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## Efficiency Evaluating and Improvement by using the Network SBM-DEA Model with Undesirable Output: A Case Study of Iranian Airlines

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### Abstract

Measuring and improving the efficiency of organizations, as one of the most important parts of the organization's performance evaluation and a tool to control it, is a contemplative issue in order to survive and advance in the area of competition. Transportation is also a key element in the growth and development of countries that creates an unbreakable link among the various sections of the economy. Therefore, evaluating and improving the efficiency of the transportation industry, especially air transportation, as a fast and expensive method, is of particular importance in today's world. The purpose of this article was to evaluate and improve the efficiency of Iranian airlines, that for this purpose, the Data Envelopment Analysis (DEA) Method, as the most popular method of evaluating efficiency, has been selected. In recent years, network envelopment data analysis models have been developed, taking into account various parts of the organization's activity in computing efficiency. Also, undesirable outputs have been considered as the reality of the outside world in these models. In this article, the Slack-Based Measure (SBM) model as one of the newest data envelopment analysis models has been considered that simultaneously sought to increase outputs and reduce inputs, in a network form and associated with undesirable output as the basis of efficiency computations. The efficiency of 14 Iranian airlines was evaluated using the mentioned model. The four airlines of Pouya Air, Atrak, Taban and Mahan were recognized as efficient airlines, and other airlines received efficiency score lower than one and were recognized as inefficient. Also, stages` efficiency values, in addition to the overall efficiency of airlines, were presented as a more precise tool of identifying the source of inefficiency. The excess of inputs` values, the shortage of desirable outputs, and the excess of undesirable outputs were also calculated to remove the inefficiency of the airlines.

**Keywords:** Airlines; Data Envelopment Analysis (DEA); Efficiency; Slack-Based Measure (SBM); Undesirable Output.

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## **1. Introduction**

In order to administrate the organization, managers need to implement four main management duties, including planning, organizing, directing and controlling. It is impossible to control an organization without evaluating its performance. Hence, the issue of performance evaluation has always been of particular consideration to managers and organizational researchers. One of the most important indicators of evaluating the performance of organizations is efficiency that, along with other indicators, depicts the performance of the organization for its managers. Technical efficiency as one of the efficiency types indicates the ability of a unit to maximize the producing products or delivering services, with regard to the specified sources and factors of production. In other words, the organization's efficiency is measured by its empowerment to minimize the waste of resources in producing products or delivering services. One of the areas requiring attention and investigation due to its importance and unique role in the national economy is the transportation industry. In this regard, air transportation is a fast and expensive way of displacing passengers and goods, so that the International Air Transport Association (IATA) announced that in 2015, 3.6 billion people were displaced by airplanes and more than 6 trillion dollars of the goods` value, approximately equal to 35 percent of the global trade value, has been allocated to the air freight transportation sector. Therefore, the evaluation and improvement of the efficiency of air transportation industry, especially the airlines, requires special attention.

Various parametric and nonparametric methods are used to measure efficiency. In the parametric methods, efficiency is measured by the specific production function that is estimated using various statistical and econometric methods (such

as regression method). In the nonparametric methods, there is no need to estimate the production function, and there is no limitation on the number of inputs and outputs, and the relative efficiency of the units is evaluated by comparing them with each other. The method of data envelopment analysis is a generalization of nonparametric methods. Indeed, data envelopment analysis method is one of the most widely used semi-parametric methods of computing efficiency that was invented by Farrell [1] and developed by Charnes et al. [2], by the CCR model under constant return to scale, as well as by Banker et al. [3] by the BCC model under variable return to scale, and afterwards by additive [4] and SBM [5] models [6], [7], [8]. Data Envelopment Analysis is a linear programming technique with the aim of comparing and evaluating the efficiency of a number of similar decision-making units and with the same conditions. In this method, a number of decision-making units are introduced as an efficient unit, and with their help, the efficiency frontier is formed as the criterion for evaluating other units [9]. The data envelopment analysis models in respect of the type of return to scale, the type of outputs and inputs orientation, the envelopment and multiplier form, have various types.

Today's organizations are a network of various activities and sections, but the traditional data envelopment analysis models ignore various stages and sections of an organization in efficiency computations. Obviously, these models can be independently used to compute the efficiency of each stage, but they do not show the relationship between the overall system efficiency and the efficiency of the system stages. Hence, in recent years, data envelopment analysis models have been developed, taking into account various stages and sections of an organization's activity in computing efficiency, and have been upgraded from a simple type that computed only the efficiency of one stage

