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Assessing the Undesirable Cost Imposed on the System with Fuzzy DEA

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Abstract
In some companies, undesirable costs are imposed for provision of services. It is required that current outputs of decision making units are produced at minimal undesirable cost. In this paper, a new method is proposed for evaluating the undesirable cost imposed on the units. Moreover, data is considered in fuzzy number form, due to the observed input-output data are sometimes imprecise in real applications. Applying the α-level based approach, the proposed mode is solved to measure minimum undesirable cost. In this method, data information is supposed as triangular fuzzy numbers. Finally, the proposed method is illustrated by a real application.

1. Introduction
Data Envelopment Analysis (DEA) is one of the non-parametric methods that have been recently used to evaluate the efficiency and the performance of the set of decision making units (DMUs). In real application, DMUs could be evaluated in terms of costs. The ability of a DMU to minimize its costs in order to produce outputs is considered as cost efficiency. The concept of cost efficiency was first introduced by Farrell [1], and then developed by Fare, Grosskopf, and Lovell [2] using linear programming model. Fare

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et al. considered the cost efficiency of a DMU minimum production cost to actual observed cost ratio. Tone [3] improved the cost efficiency model introduced by Fare et al. by evaluating DMUs in cost-based production possibility set.

In some real applications, undesirable costs are existed in numerous factors such as companies for provision of services. It is required that current outputs of units are produced at minimal undesirable cost. In practice, however the observed values of the input and output data are often imprecise and fuzzy set theory is a useful strategy to take into account the uncertainty of the data.

In recent years, some authors have studied cost efficiency models with imprecise data in the fuzzy environment [4-7]. In this paper, a new method is proposed for assessing the undesirable cost imposed on the DMUs in fuzzy environment. Based on the α-level based approach, a new fuzzy DEA model is proposed to measure minimum undesirable cost. Moreover, data is considered in triangular fuzzy number form. Besides, new definitions of efficiency and efficient unit are suggested based on the undesirable costs. At the end, this method is illustrated with a real application.

2. Methodology

Assume that the performance of n observations on the DMUs is evaluated that each DMU use m fuzzy desirable inputs to produce s fuzzy desirable outputs. Moreover, it is supposed that numbers of T undesirable outputs are existed in the production process. Now, let $\tilde{x}_j^d = (\tilde{x}_{1j}^d, \tilde{x}_{2j}^d, ..., \tilde{x}_{mj}^d)$ be the fuzzy desirable input vector used and $\tilde{y}_j^d = (\tilde{y}_{1j}^d, \tilde{y}_{2j}^d, ..., \tilde{y}_{sj}^d)$ be the fuzzy desirable outputs vector produced by DMU_j. In addition, let $\tilde{y}_j^u = (\tilde{y}_{1j}^u, \tilde{y}_{2j}^u, ..., \tilde{y}_{Tj}^u)$ be the fuzzy

undesirable outputs vector, and $\tilde{p}^j = (\tilde{p}_{1j}, \tilde{p}_{2j}, ..., \tilde{p}_{Tj})$ be the vector of fuzzy undesirable outputs costs imposed by this DMU. Under the assumption of constant returns to scale, the minimum undesirable cost imposed on the DMU_k can be measured by the optimal objective function value of the following fuzzy model.

$$UC_k^* = \min \sum_{t=1}^{T} \tilde{p}_{tk} y_t$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij}^d \leq \tilde{x}_{ik}^d \quad ; \forall i$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj}^d \geq \tilde{y}_{rk}^d \quad ; \forall r \qquad (1)$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{tj}^u \leq y_t \quad ; \forall t$$

$$\lambda_j \geq 0 \qquad ; \forall j$$

$$y_t \geq 0 \qquad ; \forall t$$

Where, notation '~' shows that the data are fuzzy numbers. Suppose that the used fuzzy data in model (1) are expressed in triangular fuzzy numbers form. Model (1) is a fuzzy DEA problem and there are several methods to solve this model. It can be solved by applying the α-level based approach. In this paper, the presented method by Saati et al [8], is used to solve the proposed fuzzy DEA problem with consideration of the concept of α-cut of fuzzy numbers.

