definitions

A production plan is a specific combination of inputs and outputs such as $(\bar{x},\bar{y})$ where $\bar{y}$ can be produced by consuming $\bar{x}$ which may be possible or impossible. The set of all possible production plane called production possibility set (PPS), here it is denoted by T. Some commonly assumed properties of T are as follows:

Convexity: If $(x,y) \in T$ and $(x',y') \in T$, then $\lambda(x,y)+(1-\lambda)(x',y') \in T$ for any $\lambda \in [0,1]$.

Monotonicity: If $(x,y) \in T$, $x' \geq x$ and $y' \leq y$, then $(x',y') \in T$.

Inclusion of observations: Each observed DMU $(x_j,y_j) \in T$.

Constant returns to scale (CRS): If $(x,y) \in T$, then $(\lambda x,\lambda y) \in T$ for any $\lambda \geq 0$.

Minimum extrapolation: T is the intersection of all sets satisfying the above assumptions.

Suppose that there are n DMUs where each DMU consumes different amount of m inputs to produce different amount of s outputs. Let $x_j \in \mathbf{R}^m$ and $y_j \in \mathbf{R}^s$ show the input and output vectors corresponding to DMU $_j$, respectively. A virtual DMU is given by $(x(\lambda),y(\lambda))$ where:

$x(\lambda)=\sum_{j=1}^{n}\lambda_j x_j,\ Y(\lambda)=\sum_{j=1}^{n}\lambda_j y_j,$
$\lambda \in S$

S is a technology set. The PPS with the CRS assumption for $\{(x_j,y_j)\}_{j=1}^{n}$ is:

$$T=\left\{(x,y) \mid x \geq x(\lambda), y \leq y(\lambda), \lambda \in S = \{\lambda \in \mathbf{R}^n : \lambda_j \geq 0, \forall j\}\right\}$$

The CCR model for evaluating (x_o,y_o) is as follows:

$$\theta^o = \text{Min} \quad \theta$$
$$S.t \quad (\theta x_o, y_o) \in T$$

(α-cut). Let A be a fuzzy set in X and $\alpha \in [0,1]$. The α-cut of the fuzzy set A is the crisp set A_α given by $A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\}$.

(Binary fuzzy relation). A binary fuzzy relation **R**(X,Y) on X× Y is defined as **R**(X,Y)=$\{((x,y),\mu_{\mathbf{R}}(x,y)) : (x,y) \in X \times Y\}$ where $\mu_{\mathbf{R}} : X \times Y \longrightarrow [0,1]$ is a grade of membership function. If X = Y then **R**(X,X) is called a binary fuzzy relation on X.

3. Basics Idea and the Proposed Model

In general, a production plan is possible if it dominates a virtual DMU based on a preference order i.e. (X,Y) is possible whenever provided the following constraints hold for at least one λ in S.

$$X \geq X(\lambda) \text{ and } Y \leq Y(\lambda) \qquad (1)$$

So, the possibility of (x,y) depends to the observed input-output data indirectly. The observed values of the input and output data in real-world problems are sometimes imprecise. Therefor, it may not be reasonable to require that the constraints define in terms of a crisp preference order. For dealing this issue, Sengupta proposed to express each constraint i of by a fuzzy constraint.

(Fuzzy Monotonicity). If $(\mathrm{x},\mathrm{y}) \in \mathrm{T}$, $\mathrm{x}' \succeq \mathrm{x}$ and $\mathrm{y} \succeq \mathrm{y}'$, then $(\mathrm{x}',\mathrm{y}') \in \mathrm{T}$. Where $\succeq$ is a fuzzy relation and called " fuzzy grater than or equal to".