to its network type, and measured the efficiency of various stages of an organization's activity. Thus, the possibility to accurately identify the source of inefficiency of the organization, in order to remove inefficiency and improve system performance is provided [10], [11]. Standard models of data envelopment analysis by reducing inputs or increasing outputs, following the depiction of the under evaluation decision-making unit, are in the efficiency frontier. But when undesirable outputs are present, the standard models of data envelopment analysis cannot reduce them. The methods that make undesirable outputs participate in data envelopment analysis models and are divided into two general groups of direct and indirect methods [12]. Direct methods use the main values of undesirable output along with the modification of the principles related to the structure of the technology set. Direct methods include three general methods; direct methods, with the weak disposability axiom for undesirable outputs [13], [14], [15]; direct method, with the extended strong disposability axiom for undesirable outputs [16] and direct method with weak or extended strong disposability axiom for undesirable outputs in terms of their technical nature [17]. The difference between the weak and strong disposability axiom is that in the weak disposability, it is assumed that there is no possibility for the reduction of undesirable outputs freely, and the reduction of undesirable outputs will be accompanied by the reduction of the same amount of desired output. However, in the strong disposability, it is assumed that there is the possibility of freely reducing undesirable outputs, without reducing desirable outputs, or reducing them trivially using modern technologies. In contrary, indirect methods change the main values of undesirable output through

using a strictly descending function in standard models of data envelopment analysis [12].

In this article, airlines` delay has been considered as an undesirable output. Regarding the nature of the airlines` activities, the way of undesirable output of delay participation was considered as the direct method with the extended strong disposability axiom. Delay in flight is originated from a variety of factors, such as adverse weather, the inability of management for the modernization and development of fleet and even political and security issues. Delays may make airlines required to pay compensation to the passengers, but a remarkable point about the delay is its importance in the customer`s view. Various researches indicated that flight without delay is a symbol of quality for the customer. Hence, delay increase in flights will reduce the demand for air travels, [18] reduction in investment in the airlines industry, market share, profitability and, ultimately airlines efficiency. Obviously, delay as an undesirable output in the airlines requires the reduction by adopting appropriate policies and procedures.

So far, many articles have been presented in the area of airlines` data envelopment analysis, but a limited number of articles have used the SBM model in the airlines. These articles are; Lozano and Gutiérrez [19], Chang and Yu [20], Chang et al. [21], Lozano and Gutiérrez [22], Tavassoli et al. [23], Li et al. [24], Chou et al. [25], Cui and Li [26], Cui et al. [27], Li et al. [28], Yu et al. [29], Wang et al. [30], Xu and Cui [31], Yu et al. [32], Zhang et al. [33], Cui et al. [34] and Cui and Li [35].

The purpose of this article was based on the evaluation and ranking the Iranian airlines using a type of data envelopment analysis model. Considering that it is possible to change inputs and outputs` values simultaneously in the airlines`

companies, the SBM model, that is the newest type of data envelopment analysis models, with the capability to increase outputs and reducing inputs simultaneously, was selected as the base model. This model, along with the two characteristics of the existence of network, the existence of undesirable output as the realities of external world was used as a model suitable for computing the efficiency and empowerment of airlines in identifying the ratio and source of inefficiency and removing it.

The innovation of this paper is that for the first time delay was used as an undesirable output in the Network SBM model [36]. Among the articles that have used the SBM model on the airline, the article of Li et al. [28], Cui and Li [26] and Cui and Li [35] had a kind of Network SBM model with undesirable output, but Li et al. [28] and Cui and Li [35] have used GHG and Cui and Li [26] has used CO<sub>2</sub> as undesirable output. They were different from this article in terms of parameters type and the shape of network model. Cui and Li [26] have applied directly the principle of Kuosmanen [37] to build their Network SBM model with weak disposability. Cui and Li [35] according to Network DEA models Tone and Tsutsui [36] and Avkiran and McCrystal [38] have proposed a model of Network Environmental Slack-Based Measure. None of them used delay as undesirable output.

Using the "SBM network model with undesirable outputs", the efficiency of 14 Iranian airlines was computed and ranked. Also, the efficiency values of the stages were obtained from solving the network model, based on which the possibility of defining more effective improvement projects by industry experts for the inefficient stages and sections of the airlines was provided. Also, according to the excess of inputs' values, the shortage

of desirable outputs and the excess of undesirable outputs resulting from solving the model, the possibility to improve the efficiency of the airlines was created.

In the following, in the second section, the SBM model was described and then a SBM model with extended strong undesirable outputs disposability and a network SBM model with extended strong undesirable outputs disposability was explained. In the third section, the structure of the under study airlines' model, the shape of the network and the efficiency model of the airlines were explained. In the fourth section, the values of the model parameters were presented. In the fifth section, the efficiency values for 14 Iranian airlines was computed using a SBM model with extended strong undesirable outputs disposability in two non-networked and network states. Also, the table of variables obtained from solving the model and suggestion for improvement was presented. In the sixth section, the interpretation of the results of article model and its advantage were expressed. In the seventh section, a brief review of the whole article, the importance and conclusions obtained from the implementation of the model were presented to assess, rank and improve the efficiency of the airlines, and a suggestion was proposed for future researchers.