By considering α-cuts of constraints, and objective function, new variables are introduced in the intervals of α-cuts of triangular fuzzy numbers, as follows:

$$x_{ij}'^d \in [\alpha x_{ij}^M + (1-\alpha)x_{ij}^L, \alpha x_{ij}^M + (1-\alpha)x_{ij}^U]; \forall i, j$$

$$y_{rj}'^d \in [\alpha y_{rj}^M + (1-\alpha)y_{rj}^L, \alpha y_{rj}^M + (1-\alpha)y_{rj}^U]; \forall r, j$$

$$y_{tj}'^u \in [\alpha y_{tj}^M + (1-\alpha)y_{tj}^L, \alpha y_{tj}^M + (1-\alpha)y_{tj}^U]; \forall r, j \qquad (2)$$

$$p_{tk}' \in [\alpha p_{tk}^M + (1-\alpha)p_{tk}^L, \alpha p_{tk}^M + (1-\alpha)p_{tk}^U]; \forall t$$

By replacing the new variables, model (1) is changed to the nonlinear programming problem. In order to linearize this model, following new variables are considered.

$$\bar{x}_{ij}^d = \lambda_j x_{ij}'^d \; ; \; \forall i, j$$

$$\bar{y}_{rj}^d = \lambda_j y_{rj}'^d \; ; \; \forall r, j$$

$$\bar{y}_{tj}^u = \lambda_j y_{tj}'^u \; ; \; \forall t, j \tag{3}$$

$$\bar{P}_{tk} = p_{tk}' y_t \; ; \; \forall t$$

The transformed problem now becomes as follows:

$$UC_k^* = \min \sum_{t=1}^{T} \bar{p}_{tk}$$

$$s.t \quad \sum_{j=1}^{n} \bar{x}_{ij}^d \le x_{ik}'^d \quad ; \forall i$$

$$\sum_{j=1}^{n} \bar{y}_{rj}^d \ge y_{rk}'^d \quad ; \forall r \tag{4}$$

$$\sum_{j=1}^{n} \bar{y}_{tj}^u \le y_t \quad ; \forall t$$

$$\lambda_j(\alpha x_{ij}^M + (1-\alpha)x_{ij}^L) \le \bar{x}_{ij}^d \le \lambda_j(\alpha x_{ij}^M + (1-\alpha)x_{ij}^U)$$
$$; \forall i, j$$

$$\lambda_j(\alpha y_{rj}^M + (1-\alpha)y_{rj}^L) \le \bar{y}_{rj}^d \le \lambda_j(\alpha y_{rj}^M + (1-\alpha)y_{rj}^U)$$
$$; \forall r, j$$

$$\lambda_j(\alpha y_{tj}^M + (1-\alpha)y_{tj}^L) \le \bar{y}_{tj}^u \le \lambda_j(\alpha y_{tj}^M + (1-\alpha)y_{tj}^U)$$
$$; \forall t, j$$

$$\alpha x_{ik}^M + (1-\alpha)x_{ik}^L \le x_{ik}'^d \le \alpha x_{ik}^M + (1-\alpha)x_{ik}^U$$
$$; \forall i$$

$$\alpha y_{rk}^M + (1-\alpha)y_{rk}^L \le y_{rk}'^d \le \alpha y_{rk}^M + (1-\alpha)y_{rk}^U$$
$$; \forall r$$

$$y_t(\alpha P_{tk}^M + (1-\alpha)P_{tk}^L) \le \bar{P}_{tk} \le y_t(\alpha P_{tk}^M + (1-\alpha)P_{tk}^U)$$
$$; \forall t$$

$$\lambda_j \ge 0 \qquad ; \forall j$$

$$y_t \ge 0 \qquad ; \forall t$$

Where, α is a parameter belonging to the interval [0,1]. Model (2) is a parametric DEA model which can be solved for

measuring the minimum undesirable cost imposed on the DMU under consideration.

Definition 2.1- The efficiency of DMU_k, is defined as the ratio of the minimum undesirable cost imposed on the DMU_k to the observed undesirable cost of DMU_k.