(Fuzzy greater than or equal to). Let $Y \subseteq \mathbf{R}^{+}$ be a set and $y_1, y_2 \in Y$. The fuzzy relation $\succeq$ on Y is defined with membership function $\mu_{\succeq}$ as follows where β is the maximum acceptable tolerance, as determined by the decision maker.

$$\mu_{\succeq}(z,y) = \begin{cases} 1, & z \geq y; \\ 1 + \frac{z-y}{\beta}, & y - \beta \leq z \leq y; \\ 0, & z \leq y - \beta . \end{cases} \quad (2)$$

For $\alpha \in [0,1]$, $\mu_{\succeq}(z,y) \geq \alpha$ if and only if $z \geq y - (1-\alpha)\beta$. The proof of the proposition is easy and so is omitted.

(Fuzzy Pareto order)[shahghobadi] Let $\mathrm{X} = (x_1, x_2, \ldots, x_n)$, $\mathrm{Y} = (y_1, y_2, \ldots, y_n) \in \mathbf{R}^{n+}$.

In Fuzzy parto preference, $\mathrm{X} \succeq \mathrm{Y}$ if and only if $x_i \succeq y_i$ for $i = 1,2,\ldots,n$. If $\mu_{\succeq i}$ denotes the membership function of $x_i \succeq y_i$ for $i = 1,2,\ldots,n$, then $\mu(\mathrm{X},\mathrm{Y}) = \mathrm{Min}_i\, \mu_{\succeq i}(x_i, y_i)$. Based on this definition, we proposed the *fuzzy parato* on the PPS input-output system:

$(\mathrm{x},\mathrm{y}) \succeq (\mathrm{w},\mathrm{z}) \Leftrightarrow \mathrm{w} \succeq \mathrm{x}, \mathrm{y} \succeq \mathrm{z}$

$(\mathrm{x},\mathrm{y}), (\mathrm{w},\mathrm{z}) \in \mathrm{T}$.

Considering fuzzy Monotonicity with the technology set S, the PPS for $\{(\mathrm{X}_j, \mathrm{Y}_j)\}_{j=1}^{n}$ is: $\widetilde{\mathrm{T}} = \{(\mathrm{x},\mathrm{y}) |\, \mathrm{x} \succeq \mathrm{x}(\lambda), \mathrm{y} \preceq \mathrm{y}(\lambda), \lambda \in \mathrm{S}\}$

Let I, O denote the indices sets of inputs and outputs, respectively.

(The degree of feasibility). We define the degree of feasibility for any $(\mathrm{x},\mathrm{y}) \in \mathrm{T}$ as follows:

$$\mu(\mathrm{x},\mathrm{y}) = \sup_{\lambda \in \mathrm{S}} \min\{\min_{\mathrm{I}} \mu_i(x_i, \mathrm{X}_i(\lambda)), \min_{\mathrm{O}} \mu_r(\mathrm{Y}_r(\lambda), y_r)\}$$

$(\mathrm{x},\mathrm{y}) \in \widetilde{\mathrm{P}}$ with the grade of membership $\mu(\mathrm{x},\mathrm{y})$. Obviously, $(\mathrm{x},\mathrm{y}) \in \widetilde{\mathrm{P}}$ when the minimum value, in the above expression, will be positive at least for a $\lambda \in \mathrm{S}$.

Height *[$\widetilde{P}$]*=P .

Proof. First, suppose that $(\mathrm{x},\mathrm{y}) \in \widetilde{\mathrm{P}}$ and $\mu(\mathrm{x},\mathrm{y}) = 1$. Regarding definition 6 there is a $\lambda \in \mathrm{S}$ such that the constraints $\mathrm{x} \succeq \mathrm{X}(\lambda), \mathrm{y} \preceq \mathrm{Y}(\lambda)$ are hold precisely, i.e $\mathrm{X} \geq \mathrm{X}(\lambda), \mathrm{Y} \leq \mathrm{Y}(\lambda)$. Then, $(\mathrm{X},\mathrm{Y}) \in \mathrm{P}$. Conversely, suppose that $(\mathrm{x},\mathrm{y}) \in \mathrm{P}$, so there is a $\bar{\lambda} \in \mathrm{S}$ where $\mathrm{x} \geq \mathrm{X}(\lambda), \mathrm{y} \leq \mathrm{Y}(\lambda)$. Hence $\mathrm{D}(\bar{\lambda}) = 1$, $\mu(\mathrm{X},\mathrm{Y}) = 1$ and finally $(\mathrm{x},\mathrm{y}) \in \widetilde{\mathrm{P}}$.