## 2. Materials and methods

### 2.1. SBM<sup>1</sup> model

Ton [5] presented the SBM model in 2001, which has been formed based on the excess of  $s_i$  inputs and the shortage of  $t_r$  outputs. In this model,  $\omega$  is the efficiency score,  $x_{ij}$  is the input value  $i$  of the  $j$  unit,  $y_{rj}$  is the output value  $r$  of the  $j$  unit,  $x_{i0}$  is the input value  $i$  of the unit under evaluation,  $y_{r0}$  is the output value  $r$  of the unit under evaluation,  $\lambda_j$  is the weight unit of  $j$ ,  $m$ ,  $k$  and  $n$ ; the number of inputs and

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1. Slack-based model

outputs and the number of units. The efficiency on the basis of this model has been computed as follows.

$$\text{Min } \omega = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i}{x_{i0}}}{1 + \frac{1}{k} \left( \sum_{r=1}^k \frac{t_r}{y_{r0}} \right)} \quad 1$$

$$\text{st.} \quad \sum_{j=1}^n \lambda_j x_{ij} + s_i = x_{i0} \quad i = 1, \dots, m \quad 1-1$$

$$\sum_{j=1}^n \lambda_j y_{rj} - t_r = y_{r0} \quad r = 1, \dots, k \quad 1-2$$

$$\sum_{j=1}^n \lambda_j = 1 \quad j = 1, \dots, n \quad 1-3$$

$$\lambda_j, s_i, t_r \geq 0 \quad 1-4$$

The value of the objective function is true in the range of [0, 1]. When  $\omega^* = 1$ ,  $DMU_0$  is efficient, and this is when the auxiliary variables of  $s_i$  and  $t_r$  are equal to zero. This means that no input is wasted and there is no shortage of output.

[5]

### 2.2. SBM model with extended strong undesirable outputs disposability

In 2004, Ton [39] also presented an efficiency model with the presence of undesirable outputs, with the extended strong disposability on the basis of  $T_c^{INP}$  technology. In this model,  $c_b$  is the excess undesirable outputs  $w_{bj}$  is the undesirable output value  $b$  of the  $j$  unit, and  $w_{b0}$  is the undesirable output value  $b$  of the unit under evaluation and  $q$  is the number of undesirable outputs. The efficiency on the basis of this model was as follows.

$$\text{Min } \omega = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i}{x_{i0}}}{1 + \frac{1}{k+q} \left( \sum_{r=1}^k \frac{t_r}{y_{r0}} + \sum_{b=1}^q \frac{c_b}{w_{b0}} \right)} \quad 2$$

$$\text{st.} \quad \sum_{j=1}^n \lambda_j x_{ij} + s_i = x_{i0} \quad i = 1, \dots, m \quad 2-1$$

$$\sum_{j=1}^n \lambda_j y_{rj} - t_r = y_{r0} \quad r = 1, \dots, k \quad 2-2$$

$$\sum_{j=1}^n \lambda_j w_{bj} + c_b = w_{b0} \quad b = 1, \dots, q \quad 2-3$$

$$\sum_{j=1}^n \lambda_j = 1 \quad j = 1, \dots, n \quad 2-4$$

$$\lambda_j, s_i, t_r, c_b \geq 0 \quad 2-5$$

### 2.3. Network SBM model with extended strong undesirable outputs disposability

In the following, the model was considered as a two-stage network. In this model,  $z_{dj}$  are the intermediate values in the network (the output of the first stage and the input of the second stage). The unit weight of  $j$  in the first stage is shown by  $\lambda_j$ , and the unit weight of  $j$ , in the second stage is shown by  $\mu_j$ .

$$\text{Min } \omega = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i}{x_{i0}}}{1 + \frac{1}{k+q} \left( \sum_{r=1}^k \frac{t_r}{y_{r0}} + \sum_{b=1}^q \frac{c_b}{w_{b0}} \right)} \quad 3$$

$$\text{st.} \quad \sum_{j=1}^n \lambda_j x_{ij} + s_i = x_{i0} \quad i = 1, \dots, m \quad 3-1$$

$$\sum_{j=1}^n \lambda_j y_{rj} - t_r = y_{r0} \quad r = 1, \dots, k \quad 3-2$$

$$\sum_{j=1}^n \lambda_j w_{bj} + c_b = w_{b0} \quad b = 1, \dots, q \quad 3-3$$

$$\sum_{j=1}^n (\lambda_j - \mu_j) z_{dj} = 0 \quad d = 1, \dots, p \quad 3-4$$

$$\sum_{j=1}^n \lambda_j = 1 \quad j = 1, \dots, n \quad 3-5$$

$$\sum_{j=1}^n \mu_j = 1 \quad j = 1, \dots, n \quad 3-6$$

$$\lambda_j, \mu_j, s_i, t_r, c_b \geq 0 \quad 3-7$$

The efficiency of stages;  $\omega_h$  based on the above model, has been also computed from the following equation.

$$\omega_h = \frac{1 - \frac{1}{m} \sum_{i=1}^m \frac{s_i}{x_{i0}}}{1 + \frac{1}{k+q} \left( \sum_{r=1}^k \frac{t_r}{y_{r0}} + \sum_{b=1}^q \frac{c_b}{w_{b0}} \right)}$$

[40], [41], [36]

In this way, the undesirable output and network were added to the base model of Ton.

