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# Fuzzy Network DEA Model for Urban Wastewater Treatment Plants

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#### Abstract

Domestic wastewater treatment covers the processes that used to treat waters that have been polluted by commercial or anthropogenic activities prior to its discharge into the environment or its re-use. Data Envelopment Analysis was applied to evaluate six urban wastewater treatment plants efficiency in three stages with fuzzy data and discretionary and nondiscretionary inputs in Iran. Stream flow, Chemical Oxygen Demand and Total Suspended Solids were considered as input variables. In this paper a fuzzy network Data Envelopment Analysis model is proposed and solved with GAMS software. Results showed that showed that Bandar Torkman has the best performance between these waste water treatment plants and Babol and yasreb waste water treatment plants do not operate well.

**Keywords:** Data Envelopment Analysis; urban wastewater treatment; fuzzy data; efficiency in three stage system; discretion and non-discretion inputs

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#### **1. Introduction**

In this past decade, there has been considerable improvement for the association and process of urban waste water treatment plants (UWWTP) in Iran's capital cities. Although there has been universal agreement that the UWWTP's should be made more capable, obvious and both commercially and customer focused, there is no comprehensive Consensus on how this capacity be achieved.

Wastewaters include so many contaminants such as biological oxygen demand (BOD), chemical oxygen demand (COD), total suspended sediment (TSS) microbial pathogens and other chemical physical compounds. Several and biological technologies have been developed to create treated water that meets governmental supplies and causes the lowest impact when discharged into the environment. Examples include: activated sludge; aerated lagoon; trickling filter; and rotating biological contactor [1,2]. Biological treatment is a natural process; organic matter in the wastewater will naturally decay in presence of microorganism as receiving bodies in the wastewater. High organic loads in the wastewater will upset the biosynthesis of bodies receiving and cause other undesirable effects. Biological treatment is engineered to accelerate natural decay processes and neutralize the waste before discharge to receiving waters in the safe environment [3,4].

In an aerobic process, air or oxygen is supplied to microorganism which has contact with the wastewater. The microorganisms utilize organic matters and stabilize the waste water [5].

Major sources of wastewater are from domestic and industrial use. Minimum water requirement is expected to be 50 liters per capita per day [6]. The average value of domestic water consumption per capita is 100 liters [7]. Reusing and recycling and of treated wastewater is a new idea. Public uncertainty on the safety of recycling treated wastewater can only be justified if the treatment ability of the wastewater systems were optimized.

The present investigation is focused on the performance of UWWTP's. Based on the process efficiencies were obtained using the three-stage systems with fuzzy data. Data Envelopment Analysis (DEA) is a famous non-parametric technique that has been used to assess the efficiency of Decision Making Units (DMUs) in many different sectors [8]. network DEA not only use inputs and produce output but also produce and consume intermediate products [9-12]. These new models allow the researcher to learn the inside of the common black box technology [13]. They accomplish this by providing a very general framework for specifying the inner workings of the black box [9,14]. Usually black box system is depicted as Figure1.



Figure1. Usual DEA considers the system as a black box

There are a number of circumstances in which a system consists of multiple networked processes for examples banking industry, energy management organization or waste water treatment plant (WWTP) [15,16]. In these conditions difficulty turns into more complexes when the data for inputs and outputs used are fuzzy. These models are able to incorporate the operations and interdependence of the component processes of a system in measuring efficiencies. Many WWTP are in operation but a great number of them are not working with high performance [17-19]. Considerable variations in the composition of the wastewater arising have created difficulties in ensuring the efficiency and effectiveness of the WWTP [20].

The present investigation was conducted in order to evaluate the efficiency of the 6 UWWTP in north of Iran. For this purpose, fuzzy network DEA is used for modeling process. The plant with poor operation could be promoted for the removal of the pollutants of real world cases.

The reviewed literature demonstrates that network DEA is an effective method for compute the efficiencies of systems. Therefore, this approach may provide a solution to the problem of comparing different WWTPs operation technologies. In this article, a model is developed to compute the efficiency of a three-stage system in different UWWTPs when the data is fuzzy. The property of the system efficiency in each UWWTPs is being equal to the product of the three process efficiencies.