We define $\widetilde{\mathrm{P}}^{\alpha}$ for $\alpha \in [0,1]$ as follows:

$$\widetilde{\mathrm{P}}^{\alpha} = \{(\mathrm{X},\mathrm{Y}) |\quad \mu_i(x_i, \mathrm{X}_i(\lambda)) \geq \alpha, i \in \mathrm{I}, \quad \mu_r(\mathrm{Y}_r(\lambda), y_r) \geq \alpha, r \in \mathrm{O}, \} \quad (3)$$

For each $\alpha \in [0,1]$, $\widetilde{\mathrm{P}}^{\alpha} \subseteq \widetilde{\mathrm{P}}_{\alpha}$, where $\widetilde{\mathrm{P}}_{\alpha}$ is an α −cut of $\widetilde{P}$.

Proof. It is a direct result of the definition 6.

Let $\alpha \in [0,1]$. If S is compact, then $\widetilde{\mathrm{P}}^{\alpha} = \widetilde{\mathrm{P}}_{\alpha}$.

Proof. Assume that $(\mathrm{X},\mathrm{Y}) \in \widetilde{\mathrm{P}}_{\alpha}$. Hence, $\sup_{\lambda \in \mathrm{S}} \mathrm{D}(\lambda) \geq \alpha$. We have from mathematical analysis that $\mathrm{D}(\lambda)$ is continues. In addition, S is compact, then there is a $\lambda^* \in \mathrm{S}$ such that $\mathrm{D}(\lambda^*) \geq \alpha$. Regarding ??, $(\mathrm{X},\mathrm{Y}) \in \widetilde{\mathrm{P}}^{\alpha}$. This together Lemma complete the proof.

By selecting $\widetilde{\mathrm{P}}^{\alpha}$, $\alpha \in [0,1]$, as the PPS, the CCR model will be transformed to the following model that we call it α −CCR model:

$$\theta^{o}_{(\mathrm{p},\mathrm{q})}(\alpha) = \min\theta$$
$$S.t \quad (\theta \mathrm{x}_o, \mathrm{y}_o) \in \widetilde{\mathrm{P}}^{\alpha}$$
$$\lambda \in \mathrm{S}$$

From Proposition 1 we have the following DEA model which its optimal solution is called α − efficiency score.

$$\begin{aligned} \theta^{o}_{(\mathrm{p},\mathrm{q})}(\alpha) = \quad & \min\theta \\ & S.t \\ & \textstyle\sum_{j=1}^{n} \lambda_j \mathrm{x}_j - (1-\alpha)\mathbf{p} \leq \theta \mathrm{x}_o \\ & \textstyle\sum_{j=1}^{n} \lambda_j \mathrm{y}_j + (1-\alpha)\mathbf{q} \geq \mathrm{y}_o \\ & \lambda \geq 0 \end{aligned} \quad (4)$$

Where $\mathbf{p} = (p_1, p_2, \ldots, p_m)^T$ and $\mathbf{q} = (q_1, q_2, \ldots, q_s)^T$ are the constant vectors of tolerance for inputs and outputs constraints and $\alpha \in [0,1]$.