### 3. Empirical study

Iranian airlines have been established since 1966, including 22 governments, private and military lines. In this article, due to the fact that the data of 14 Iranian airlines including Pouya Air, Atrak, Meraj, Naft, KishAir, GheshmAir, Taban, Airtour, Caspian, AtaAir, Mahan, Zagros, Aseman and IranAir were fully available, the efficiency of these airlines was

measured using the “Network SBM model with extended strong undesirable outputs disposability”.

#### 3.1. Airline model Structure

In the model used in this article, inputs include: Number of Employees (NE), Fleet Size (FS), outputs include: Revenue Passenger Kilometers (RPK) and Revenue Ton Kilometers (RTK) as desirable outputs, and the airlines delay as undesirable output and intermediaries include: Available Seat Kilometers (ASK) and Available Ton Kilometers (ATK), as have been shown in Table 1.

In Table 2, a brief definition has been presented in relation to each one of the model parameters.

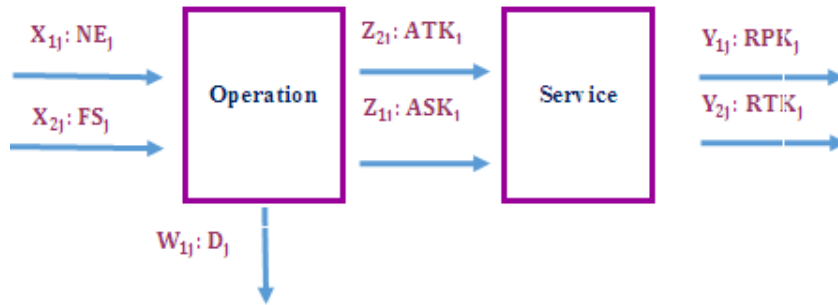
The shape of airline network has also been shown in Figure 1.

#### 1- Parameters of research airline model

Inputs	Desirable outputs	Undesirable outputs	Intermediates
X1 :Number of employees	RPK: Y1	W1 :Delay	Z1 :ASK
X2 :Fleet Size	RTK: Y2		Z2 :ATK

#### 2- Parameters description of research airline model

Parameters	Description
<b>NE</b>	<b>Number of employees:</b> The number of full time employees of the airlines
<b>FS</b>	<b>Fleet Size:</b> The number of available aircrafts including aircrafts at the ownership of airlines and leased aircrafts of the airlines.
<b>RPK</b>	<b>Revenue Passenger Kilometers:</b> The total number of carried passengers multiplied to the distance travelled in the air travel in terms of kilometer during one year.
<b>RTK</b>	<b>Revenue Ton Kilometers:</b> The total amount of carried freight multiplied to the distance travelled in the air travel in terms of ton during one year.
<b>D</b>	<b>Delay:</b> Total number of delays in airlines` flights during one year.
<b>ASK</b>	<b>Available Seat Kilometers:</b> Total aircrafts seat capacity multiplied to distance travelled in air travel in terms of kilometer during one year.
<b>ATK</b>	<b>Available Ton Kilometers:</b> Total freight capacity of aircrafts multiplied to distance travelled in air travelling in terms of ton during one year.



### 1- network airline

Accordingly, the network desired for this article consisted of two stages of operation and service.

#### Operation stage

The operation stage requires full use of resources including employees and aircrafts to increase passenger and freight capacity, that is ASK and ATK. In other words, operational efficiency would be the ability of capacity creation.

The number of employees and the number of aircrafts have been considered as inputs, and passenger and freight capacity, namely ASK and ATK, have been considered as outputs.

Also, considering that the delay is the result of the performance of operational sector, the delay of airlines was also considered as an undesirable output of the operational stage.

Efficient operation requires the creation of the highest amount of ASK and ATK, when the amount of inputs are predetermined, or minimizes the number of employees and the number of aircrafts when the passenger and freight capacity are predetermined.

#### Service stage

The service stage requires meeting the demand for passenger and freight travel from the origins to destinations in a safe, timely, convenient and comfortable manner, requiring the supply of aircrafts,

enough seat and freight capacity for carrying passengers and freight. In other words, service efficiency would be the ability to apply capacity.