Definition 2.2- DMU_k is defined as the efficient unit when the following condition is satisfied:

$$\sum_{t=1}^{T} p_{tk}^L y_{tk}^L \le \sum_{t=1}^{T} \bar{P}_{tk}^* \le \sum_{i=1}^{m} p_{tk}^U y_{tk}^U \tag{5}$$

Where, $(\sum_{t=1}^{T} p_{tk}^L y_{tk}^L, \sum_{t=1}^{T} p_{tk}^M y_{tk}^M, \sum_{t=1}^{T} p_{tk}^U y_{tk}^U)$ is the observed undesirable cost, and $\sum_{t=1}^{T} \bar{P}_{tk}^*$ is the minimum undesirable cost obtained by parametric linear model (4).

3. Application
In this section, we apply the proposed method for evaluating the undesirable cost imposed on 20 roads of East Azerbaijan Province in Iran. These roads are evaluated on the basis of road safety index.
In this study, input-output factors are considered as follows:

Desirable inputs:
i. Road Infrastructure Indicator (RI): this indicator is considered for numerous factors, including the proportion of traffic flow volume with the capacity of the road, safety status the height of the bridges and underpasses located on the roads, qualitative status of the shoulder of the road, intersection and exchange of cross-roads with the desired road, and geometric design influence of the desired road (direction, longitudinal and transverse slopes, horizontal radius of the horizontal and vertical arch, visibility distances)

ii. Safety Equipment (SE): status of road marking in accordance with the rules, compliance status of regulations with road signs (warning signs, law enforcement and intelligence), and long-distance safety shields (Gard rill)

iii. Intelligent Systems (IS): the quantitative and qualitative situation of video surveillance cameras and record offenses along the road, road traffic notices(traffic, weather conditions and road obstruction) by relevant sources, and the use extent of intelligent road transport systems

Desirable output:

i. Driving Indicator (DI): driving directions of drivers (Observance of Speed, Priority,

Seat Belt and Other Regulations), and compliance with the rules of carriage (tonnage and loading) on the road

Undesirable output:

i. Fatality Indicator (FI): number of road deaths due to accidents

ii. Injured of Road Accidents (IRA): number of injured person of road accidents

We apply the proposed method for evaluating 20 DMUs, which each DMU uses three inputs to produce three outputs with 3 inputs, and 1 output are desirable, and 2 outputs are undesirable. The data set for this application are shown in Table 1.

Table 1. data set

DMU	Desirable inputs			Desirable outputs	Undesirable outputs	
	RI	SE	IS	DI	IRI	FI
1	(4.929,1.297)	(5.026,0.742)	(4.445,0.813)	(4.394,0.142)	(80,7)	(6,1)
2	(4.354,1.898)	(4.626,0.826)	(4.076,1.134)	(4.326,0.242)	(70,11)	(5,2)
3	(4.511,1.659)	(5.128,1.046)	(4.463,1.205)	(5.197,0.119)	(38,18)	(2,1)
4	(4.791,1.383)	(5.103,0.535)	(4.332,1.074)	(5.197,0.119)	(17,4)	(2,1)
5	(4.266,1.128)	(3.531,0.531)	(3.355,0.959)	(3.786,0.076)	(132,60)	(8,2)
6	(4.838,0.868)	(4.605,0.727)	(3.742,0.8)	(4.16,0.298)	(41,2)	(2,1)
7	(4.605,1.163)	(4.674,0.326)	(3.457,0.515)	(2.129,0.497)	(37,12)	(5,1)
8	(4.427,1.169)	(3.757,0.417)	(2.992,0.756)	(2.882,0.076)	(63,3)	(2,1)
9	(4.779,0.99)	(4.812,0.13)	(3.607,0.665)	(4.589,0.337)	(37,6)	(7,6)
10	(4.072,0.929)	(3.401,0.381)	(2.914,0.678)	(3.934,0.54)	(186,8)	(10,1)
11	(4.672,0.858)	(4.27,0.622)	(3.786,1.16)	(4.521,0.795)	(33,6)	(1,1)
12	(4.861,0.455)	(4.686,0.196)	(3.615,0.673)	(4.355,0.103)	(68,14)	(2,0)
13	(4.942,1.548)	(5.206,0.342)	(3.856,1.366)	(4.784,0.532)	(19,6)	(2,1)
14	(4.547,1.153)	(4.811,0.321)	(3.414,0.472)	(3.934,0.524)	(47,26)	(2,1)