4. Procedure Ranking and Selecting Tolerance Vector

In this section, we introduce a procedure for complete ranking of all DMUs using proposed model. Our basic criteria for ranking of DMUs is only their efficiency scores. Since the efficiency score of a efficient unit is equal to 1 therefore, only inefficient DMUs are ranked. But in the proposed model when $\alpha < 1$ all DMUs are placed inside PPS and then scores of efficient DMUs can be distinguished from each other. For this reason the following numerical approach has been proposed:

1. Select a fixed pair appropriate vectors (p, q).
2. Compute $\theta^{o}_{(\mathrm{p,q})}(\alpha_j)$ for arbitrary values of $\alpha_j \in [0,1], j = 1,2,\ldots,k$.
3. Compute the $\rho^{o}_{(\mathrm{p,q})}$ for $o \in$ J, where $\rho^{o}_{(\mathrm{p,q})} = \frac{\sum_{j=1}^{k} \alpha_j \theta^{o}_{(\mathrm{p,q})}(\alpha_j)}{\sum_{j=1}^{k} \alpha_j}$ for $o \in$ J.
4. Rank all DMUs according to the obtained results of the Stage 3.

You can choose $\mathbf{p} \geq 0$ and $\mathbf{q} \geq 0$ according to the expert opinion, we propose the following approach. If x_i denote the ith input, assume that it is a stochastic variable which has normal distribution with the mean $\bar{m}(x_i) = \frac{\sum_{i=1}^{m} x_{ij}}{m}$ and the standard deviation $\sigma(x_i) = \sqrt{\frac{\sum_{i=1}^{m}(x_{ij}-\bar{m})}{m-1}}$ for $i \in$ I. We have from stochastic?? that $\bar{m}(x_i) - 2\sigma(x_i) \leq x_i \leq \bar{m}(x_i) + 2\sigma(x_i)$ in 95 percent of times. So, we propose to choose p_i such that in addition $\sum_{j=1}^{n} \lambda_j x_{ij} - (1-\alpha)p_i \geq \bar{m}(x_i) - 2\sigma(x_i)$ for each $\alpha \in [0,1]$, $\sum_{j=1}^{n} \lambda_j x_{ij} - (1-\alpha)p_i \geq 0$. Then, we take

$$p_i \leq \begin{cases} \max\{\min_j\{x_{ij}\} - \bar{m}(x_i) + 2\sigma(x_i), 0\}, & \bar{m}(x_i) - 2\sigma(x_i) \geq \min_j\{x_{ij}\}, \\ & \bar{m}(x_i) - 2\sigma(x_i) \leq 0 \end{cases} \quad (5)$$

for $i \in$ I. Similarity, we take

$$q_r \leq \max\{\bar{m}(y_r) + 2\sigma(y_r) - \max_j\{y_{rj}\}, 0\} \quad (6)$$

for $r \in$ O where $\bar{m}(y_r) = \frac{\sum_{r=1}^{m} y_{rj}}{s}$ and $\sigma(y_r) = \sqrt{\frac{\sum_{r=1}^{s}(y_{rj}-\bar{m}(y_r))}{s-1}}$.

5. Numerical Example

In this section, we provide two numerical examples to illustrate advantages and application of the proposed method. As mentioned earlier, the tolerance vectors p, q could be selected according to the expert opinion. However, we select them regarding to the data dispersion according to the proposed method. A simple numerical example with 8 DMUs that consume tow inputs to produce one output is given below. The data are displayed in Table 1.

Table 1: Data for 8 DMUs

DMU	Input1	Input2	Output
A	2	9	1
B	2	6	1
C	3	4	1
D	4	3	1
E	5	4	1
F	7	2	1
G	7	3	1
H	8	2	1

Using proposed approach for selecting tolerance vector we have $(p_1, p_2, q_1) = (2,2,0)$. Table 2 shows the α −CCR efficiency scores for $\alpha = 1, 0.9, 0.8, 0.7$, and 0.6 . Also the ranking results of all DMUs, and ρ^o are given in this table. In addition, in Table 2 we clearly see the difference between the components of each columns which leads to complete ranking of DMUs.