Seat and freight capacity, namely ASK and ATK, have been considered as the inputs of airlines; and carried passenger and freight, namely RPK and RTK, have been considered as outputs of the airlines' service stage.

Efficient service requires minimizing seat and freight capacity, if the passenger and freight travelled is a specific amount, or maximizing the passenger and freight travelled, if the seat and freight capacity is a specific amount.

Therefore, seat and freight capacity, namely ASK and ATK, are as the linking (intermediaries) activities of the operation and service stages.

### 3.2. The efficiency model

#### 3.2.1. SBM model

According to model (2), the SBM model with extended strong undesirable outputs disposability, with the participation of inputs and outputs has been rewritten as follows.

$$\text{Min } \omega = \frac{1 - \frac{1}{2} \left( \frac{S^{NE}}{NE_0} + \frac{S^{FS}}{FS_0} \right)}{1 + \frac{1}{2+1} \left( \frac{\tau^{RPK}}{RPK_0} + \frac{\tau^{RTK}}{RTK_0} + \frac{c^D}{D_0} \right)} \quad 4$$

$$\text{st.} \quad \sum_{j=1}^{14} \lambda_j NE_j + S^{NE} = NE_0 \quad 4-1$$

$$\sum_{j=1}^{14} \lambda_j FS_j + S^{FS} = FS_0 \quad 4-2$$

$$\sum_{j=1}^{14} \lambda_j RPK_j - \tau^{RPK} = RPK_0 \quad 4-3$$

$$\sum_{j=1}^{14} \lambda_j RTK_j - \tau^{RTK} = RTK_0 \quad 4-4$$

$$\sum_{j=1}^{14} \lambda_j D_j + c^D = D_0 \quad 4-5$$

$$\sum_{j=1}^{14} \lambda_j = 1 \quad 4-6$$

$$\lambda_j, S^{NE}, S^{FS}, \tau^{RPK}, \tau^{RTK}, c^D \geq 0 \quad 4-7$$

$$\sum_{j=1}^{14} \mu_j RTK_j - \tau^{RTK} = RTK_0 \quad 5-4$$

$$\sum_{j=1}^{14} \lambda_j D_j + c^D = D_0 \quad 5-5$$

$$\sum_{j=1}^{14} (\lambda_j - \mu_j) ASK_j = 0 \quad 5-6$$

$$\sum_{j=1}^{14} (\lambda_j - \mu_j) ATK_j = 0 \quad 5-7$$

$$\sum_{j=1}^{14} \lambda_j = 1 \quad 5-8$$

$$\sum_{j=1}^{14} \mu_j = 1 \quad 5-9$$

$$\lambda_j, \mu_j, S^{NE}, S^{FS}, \tau^{RPK}, \tau^{RTK}, c^D \geq 0 \quad 5-10$$

**3.2.2. Network SBM model**

Based on model (3), the Network SBM model with extended strong undesirable outputs disposability, with the participation of inputs, intermediaries, and outputs was as follows.

$$\begin{aligned} & \text{Min } \omega \\ & = \frac{1 - \frac{1}{2} \left( \frac{S^{NE}}{NE_0} + \frac{S^{FS}}{FS_0} \right)}{1 + \frac{1}{2+1} \left( \frac{\tau^{RPK}}{RPK_0} + \frac{\tau^{RTK}}{RTK_0} + \frac{c^D}{D_0} \right)} \end{aligned} \quad 5$$

$$\text{st.} \quad \sum_{j=1}^{14} \lambda_j NE_j + S^{NE} = NE_0 \quad 5-1$$

$$\sum_{j=1}^{14} \lambda_j FS_j + S^{FS} = FS_0 \quad 5-2$$

$$\sum_{j=1}^{14} \mu_j RPK_j - \tau^{RPK} = RPK_0 \quad 5-3$$

The efficiency of the first stage;  $\omega_1$  and the efficiency of the second stage;  $\omega_2$  were also obtained from the following equations.

$$\omega_1 = \frac{1 - \frac{1}{2} \left( \frac{S^{NE}}{NE_0} + \frac{S^{FS}}{FS_0} \right)}{1 + \frac{1}{1} \left( \frac{c^D}{D_0} \right)}$$

$$\omega_2 = \frac{1}{1 + \frac{1}{2} \left( \frac{\tau^{RPK}}{RPK_0} + \frac{\tau^{RTK}}{RTK_0} \right)}$$

**4. The data**

The values of inputs, intermediaries and outputs of the model were aggregated in the Table below based on the data available on Iran Civil Aviation Organization (ICAO) website for 2016.