## 2. The structure of data envelopment analysis

#### 2.1 The basic DEA model

Efficiency estimating and ranking of companies in various industrial sectors is one of the most significant tools for managers and decision makers. One of the most important models which often apply by researchers for efficiency estimating and ranking of units is DEA model. DEA is a non-parametric method for measuring and evaluating the efficiency of a set of decision making units (DMUs) for systems with multiple inputs to produce multiple outputs. "Measuring the efficiency of decision making units" is the most powerful DEA paper. In the efficiency literature, the activity units are generally referred to by the term "Decision Making Units" or DMUs. In this paper, it relates to urban wastewater plants under evaluation. Charnes et al., developed DEA model for measuring the relative efficiency of DMU k in multiplier form is as follows [21]:

$$E_{k} = Max \sum_{r=1}^{s} u_{r} \cdot y_{rk}$$
  
s.t. 
$$\sum_{i=1}^{m} v_{i} \cdot x_{ik} = 1$$
(1)
$$\sum_{r=1}^{s} u_{r} \cdot y_{rj} - \sum_{i=1}^{m} v_{i} \cdot x_{ij} \le 0$$
$$j = 1, \dots, n \quad u_{r}, v_{i} \ge \varepsilon$$

Let  $x_{ij}$  and  $y_{rj}$  denote the *i*th input,  $i = 1, 2, \dots, m$ , and *r*th output,  $r = 1, 2, \dots, s$ of the *i*th DMU, j = 1, 2, ..., n, respectively, where  $u_r$  and  $v_i$  are virtual multipliers and  $\varepsilon$  is a small non-Archimedean number imposed for avoiding ignorance of any factor. Above model is usually referred to as the CCR model and the below model is the Envelopment form:

$$E_{k} = \min \theta$$
s.t 
$$\sum_{j=1}^{n} \lambda_{j} \cdot x_{ij} \leq \theta \cdot x_{ik}$$

$$\sum_{j=1}^{n} \lambda_{j} \cdot y_{rj} \geq y_{rk}$$

$$j = 1, ..., n , \lambda_{j} \geq 0$$
(2)

#### 2.2 Network DEA model

Most traditional DEA models treat their reference technologies as black boxes and calculate their efficiencies by considering their initial inputs and their ultimate outputs. It is not able to deal with a system with more than one process, but network models allow us to look into these boxes and to estimate managerial performance for systems with more than one process. Therefore, all intermediate measures take in to account in the process of changing the inputs to outputs Later researchers apply the two-stage DEA model of Seiford to evaluate the efficiencies of the entire and the two sub-processes process separately. An iterative method for twostage model proposed and were applied to 20 Major League Baseball teams during the 2009 season to measure the performance [22]. A relational model was proposed for two-stage to measure the efficiency of 24 non-life insurance companies in Taiwan [23]. In a real world situation we have network data envelopment analysis which using the DEA model to measure the relative efficiency of a system, with considering operations of the component processes.

A serial network DEA model includes DMUs with three internal procedures which are connected with intermediate measures. The conventional three-stage DEA research used a Model to measure the overall efficiency and the efficiencies of the entire process and the three sub processes separately. The first stage uses initial inputs to produce first intermediate and second stage uses first intermediate as an input to produce second intermediate, at last third stage uses the second intermediate measures to produce final outputs.

## 2.3 Fuzzy DEA with exogenous inputs

A standard DEA model is not able to deal with uncertainty in data and with considering exist uncertainty in real world data, there is many efforts to develop DEA in uncertainty conditions. One of them is to develop DEA model based on fuzzy approach. The notion of fuzziness introduced in. [24,25]. The basic idea is to apply the a-cuts and Zadeh's extension principle.

The data in DEA models assume to be deterministic values. However, the observed value of the input and output data are sometimes imprecise or vague. The applications of fuzzy set theory in DEA are usually categorized into four groups [8]:

- The tolerance approach
- The  $\alpha$ -level based approach
- The fuzzy ranking approach
- The possibility approach.