In this example, 33 national university in Iran were placed under investigation during 2011-2012. Each university with 9 inputs and 5 outputs in this evaluation are considered. Inputs and outputs are as follows:

Inputs: faculty member's I_1, accepted students I_2, grade of students I_3, physical space per capita I_4, welfare services space per capita I_5, The current budget for education I_6, scientific research I_7, dedicated revenue I_8, average rank of entrance exam I_9. Outputs: index of graduates O_1, index of papers O_2, index of books O_3, MS access pass O_4, the mean of passing grade O_5.The names of these universities are: Urmia, Isfahan, Alzahra, Boali sina, Tabriz, Tehran, Razi, Sh.Balochestan, Sh.Bahonar, Sh.Behshti, Sh.Chamran, Shiraz, Ferdosi, Gilan, Mazandaran, Yazd, Arak, Ilam, Birjand, E.Khomaini, Persion Gulf, Zabul, Zanjan, Semnan, Shahrod, Qom, Kashan, Kordestan, Lorstan, M.Ardebili, Valie asr, Hormozgan, Yasouj, respectively marked with 1 to 33. The data of these universities displayed in Table 3.

Table 2 :The obtained results for example 1

	α-efficiency						α-ranking					
DMU	α =1	0 α =.9	0 α =.8	0 α =.7	0 α =.6	ρ^o	1 α =1	0 α =.9	0 α =.8	0 α =.7	0 α =.6	ρ^o
A	1	0.9000	0.8000	0.7000	0.6000	0.8250	11	95	96	96	97	6
B	1	0.9400	0.8800	0.8200	0.7600	0.8950	11	33	33	33	33	3
C	1	0.9400	0.8857	0.8286	0.7714	0.9000	11	11	11	11	11	1
D	1	0.9400	0.8857	0.8286	0.7714	0.9000	11	11	11	11	11	1
E	.7778	0.7300	0.6889	0.6444	0.6000	0.7000	18	68	68	68	67	8
F	1	0.9400	0.8769	0.8154	0.7538	0.8923	11	14	14	14	14	4
G	.8125	0.7600	0.7125	0.6625	0.6125	0.7250	17	17	17	17	16	7
H	1	0.9000	0.8143	0.7571	0.7000	0.8529	11	25	25	25	25	5

Table 3: Input and output data of 33 national university of Iran

	I1	I2	I3	I4	I5	I6	I7	I8	I9	O1	O2	O3	O4	O5
DMU1	0.10	0.27	0.73	0.59	0.40	0.23	0.38	0.20	2.57	0.38	0.13	0.05	0.26	0.91
DMU2	0.26	0.41	0.74	0.45	0.66	0.35	0.36	0.25	4.76	0.43	0.42	0.33	0.45	0.90
DMU3	0.13	0.28	0.39	0.25	0.30	0.22	0.25	0.20	2.68	0.24	0.13	0.03	0.18	1.00
DMU4	0.12	0.30	0.54	0.68	0.37	0.22	0.44	0.18	3.82	0.26	0.12	0.13	0.26	0.91
DMU5	0.16	0.43	0.61	0.75	0.65	0.43	1.00	0.44	7.05	0.42	0.18	0.05	0.53	0.92
DMU6	1.00	1.00	0.97	0.82	0.48	1.00	0.93	1.00	18.1	1.00	1.00	1.00	1.00	0.93
DMU7	0.08	0.25	0.60	0.58	0.42	0.23	0.36	0.16	2.45	0.29	0.09	0.14	0.26	0.89
DMU8	0.07	0.43	0.98	0.22	0.62	0.40	0.38	0.21	4.24	0.24	0.04	0.05	0.22	0.92
DMU9	0.17	0.54	0.73	0.68	0.56	0.27	0.53	0.27	5.27	0.47	0.17	0.05	0.36	0.91
DMU10	0.19	0.33	0.40	0.42	0.23	0.49	0.40	0.43	5.12	0.42	0.23	0.28	0.40	0.95
DMU11	0.20	0.30	0.60	0.22	0.43	0.42	0.34	0.27	5.49	0.39	0.09	0.09	0.37	0.93
DMU12	0.20	0.40	0.54	0.89	0.63	0.59	0.47	0.37	6.70	0.44	0.69	0.05	0.49	0.93
DMU13	0.20	0.49	0.92	0.72	0.46	0.47	0.57	0.38	5.90	0.60	0.27	0.22	0.61	0.94