**3- Inputs and outputs of the mathematical research model**

DMUs		Inputs		Intermediates		Desirable outputs		Undesirable outputs
No.	Airlines	Number of employees	Fleet Size	ASK	ATK	RPK	RTK	Delay
1	Pouya Air	136	10	8803	115130	5495	42839	61
2	Atrak	201	4	235508	22151	213833	19266	296
3	Meraj	389	7	61429	86664	53466	72998	930
4	Naft	563	9	660099	65142	504873	44464	1441
5	KiskAir	792	14	2340348	243567	1916141	192803	2011
6	QeshmAir	886	22	1865737	175801	1516563	136870	2293
7	Taban	866	11	2548226	251366	2279117	193842	1973



8	Airtour	660	7	1138683	106482	1019578	90409	2061
9	Caspian	551	10	1955040	239692	1662531	172108	2403
10	ATAAir	978	15	1737736	222865	1421762	195179	3650
11	Mahan	4719	63	12623419	2E+06	8891611	913277	3118
12	Zagros	765	21	2786177	280017	1952587	176723	3290
13	Aseman	3321	35	2667835	251638	2301910	207902	5853
14	IranAir	11030	49	5264860	698252	3810524	406163	4465

**5. Results**

The results of calculating the efficiency of 14 airlines based on two models; the SBM with the extended strong undesirable output disposability and network SBM with the extended strong undesirable output disposability were calculated by Algebraic Modeling System (GAMS) software and were presented in the following Tables.

**5.1. Non-network model**

The efficiency of the 14 airlines based on the "the SBM model with extended strong

undesirable outputs disposability" was as follows: (table 4)

As it can be observed in the table 4, the five airlines of Pouya Air, Atrak, Taban, Mahan and Caspian were recognized as efficient airlines, and other airlines had an efficiency number lower than one and were ranked.

**5.2. Network model**

The efficiency score of the 14 airlines based on the "the network SBM model with extended strong undesirable outputs disposability" was as follows: (table 5)

**4- Efficiency of SBM model with undesirable outputs model**

No.	Airlines	Efficiency
1	Pouya Air	1
2	Atrak	1
7	Taban	1
9	Mahan	1
11	Caspian	1
5	KiskAir	0.91
8	Airtour	0.71
12	Zagros	0.63
10	ATAAir	0.60
6	QeshmAir	0.47
4	Naft	0.43
14	IranAir	0.30
13	Aseman	0.25
3	Meraj	0.16

**5- Efficiency of network SBM model with undesirable outputs model**

No.	Airlines	Efficiency	Efficiency of first stage	Efficiency of second stage
1	Pouya Air	1	1	1
2	Atrak	1	1	1
7	Taban	1	1	1

11	Mahan	1	1	1
9	Caspian	0.98	1	0.98
5	KiskAir	0.81	0.87	0.94
8	Airtour	0.69	0.81	0.84
12	Zagros	0.62	0.71	0.87
10	ATAAir	0.51	0.59	0.87
6	QeshmAir	0.45	0.51	0.88
4	Naft	0.39	0.95	0.41
14	IranAir	0.29	0.34	0.86
13	Aseman	0.25	0.30	0.82
3	Meraj	0.16	0.95	0.17

As it can be observed in the table 5, the four airlines of Pouya Air, Atrak, Taban and Mahan were recognized as efficient airlines, and other airlines had an efficiency number lower than one.

In the network model, with the help of the efficiency values of stages, the possibility of identifying the source of inefficiency of the units was provided more precisely. Thus, in each airline, a stage having a lower efficiency value required more attention and investigation to improve the efficiency related to that particular part of the organization, which resulted in improving the overall efficiency of the organization. Accordingly, as it can be observed in Table 5, Aseman and IranAir airlines, respectively, with the first stage

efficiency of 0.30 and 0.34 had the lowest efficiency score of the first stage (efficiency lower than 50%). Meraj and Naft airlines respectively, with the second stage efficiency of 0.17 and 0.41 had allocated the lowest efficiency score of the second stage (efficiency lower than 50%) to themselves, which required defining and implementing more effective improvement projects by experts and managers of the air industry in order to remove the existing inefficiencies.

**5.3. Efficiency comparison of non-networked and network models**

The efficiency of airlines in two non-networked and network models has been compared in the Table 6.

**6- comparison of non-networked and network models**

No.	Airlines	Efficiency of non-networked models	Efficiency of networked models
1	Pouya Air	1	1
2	Atrak	1	1
7	Taban	1	1
11	Mahan	1	1
9	Caspian	1	0.98
5	KiskAir	0.91	0.81
8	Airtour	0.71	0.69
12	Zagros	0.63	0.62
10	ATAAir	0.60	0.51
6	QeshmAir	0.47	0.45
4	Naft	0.43	0.39
14	IranAir	0.30	0.29
13	Aseman	0.25	0.25
3	Meraj	0.16	0.16