Sengupta (1992a, b) was the first to introduce а fuzzv mathematical programming approach in which fuzziness was incorporated into the DEA model by defining tolerance levels on both the objective function and constraint violations. The a-level approach is perhaps the most popular fuzzy DEA model. This is obvious by the number of a-level based papers published in the fuzzy DEA literature. In this approach the main idea is to convert the fuzzy DEA model into a pair of parametric programs in order to find the lower and upper bounds of the a-level of the membership functions of the efficiency scores[8], for the two-stage system with fuzzy [26]. Developed a pair of two-level mathematical programs to determine the fuzzy efficiency of the system. They calculate the system efficiency in addition the process efficiencies. The to relationship between the system and process efficiencies will also be explored. To measure efficiency since data were fuzzy numbers a model presented by [27]. Multiple solutions may happen by different level of  $\alpha$ -cut in their model.

A new model to measure the process efficiencies in two-stage systems with fuzzy data is presented by [28]. The first step is to compute the upper and lower limits of the  $\alpha$ -cuts of the system efficiency using the corresponding models in [27]. The second step consists in computing the upper and lower limits of the  $\alpha$ -cuts of the efficiency of each of the processes.

In real world situation, nondiscretionary factors may exist sometimes called as exogenously fixed that are away from the discretionary control of DMU managers, need to be considered. Measuring the relative efficiency of DMUs, while some of the inputs or outputs are exogenously fixed was first proposed by [29] as follow:

 $\begin{array}{lll} \max & \sum_{r=1}^{s} u_{r0} y_{r0} \\ \text{s.t} & \sum_{i \in D} v_{i0} x_{ij} = 1 \\ & \sum_{r=1}^{s} u_{r0} y_{rj} - \sum_{i \in D} v_{i0} x_{ij} - \\ \sum_{i \in ND} v_{i0} (x_{ij} - x_{i0}) \leq 0 & , \quad \forall j \ (3) \\ & u_{r0} \geq \epsilon & , \quad \forall r \\ & v_{i0} \geq \epsilon & , \quad i \in D \\ & v_{i0} \geq 0 & , \quad i \in ND \end{array}$ 

They stated that since some inputs are nondiscretionary and manager cannot reduce its consumption, the problem then is to determine reduction in the consumption of the discretionary input. They develop a general model for condition relating multiple outputs. multiple discretionary inputs and multiple nondiscretionary inputs to estimate the amount of reduction possible in the consumption of discretionary inputs by involving the performance of the most efficient DMUs [30].

The 3-stage model with non-discretionary inputs in DEA was introduced in [31]. They presented new second-stage models and compared them with simulated data.

## 3. Urban wastewater treatment plants in north of Iran

Six UWWTPs which evaluated are located at the North of Iran such as Gorgan, Kordkuy, Bandar Gaz, Bandar Torkman, Babol and Yasreb. Large variations in composition of the wastewater which was created by a number of pollutions considered difficulties in effectiveness of the each WWTPs.

Generally, the wastes consist of solid and liquid parts. The solids were removed from the wastewater by screening and grit removal chamber. The wastewater was sent to equalization tank to control the flow rate and adjust ph. Subsequently, the wastewater was transferred to the aerobic biological unit. Most of aerobic processes required enough oxygen to existing in liquid phase for comprehensive oxidation. The purpose of this unit is to biologically stabilize the oxygen-demanding material present in wastewater with aerobic microorganisms.

Fresh wastewater after passing equalization tank was introduced into the aeration tank and then the effluent was settled in a settling basin. Clear effluent from the settled sludge was separated in a settling tank and then treated waste water was discharged to the environment.

Three stages in these UWWTPs consisted of equalization tank, aeration reactor, settling tank which are shown in figure 2. Three stage systems were included inputs (X), outputs (Y) and intermediate product

 $z^{1}$  (effluent from stage 1) and  $z^{2}$  (effluent from stage 2).