DMU14	0.16	0.19	0.41	0.48	0.51	0.21	0.41	0.20	3.42	0.34	0.08	0.09	0.31	0.92
DMU15	0.10	0.36	1.00	0.39	0.59	0.30	0.53	0.25	4.98	0.40	0.08	0.05	0.35	0.93
DMU16	0.13	0.23	0.41	1.00	0.49	0.19	0.17	0.16	2.98	0.31	0.07	0.06	0.29	0.93
DMU17	0.03	0.14	0.32	0.51	0.28	0.11	0.24	0.10	1.80	0.10	0.02	0.01	0.14	0.95
DMU18	0.04	0.14	0.31	0.21	0.39	0.11	0.23	0.08	1.70	0.04	0.01	0.02	0.05	0.90
DMU19	0.06	0.20	0.29	0.56	0.51	0.11	0.29	0.10	2.16	0.24	0.03	0.01	0.13	0.94
DMU20	0.08	0.18	0.36	0.44	0.39	0.11	0.14	0.09	4.34	0.12	0.05	0.02	0.12	0.91
DMU21	0.05	0.11	0.20	0.69	0.49	0.10	0.21	0.08	1.76	0.07	0.02	0.01	0.08	0.93
DMU22	0.11	0.27	0.85	0.21	0.23	0.16	0.11	0.15	2.45	0.32	0.01	0.01	0.09	0.93
DMU23	0.08	0.20	0.37	0.58	0.44	0.15	0.23	0.13	2.84	0.18	0.04	0.02	0.14	0.91
DMU24	0.04	0.16	0.28	0.36	0.41	0.14	0.21	0.09	3.69	0.12	0.03	0.05	0.09	0.90
DMU25	0.02	0.13	0.27	0.97	1.00	0.12	0.26	0.10	2.37	0.06	0.07	0.01	0.06	0.96
DMU26	0.07	0.14	0.25	0.55	0.53	0.08	0.17	0.08	1.89	0.09	0.01	0.04	0.06	0.95
DMU27	0.06	0.14	0.27	0.41	0.17	0.11	0.09	0.12	3.55	0.14	0.06	0.03	0.17	0.93
DMU28	0.08	0.17	0.28	0.62	0.51	0.11	0.20	0.09	2.54	0.33	0.04	0.04	0.13	0.88
DMU29	0.06	0.23	0.58	0.30	0.25	0.12	0.14	0.08	1.73	0.15	0.03	0.03	0.11	0.93
DMU30	0.06	0.19	0.32	0.61	0.63	0.09	0.20	0.07	1.88	0.28	0.05	0.06	0.10	0.95
DMU31	0.04	0.03	0.26	0.42	0.34	0.10	0.11	0.07	1.05	0.06	0.01	0.03	0.07	0.92
DMU32	0.28	0.08	0.22	0.47	0.66	0.08	0.10	0.05	1.00	0.08	0.01	0.07	0.04	0.95
DMU33	0.03	0.11	0.22	0.47	0.34	0.08	0.09	0.05	1.12	0.08	0.03	0.01	0.05	0.91

The results are computed by MATLAB software and displayed in Table 4. Using proposed approach for selecting tolerance vector we have $(p_1,\dots,p_9,q_1,\dots,q_5) =.$ $(0.024, 0.028, 0.198, 0.208, 0.174, 0.079, 0.085, 0.045, 1, 0, 0, 0, 0, 0)$ Due to the large number of inputs and outputs, about 89 percent of the DMUs, are efficient using by CCR model. But, all DMUs have ranked completely by the proposed procedure.