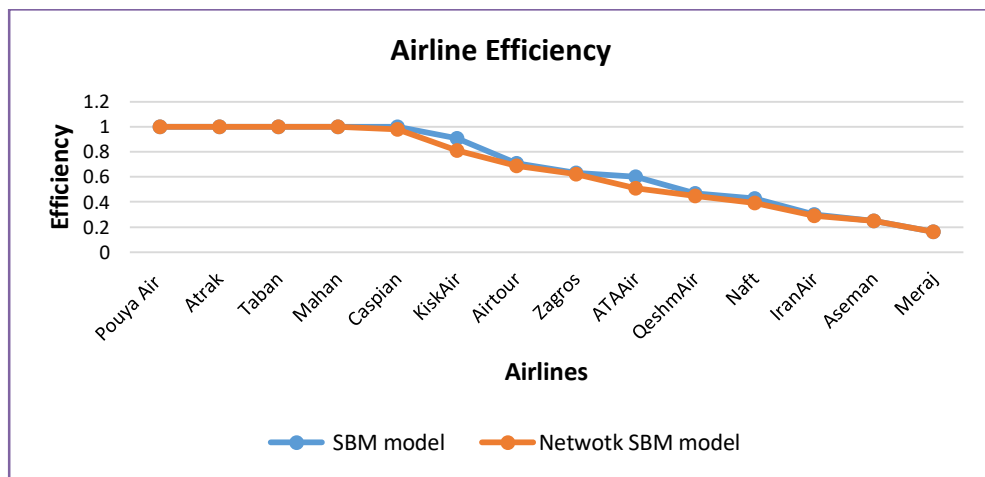
The difference in efficiency values in two non-networked and network models has been shown in Figure 2.

According to the above Table and Figure, the number of efficient units in a non-networked model was one, more than a network model. Also, in general, the efficiency values of airlines in a non-networked model were higher than the efficiency values in a network model (except for two airlines of Aseman and Meraj that their efficiency in a non-

networked and network model is equal to each other). The above points indicated the greater accuracy of the network model than the non-networked model.

**5.4. Values of decision variables in results of the network SBM model with extended strong undesirable outputs disposability**

The values of variables obtained from solving the model in the GAMS software have been summarized in the Table 7.



2- comparison of non-networked and network models

**7- The result of a network SBM model with extended strong undesirable output disposability**

Variables	DMUs													
	01	02	03	04	05	06	07	08	09	10	11	12	13	14
$\omega$	1.00	1.00	0.16	0.39	0.81	0.45	1.00	0.69	0.98	0.51	1.00	0.62	0.25	0.29
$\omega_1$	1.00	1.00	0.95	0.95	0.87	0.51	1.00	0.81	1.00	0.59	1.00	0.71	0.30	0.34
$\omega_2$	1.00	1.00	0.17	0.41	0.94	0.88	1.00	0.84	0.98	0.87	1.00	0.87	0.82	0.86
$S^{NE}$	0.00	0.00	0.00	46.17	6.82	365.56	0.00	246.91	0.00	441.48	0.00	57.40	2414.72	9159.71
$S^{FS}$	0.00	0.00	0.72	0.16	3.62	12.52	0.00	0.00	0.00	5.52	0.00	10.50	23.46	24.45
$\tau^{RPK}$	0.00	0.00	801536.55	708822.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\tau^{RTK}$	0.00	0.00	11522.87	132135.87	0.00	50875.75	0.00	14165.03	13129.63	0.00	0.00	19611.76	0.00	0.00
$c^D$	0.00	0.00	0.00	0.00	414.55	73.95	0.00	824.38	0.00	1697.43	0.00	1100.77	3868.03	2193.55
$\lambda_1$	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mu_1$	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\lambda_2$	0.00	1.00	0.66	0.47	0.24	0.09	0.00	0.52	0.00	0.22	0.00	0.00	0.00	0.00
$\mu_2$	0.00	1.00	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\mu_3$	0.00	0.00	0.13	0.15	0.01	0.06	0.00	0.23	0.15	0.00	0.00	0.00	0.00	0.00

$\lambda_7$	0.00	0.00	0.21	0.00	0.74	0.00	1.00	0.14	0.00	0.00	0.00	0.50	0.99	0.74
$\mu_7$	0.00	0.00	0.00	0.00	0.60	0.20	1.00	0.18	0.41	0.00	0.00	0.60	0.51	0.00
$\mu_8$	0.00	0.00	0.82	0.00	0.00	0.00	0.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00
$\lambda_8$	0.00	0.00	0.13	0.50	0.00	0.91	0.00	0.33	1.00	0.77	0.00	0.50	0.00	0.00
$\mu_{10}$	0.00	0.00	0.00	0.85	0.39	0.74	0.00	0.00	0.43	1.00	0.00	0.40	0.07	0.30
$\lambda_{11}$	0.00	0.00	0.00	0.03	0.02	0.00	0.00	0.00	0.00	0.01	1.00	0.00	0.01	0.26
$\mu_{11}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	1.00	0.00	0.01	0.19
$\mu_{13}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.41	0.17
$\mu_{14}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.34