**Figure 2**. Three stage system with inputs X, outputs Y and intermediate product  $z^1$  and  $z^2$ 

#### 4. A new FNDEA model and Results

In this research, we consider the 3-stage DEA with fuzzy data in the case that some inputs are nondiscretionary variables, which is extension of the model [27]. Considering those factors affecting the production in DEA, some could lie outside the control of the DMU under evaluation. For example, in a WWTP, inputs involve BOD, COD and TSS is beyond from the non-discretionary control of DMU managers and Stream flow (O) is discretionary input. Since need to be taken into consideration in measuring relative efficiency. Suppose that notation D refers to sets of discretionary inputs and ND to non-discretionary ones. In this model, we apply the  $\alpha$ -cut method to expand a new fuzzy model for calculating the efficiency of DMUs. The upper limit efficiency is achieved when the DMU k under evaluation consumes the lowest feasible amount of inputs  $(x_{ik})^l_{\alpha}$ , while the other DMUs consume the largest feasible amount  $(x_{ij})^{\mu}_{\lambda} \forall j \neq k$ . For the outputs accurately the reverse occurs, i.e. the upper limit efficiency for DMU k occurs when it produces  $(y_{rk})^{u}_{\alpha}$  and the other DMUs  $(y_{ri})_{\alpha}^{\prime} \forall j \neq k$ . Note that the intermediate products are still undetermined and can fluctuate within the lower and upper limits of their corresponding  $\alpha$ -cuts. Let  $z_{pi}^1$ ,  $z_{pi}^2$ denote the pth of first intermediate and second intermediate, p=1,...,q of the jth DMU, j=1, ..., n. In the end, the upper limits of the system efficiency  $\alpha$ -cuts are computed solving the following model. The proposed model of FNDEA for calculating the upper limit efficiency of DMU k is defined as follows:

$$\begin{split} \left(E_{k}\right)_{\alpha}^{u} &= Max \sum_{r=1}^{s} u_{r} \cdot (y_{rk})_{\alpha}^{u} \\ s.t &\sum_{i \in D} v_{i} \cdot (x_{ik})_{\alpha}^{l} = 1 \\ &\sum_{p=1}^{q} \hat{z}_{pk}^{1} - \sum_{i \in D} v_{i} \cdot (x_{ik})_{\alpha}^{l} \leq 0 \quad (4a) \\ &\sum_{p=1}^{q} \hat{z}_{pj}^{1} - \sum_{i=1} v_{i} \cdot (x_{ij})_{\alpha}^{u} + \sum_{i \in ND} v_{i} \cdot (x_{ik})_{\alpha}^{l} \leq 0 , \forall j \neq k \\ &\sum_{p=1}^{q} \hat{z}_{pj}^{2} - \sum_{p=1}^{q} \hat{z}_{pj}^{1} \leq 0 \quad ,\forall j \\ &\sum_{r=1}^{s} u_{r} \cdot (y_{rk})_{\alpha}^{u} - \sum_{p=1}^{q} \hat{z}_{pk}^{2} \leq 0 \\ &\sum_{r=1}^{s} u_{r} \cdot (y_{rj})_{\alpha}^{l} - \sum_{p=1}^{q} \hat{z}_{pj}^{2} \leq 0 \quad ,\forall j \neq k \\ &w_{p}^{1} \cdot (z_{pj}^{1})_{\alpha}^{l} \leq \hat{z}_{pj}^{1} \leq w_{p}^{1} \cdot (z_{pj}^{1})_{\alpha}^{u} \quad ,\forall j,\forall p \\ &w_{p}^{2} \cdot (z_{pj}^{2})_{\alpha}^{l} \leq \hat{z}_{pj}^{2} \leq w_{p}^{2} \cdot (z_{pj}^{1})_{\alpha}^{u} \quad ,\forall j,\forall p \\ &i = 1, ..., m \quad j = 1, ..., n \quad p = 1, ..., q \\ &r = 1, ..., s, u_{r}, v_{r}, w_{p} \geq \varepsilon \end{split}$$

After the optimal solutions  $u_r^*$ ,  $v_i^*$ ,  $w_p^{1*}$ ,  $w_p^{2*}$ ,  $\hat{z}_p^{1*}$ ,  $\hat{z}_p^{2*}$  are obtained, the upper limit efficiency of fuzzy system  $(E_k)_{\alpha}^{u}$  and the three fuzzy process efficiencies  $(E_k^1)_{\alpha}^{\mu}$ ,  $(E_k^2)_{\alpha}^{\mu}$ ,  $(E_k^3)_{\alpha}^{\mu}$ , This function is shown by the following expressions:

$$\left(E_{k}\right)_{\alpha}^{u} = \sum_{r=1}^{s} u_{r}^{*} \cdot \left(y_{rk}\right)_{\alpha}^{u} / \sum_{i=1}^{m} v_{i}^{*} \cdot \left(x_{ik}\right)_{\alpha}^{l}$$