Table 4: The obtained results for example 2

	α-efficiency						α-ranking					
DMU	α =1	**0 α =.9**	0 α =.8	0 α =.7	0 α =.6	ρ^o	1 α =1	0 α =.9	0 α =.8	0 α =.7	0 α =.6	ρ^o
DMU1	1	0.985	0.970	0.955	0.940	0.974	11	99	99	99	99	9
DMU2	1	0.993	0.986	0.979	0.972	0.988	11	33	33	33	33	3
DMU3	1	0.970	0.940	0.910	0.880	0.947	11	18	18	18	18	18
DMU4	0.950	0.914	0.879	0.845	0.814	0.889	331	330	229	227	225	29
DMU5	1	0.991	0.982	0.972	0.963	0.984	11	66	66	66	66	6
DMU6	1	0.997	0.995	0.992	0.989	0.995	11	11	11	11	11	1
DMU7	1	0.983	0.967	0.950	0.933	0.971	11	111	111	111	111	11
DMU8	1	0.964	0.927	0.891	0.855	0.937	11	220	220	220	220	20
DMU9	1	0.976	0.951	0.927	0.903	0.958	11	115	115	115	115	15
DMU10	1	0.991	0.982	0.973	0.963	0.984	11	55	55	55	55	5
DMU11	1	0.984	0.968	0.951	0.935	0.972	11	110	110	110	110	10
DMU12	1	0.993	0.986	0.979	0.972	0.988	11	22	22	22	22	2
DMU13	1	0.992	0.984	0.977	0.969	0.986	11	44	44	44	44	4
DMU14	1	0.985	0.970	0.955	0.940	0.974	11	88	88	88	88	8
DMU15	1	0.985	0.970	0.955	0.941	0.974	11	77	77	77	77	7
DMU16	1	0.983	0.965	0.948	0.931	0.970	11	113	113	113	113	13
DMU17	1	0.964	0.928	0.892	0.856	0.937	11	119	119	119	119	19
DMU18	1	0.946	0.892	0.837	0.783	0.905	11	228	228	229	229	28

DMU19	1	0.961	0.922	0.883	0.844	0.932	11	221	221	221	221	21
DMU20	.906	.852	.807	.764	.722	.822	32	32	32	31	31	
DMU21		.932	.865	.797	.730	.882	1	29	30	30	30	0
DMU22		.974	.947	.921	.895	.954	1	16	16	16	16	6
DMU23	.803	.766	.730	.695	.660	.740	33	33	33	33	33	3
DMU24		.953	.906	.859	.812	.918	1	25	25	25	26	5
DMU25		.956	.912	.867	.823	.923	1	24	24	24	24	4
DMU26	.997	.908	.830	.759	.691	.856	30	31	31	32	32	1
DMU27		.971	.942	.913	.883	.949	1	17	17	17	17	7
DMU28		.983	.966	.949	.932	.970	1	12	12	12	12	2
DMU29		.957	.913	.870	.826	.924	1	13	13	13	13	3
DMU30		.978	.956	.935	.913	.962	1	14	14	14	14	4
DMU31		.947	.894	.842	.789	.908	1	27	27	28	28	7
DMU32		.960	.920	.880	.840	.930	1	22	22	22	22	2
DMU33		.949	.897	.846	.795	.910	1	26	26	26	27	6

6. Conclusion

In this study, we investigated the the classical DEA model under fuzzy *Monotonicity* assumption. A parametric DEA model was obtained and used to evaluate relative efficiency of DMUs. It was seen that the obtained efficiency scores were very variety. We con conclude that if the observed units are interior points of PPS, then their efficiency scores will be dispersed, and it is important in ranking point of view. The proposed method was applied for a case study to rank national universities of Iran. It is shown that although almost all the DMUs were efficient by classic DEA model, they are completely ranked by the proposed method. How to choose the tolerance vector could be the subject for future researches.

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