Based on the results obtained from solving the model, the excess inputs and the shortage of desirable outputs and the excess undesirable outputs related to each airline have been identified, so that  $S^{NE}$  is the excess number of employees,  $S^{FS}$  is the excess number of airplanes,  $\tau^{RPK}$  is the shortage of carried passengers (RPK shortage),  $\tau^{RTK}$  is the shortage of carried freight (RTK shortage) and  $c^D$  is the excess number of airlines delay (as excess undesirable output). Obviously, removing the inefficiency of inefficient airlines would be possible by removing excess inputs (number of employees and airplanes), shortage of outputs (RPK and RTK amount which is actually the amount of passengers and freight carried) and the excess of undesirable output of delay whose values have been presented in this article. But how to do this required defining improvement projects by airlines industry experts and managers.

## 6. Discussion

Based on the results obtained from solving the models in the GAMS software, Pouya Air, Atrak, Taban, Caspian and Mahan (units 1, 2, 7, 9 and 11), obtained one efficiency score in the non-networked model. Also, Pouya Air, Atrak, Taban and Mahan airlines (units 1, 2, 7 and 11) obtained one efficiency score in the network model and were recognized as efficient airlines among other airlines. But other airlines in both models obtained the efficiency score less than 1 and were identified and ranked as inefficient units.

The excess inputs values, the shortage of desirable outputs and the excess undesirable outputs were also presented, which was necessary to make them zero in order to convert inefficient units into efficient ones.

In this article, based on the efficiency of stages resulting from solving the mathematical model of the research, it was possible to identify the source of inefficiency more accurately. Thus, by investigating the efficiency values of the stages, it was possible to address the stage of the organization's activity more, which showed less efficiency and probably was the main source of airline inefficiency. This point became possible by implementing more effective improvement projects in the more inefficient part of the organization, and this was the strength of network data envelopment analysis models. It is clear that such a possibility did not exist in the models that lacked a network state, and only one overall efficiency number associated with each airline was obtained that indicated the overall performance of the organization, but the performance of its various parts would not be separately investigated, and this was the large weakness of non-networked data envelopment analysis models.

## 7. Conclusions, outlooks and directions for future studies

The evaluation and improvement of efficiency as a part of the organization's control process are of particular

importance. Transportation, especially air transportation, is also an interesting topic in the growth and development of countries as a flow of linking various sections of economy. Therefore, evaluating and improving the efficiency of the country's airlines is an important issue. One of the most common methods of evaluating relative efficiency is data envelopment analysis, in which it is possible to use several inputs and outputs and there is no need to estimate the production function using various statistical methods. Hence, the purpose of this article was considered to evaluate and rank the efficiency of airlines with a kind of data envelopment analysis model in order to empower airlines to improve their efficiency. For this purpose, considering the nature of the airlines, in which there is a possibility to change the inputs and outputs` values simultaneously, a kind of SBM model as one of the newest data envelopment analysis models that seeks to increase outputs and reduce inputs simultaneously, was selected as the base model. Considering the importance of calculating the efficiency of various stages and sections of the organization, along with the overall efficiency of the organization, in order to identify the source of inefficiency more accurately, the model of this article was considered as a network. On the other hand, due to the importance of considering undesirable outputs as a reality, the "airlines` delay" being undesirable output, entered into the model with the participation method of direct kind with the extended strong disposability axiom. In fact, in this article, the SBM model, along with two features of being network and the existence of undesirable output, was used as the "SBM model with extended strong undesirable outputs disposability" to measure efficiency.

Using the mentioned model, the efficiency of 14 Iranian airlines was calculated and ranked by GAMS software. Based on the results of solving the model, the four airlines of Pouya Air, Atrak, Taban and Mahan were recognized as efficient. Also, by solving the non-networked model, the five airlines of Pouya Air, Atrak, Taban, Caspian and Mahan were recognized and ranked as efficient. The results of the two models were compared. Also, the efficiency of the stages in the network model was presented in order to identify the source of inefficiency of the units more accurately and to remove them. Excess inputs` values and the shortage of desirable outputs and the excess of undesirable outputs of the model were also presented to improve the efficiency of Iranian airlines.

In the following, according to the nature of the industry studied, adding types of uncertainties (stochastic, fuzzy, and robust) to the "SBM model with extended strong disposability for undesirable outputs" can be suggested to future researchers, so that the model of this article could be accordingly developed. Articles 42 to 46 use the concept of robust uncertainty and articles 47 to 49 use the concept of fuzzy uncertainty in their DEA models, which can be helpful in the creation and development of new ideas [42], [43], [44], [45], [46], [47], [48], [49].

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