$$\left(E_{k}^{1}\right)_{\alpha}^{u} = \sum_{p=1}^{q} \hat{Z}_{pk}^{1*} / \sum_{i=1}^{m} v_{i}^{*} \cdot \left(x_{ik}\right)_{\alpha}^{l}$$

$$\left(E_{k}^{2}\right)_{\alpha}^{u} = \sum_{p=1}^{q} \hat{Z}_{pk}^{2*} / \sum_{p=1}^{q} \hat{Z}_{pk}^{1*}$$

$$\left(E_{k}^{3}\right)_{\alpha}^{u} = \sum_{r=1}^{s} u_{r}^{*} \cdot \left(y_{rk}\right)_{\alpha}^{u} / \sum_{p=1}^{q} \hat{Z}_{pk}^{2*}$$

$$\left(E_{k}^{3}\right)_{\alpha}^{u} = \sum_{r=1}^{s} u_{r}^{*} \cdot \left(y_{rk}\right)_{\alpha}^{u} / \sum_{p=1}^{q} \hat{Z}_{pk}^{2*}$$

The expected system efficiency with three stage is more smaller quantity than the standard DEA with one stage because  $(E_k)^{\mu}_{\alpha}$  achieved from multiplying 3 number  $(E_k^1)^{\mu}_{\alpha}$ ,  $(E_k^2)^{\mu}_{\alpha}$  and  $(E_k^3)^{\mu}_{\alpha}$  which all are less than or equal to one.

The proposed model of FNDEA for calculating the lower limit efficiency of DMU k is described in Equation (5).  $(E_{i})^{t} = Min = \theta$ 

$$\begin{split} \left( E_k \right)_a^{a} &= \min \quad \theta \\ s.t \quad \theta. \left( x_{ik} \right)_a^{u} - \lambda_k. \left( x_{ik} \right)_a^{u} - \sum_{j \neq k} \lambda_j. \left( x_{ij} \right)_a^{l} \geq 0 \quad i \in D \\ \sum_{j \neq i} \lambda_j \cdot \left( x_{ik} \right)_a^{u} - \lambda_k. \left( x_{ik} \right)_a^{u} - \sum_{j \neq k} \lambda_j. \left( x_{ij} \right)_a^{l} \geq 0 \quad i \in ND \\ \sum_j \lambda_j. z_{pj}^{1} - \sum_j \gamma_j. z_{pj}^{1} \geq 0 \quad \forall p \qquad (5a) \\ \sum_j \gamma_j. z_{pj}^{2} - \sum_j \mu_j. z_{pj}^{2} \geq 0 \quad \forall p \\ \mu_k. \left( y_{rk} \right)_a^{l} + \sum_{j \neq k} \mu_j. \left( y_{rj} \right)_a^{u} \geq \left( y_{rk} \right)_a^{l} \quad \forall r \\ \left( z_{pj}^{1} \right)_a^{l} \leq z_{pj}^{1} \leq \left( z_{pj}^{1} \right)_a^{u} \quad , \forall j, \forall p \\ \lambda_j \geq 0, \quad j = 1, \dots, n, \quad i \in D \cup ND, \\ r = 1, \dots, s, \quad p = 1, \dots, q \end{split}$$

Similarly, for the lower limits we use the optimal solutions  $z_p^{1*}$ ,  $z_p^{2*}$  obtain from solving model and the optimal solutions  $u_r^*$ ,  $v_i^*$ ,  $w_p^{1*}$ ,  $w_p^{2*}$  that obtained from upper limit but this time with the opposite inputs and outputs data, means that the lower limit efficiency is achieved when the DMU *k* under evaluation consumes  $(x_{ik})_{\alpha}^{u}$  and produces  $(y_{rk})_{\alpha}^{l}$  while the other DMUs consume  $(x_{ij})_{\alpha}^{l} \forall j \neq k$  and produces  $(y_{rj})_{\alpha}^{u} \forall j \neq k$ . These functions are revealed by the following equations:

$$\left(E_{k}\right)_{\alpha}^{l} = \sum_{r=1}^{s} u_{r}^{*} \cdot \left(y_{rk}\right)_{\alpha}^{l} / \sum_{i=1}^{m} v_{i}^{*} \cdot \left(x_{ik}\right)_{\alpha}^{u}$$

$$\left(E_{k}^{1}\right)_{\alpha}^{l} = \sum_{p=1}^{q} Z_{pk}^{1*} / \sum_{i=1}^{m} v_{i}^{*} \cdot \left(x_{ik}\right)_{\alpha}^{u}$$