15	(5.156,0.628)	(4.705,0.215)	(3.867,0.925)	(3.708,0.298)	(66,11)	(3,2)
16	(5.099,0.685)	(4.805,0.257)	(3.863,0.921)	(3.63,0.766)	(38,21)	(1,1)
17	(4.616,0.7)	(4.449,0.235)	(3.575,0.633)	(2.632,0.626)	(37,11)	(4,1)
18	(4.688,0.629)	(4.725,0.551)	(3.506,0.658)	(3.866,0.14)	(87,2)	(4,1)
19	(4.409,0.986)	(4.811,0.321)	(3.354,0.728)	(3.866,0.14)	(44,15)	(4,2)
20	(5.206,0.968)	(4.66,0.486)	(5.012,2.012)	(4.16,0.298)	(73,12)	(3,1)

The used fuzzy data in Table 1 are in triangular fuzzy numbers form and as $\tilde{A} = (m, \alpha)$, where m is the mean value of $\tilde{A}$, and α is the left and right spreads.

In this study, the models were solved with GAMS software with CPLEX solver. Furthermore, the value of minimum undesirable cost imposed on the DMUs was obtained by the optimal objective function value of the model (4). This model is a parametric linear programming problems, which was solved for the given different values of α, and the results are shown in Table 2.

Furthermore, column 2 in Table 2 shows that the fuzzy observed undesirable cost of DMUs has a triangular fuzzy numbers form. The value of minimum undesirable cost strictly increases when α in interval [0, 1] is increased, for each DMU, as shown in Table 2.

Table 2. Results of model (4)

DMU	observed undesirable cost	minimum undesirable cost			
		α=0.25	α=0.5	α=0.75	α=1
1	(15200,28730,44528)	2336.718	3828.824	5530.158	6813.786
2	(9880,24465,42944)	2258.782	3724.313	5412.158	6708.338
3	(3310,11356,24024)	2783.76	4550.215	6556.318	11356
4	(2960,8059,14784)	2783.76	4550.215	6556.318	8059
5	(17460,42284,81488)	2032.331	3319.554	4779.65	13557.43
6	(4260,11827,20592)	2145.42	3552.489	5183.77	6450.922
7	(10490,19284,31416)	957.1687	1665.534	2543.67	3301.445
8	(5310,15281,26664)	1539.645	2518.893	3632.636	4469.124
9	(3860,24674,51392)	2363.287	3915.178	5715.723	24674
10	(29690,56152,85096)	1923.33	3245.156	4840.009	56152
11	(1350,7876,16456)	2139.017	3652.129	5484.164	7876
12	(7320,16066,27808)	2331.404	3811.553	5493.045	6865.073
13	(2960,8373,15840)	2389.856	4001.532	5901.289	8373
14	(3360,12769,28512)	1929.87	3252.242	4825.328	6100.463
15	(5060,18447,35728)	1899.077	3152.159	4610.263	5750.004
16	(850,8661,21736)	1665.269	2875.825	4362.843	5629.049

17	(8230,16589,28072)	1178.578	2053.907	3140.967	4081.449
18	(11180,24439,38896)	2049.772	3362.067	4860.855	5995.015
19	(6070,17688,34056)	2049.772	3362.067	4860.855	5995.015
20	(7670,19546,34760)	2145.42	3552.489	5183.77	6450.922

As shown in Table 2, the minimum cost of DMU_{11}, DMU_{16} satisfy in relation (5) for all the values of α which shows that these two DMUs is always efficient unit based on the Definition 2. Besides, $DMU_{3}, DMU_{4}, DMU_{9}, DMU_{13}$ are efficient unit for values of parameters α=0.5, α=0.75, α=0.1. as well as DMU_{6}, DMU_{14} are efficient unit for values of parameters, α=0.75, α=0.1.

4. Conclusion

This paper, suggests a new method to evaluate the undesirable cost imposed on the DMUs. This method is applied in fuzzy environment, as well as data information is supposed in triangular fuzzy numbers form. In this regard, the proposed mode is solved to measure minimum undesirable cost, based on the α-level based approach. Moreover, a new definition is proposed for efficient units, in terms of undesirable costs. Finally, an example is presented to show the applicability and efficiency of the proposed method.

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