$$\left(E_{k}^{2}\right)_{\alpha}^{l} = \sum_{p=1}^{q} Z_{pk}^{2*} / \sum_{p=1}^{q} Z_{pk}^{1*}$$

$$\left(E_{k}^{3}\right)_{\alpha}^{l} = \sum_{r=1}^{s} u_{r}^{*} \cdot \left(y_{rk}\right)_{\alpha}^{l} / \sum_{p=1}^{q} Z_{pk}^{2*}$$
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This paper presents a fuzzy 3-stage DEA (FNDEA) model with discretionary and non-discretionary factors in the input for six UWWTPs. Due to the non-linear constraint, this model has been run by the GAMS software. Results solved for different UWWTPs or DMUs by GAMS 23.6 software.

The discretion and non-discretion inputs are used. Q considered as discretion input, BOD, COD and TSS were calculated as non-discretion variables. The first step to compute the efficiencies of systems with fuzzy data are to obtain maximum and minimum limits of each non-discretion input for six UWWTPs during 1 year which are shown in figure 3,4 and 5.



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Figure 3. Maximum and minimum COD from different waste water treatment plants



Figure 4. Maximum and minimum BOD from different waste water treatment plants



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Figure 5. Maximum and minimum TSS from different waste water treatment plants

By applying these Models, the bounds of the fuzzy system efficiency and three fuzzy process efficiencies are obtained. Table 1 and 2 are illustrated the results of each compartment for upper efficiency and lower, correspondingly. In accordance with tables 1 and 2, the aerobic tank (stage 2) provided the better performance criteria than the other compartments. Factors that throw in affecting the efficiency of WWTP were wastewater flow rate, temperature, pH and cold weather. During cold weather the efficiency of WWTP is poor and for overcome this problem, it is important to decrease surface over flow rate. When the temperature is low settling tank does not operate well. In aerobic tank aeration rate is crucial, for this use of new aeration equipment is necessary. For example, Babol and yasreb waste water treatment plants do not operate well, because of the old equipment in aeration compartment. These two UWWTP should be upgraded.

Tabl	e 1.	Results	of	each	stage fo	r upper	efficiency	in in	waste	water	treatment	plants
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waste water treatment plants	Equalization tank	Aerobic tank	Settling tank
Bandar Torkman	0.72	0.92	0.72
Bandar Gaz	0.52	0.89	0.55
Gorgan	0.43	0.98	0.59
Kordkuy	0.52	0.98	0.57
yasreb	0.40	0.67	0.64
Babol	0.46	0.56	0.88

waste water treatment plants	Equalization tank	Aerobic tank	Settling tank	
Bandar Torkman	0.43	0.54	0.43	
Bandar Gaz	0.48	0.55	0.36	
Gorgan	0.31	0.71	0.43	
Kordkuy	0.3	0.60	0.33	
yasreb	0.25	0.57	0.40	
Babol	0.27	0.34	0.53	

Table 2. Results of each stage for lower efficiency in waste water treatment plants

These WWTPs were evaluated in a period of twelve months (June 2014 and January 2015). Lower and upper efficiencies were calculated which is illustrated in figure 6. Furthermore, this fuzzy 3-stage DEA model illustrated that Bandar Torkman WWTP had the best performance.





### 5. Conclusions

Great variations in composition of the wastewater which was created by several considered pollutions difficulties in ensuring the efficiency and success of the WWTPs. DEA was applied to evaluate the UWWTPs efficiency in three stage system such as equalization tank, aerobic tank and settling tank with fuzzy data. The discretion and non-discretion inputs were used. Due to the non-linear constraint, this model had been run by the GAMS software. Results solved for different cuts of  $\alpha$  to check while  $\alpha$  change how the efficiency scores of the DMUs change.

The first intermediate products, which are the outputs of the first process and also the inputs of the second process and second intermediate products, are the outputs of the second process and also the inputs of the third process.

BOD, COD, Stream flow and TSS data were considered as input variables. In addition, this fuzzy 3-stage DEA model demonstrated that Bandar Torkman WWTP had the best performance. Babol and yasreb waste water treatment plants do not operate well, because of the old equipment in aeration compartment.

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