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# Efficiency Analysis in Multi-Stage Network DEA-R Models

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## Abstract

In many organizations and financial institutions, it is in many cases more cost and time efficient to access ratio data. Therefore, it is of great importance to evaluate the performance of decision-making units (DMUs) which only have access to ratios of inputs to outputs or vice versa (for instance, ratio of employees to students, ratio of assets to liabilities and ratio of doctors to patients). In this paper, we will propose two-stage network DEA-R model with multi-objective linear programming (MOLP) structures. Then, introducing a production possibility set (PPS) in each network stage, we will compare efficiency values in network DEA and DEA-R. In the end, through an applied study on 22 medical centers which treat special patients in three stages, we will suggest an output-oriented multi-stage network DEA-R model under assumption of CRS technology. The medical centers are evaluated in all three stages based on overall network efficiency. The results of the analysis are presented and a future research in this field is discussed in the final section of the paper.

**Keywords:** *DEA, Network DEA, DEA-R models, efficiency.*

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## **1. Introduction**

Data envelopment analysis (DEA) is a non-parametric method for evaluating performance of a set of decision-making units (DMUs). Measuring efficiency, as an important factor in evaluation of a company or organization's performance, has always been a topic of interest among researchers. In 1957, Farrell proposed the idea of an efficient piecewise linear frontier as an alternative to estimating production function. However, there was an apparent issue with the number of inputs and outputs in the DMUs (Farrell, 1957). Charnes et al. (1978) extended Farrell's non-parametric method for a multiple inputs and outputs system through mathematical programming. Their model came to be known as CCR models. Later on, several articles and models were presented in this area, out of which we can mention the infamous BCC model introduced by Banker et al. (1984) for variable returns to scale (VRS) production technology.

Further developing the cone-ratio DEA model, Fare and Grosskopf (1996, 2000) presented network DEA models. Chen et al. (2006) introduced the DEA game model for measuring efficiency in supply chains. Kao and Hwang (2008) studied efficiency decomposition in two-stage processes for 24 non-life insurance companies in Taiwan. Their model is one of the most known two-stage DEA network models at all. Combining game approach with efficiency decomposition in two-stage DEA models, Liang et al. (2008) evaluated 30 commercial banks using centralized and non-cooperative models. Chen et al. (2009) studied the subject of additive efficiency in two-stage DEA under both constant and variable returns to scale assumptions (CRS / VRS) and rated the DMUs based on the results. Kao (2009) extended his previous paper and evaluated efficiency decomposition in network DEA for parallel and series systems using the same case (24

insurance companies in Taiwan). Cook et al. (2010) presented three-stage and multi-stage network DEA models with parallel processes based on additive efficiency decomposition. Chen and Yan (2011) introduced three network DEA models for evaluating performance of supply chain while considering internal resource waste. They proposed production possibility sets for supply chain based on centralized, decentralized and mixed mechanisms.

Chen et al. (2012) studied a new methodology for performance evaluation through two-stage network DEA in order to evaluate environmental conditions among 23 companies. Li et al. (2012) presented DEA models for extended two-stage network structures with additional inputs in 3 regions in China. One of the advantages of their work was an extension of the model introduced in (Liang et al., 2008).

Cook and Zhu (2014) published an extensive study on the internal structure of network DEA. Kao (2014a) studied efficiency decomposition for multi-stage systems in DEA and proposed the subject of shared resources. He also did in (Kao, 2014b) a review on network DEA models based on two-stage, parallel, series, mixed and dynamic structures. Despotis et al. (2015) introduced a multi-objective programming approach to network DEA. A network DEA model with two-stage processes was studied using formulation of a bi-objective problem and finding its Pareto optimal solutions and overall efficiency in (Despotis et al., 2016a). A new approach to network DEA for multi-stage processes using the composition paradigm was introduced in (Despotis et al., 2016b). They first presented a bi-objective problem and then calculated the overall efficiency using a two-stage method. They also applied the "weak-link" approach to network DEA for two-stage processes and adjusted the network DEA process by defining a subsystem.

Guo et al. (2016) obtained overall efficiency by selecting suitable weights and applying them to a multi-objective programming problem within the framework of network DEA for two-stage.

One of the first ratio DEA models (DEA-R) was introduced in (Despic et al., 2007). The authors extended the subjects of arithmetic, geometric and harmonic efficiency to DEA, DEA-R and multiplicative models based on the relationships between arithmetic, geometric and harmonic means. Furthermore, they revealed the efficiency value in input-oriented DEA-R to be greater than or equal to efficiency value in DEA. Studying 21 medical centers, Wei et al. (2011a) proposed the subject of pseudo-inefficiency in DEA. In another study (Wei et al., 2011b), they evaluated the relationship between CCR models in DEA and DEA-R through comparing optimal weights in DEA and DEA-R. Wei et al. (2011c) presented output-oriented DEA-R models with weight restrictions and made a comparison between them and DEA-R models with harmonic. Liu et al. (2011) suggested DEA models without explicit inputs; they proposed the subject of ratio data and corresponding production possibility sets (PPS) under general conditions. Mozaffari et al. (2014a) analyzed the relationship between DEA models without explicit inputs and DEA-R models. A cost and revenue efficiency in DEA and DEA-R models is studied in (Mozaffari et al., 2014b).

Olesen et al. (2015) proposed the subject of efficiency analysis based on ratio measure. They did a detailed study on production possibility set and dependence of inputs and outputs on ratio data under constant and variable returns to scale technology assumptions. This article differs from their study in terms of dependence of inputs and outputs - ratio

data are available in this study and they are defined as independent.

The rest of the paper is organized as follows. In Section 2, two-stage DEA models and basic concepts of DEA-R are introduced. Section 3 presents output-oriented multi-stage DEA-R models along with their production possibility sets (PPS). A three-stage DEA-R model is formulated in Section 4. A case study that demonstrates and compares stage efficiencies and overall efficiencies using the structure of multiple objective linear programming is presented in Section 5. Final section of the paper contains conclusions and suggestions for future research.

## 2. Theoretical background

In this section, selected two-stage serial DEA models are formulated to the extent necessary for further analysis, and basic ideas of DEA-R models are presented.

### 2.1 Two-stage DEA models

Let us consider a two-stage structure, where in the first stage,  $m$  inputs ( $X_j = (x_{1j}, \dots, x_{mj})$ ) produce  $b$  outputs ( $Z_j = (z_{1j}, \dots, z_{bj})$ ). In the second stage, the outputs ( $Z_j = (z_{1j}, \dots, z_{bj})$ ) are turned into inputs, which in turn produce  $s$  final outputs ( $Y_j = (y_{1j}, \dots, y_{sj})$ ). The inputs, intermediate characteristics and the final outputs of the  $j$ -th DMU are  $X_j = (x_{1j}, \dots, x_{mj})$ ,  $Z_j = (z_{1j}, \dots, z_{bj})$ , and  $Y_j = (y_{1j}, \dots, y_{sj})$  respectively. A model for evaluation of the DMU<sub>*o*</sub> in the first stage based on the idea presented in (Despotis et al., 2016) is introduced as follows:

$$\begin{aligned} \text{Minimize} \quad & e_1 = \frac{\sum_{i=1}^m \varphi_i x_{io}}{\sum_{f=1}^b \eta_f z_{fo}} \\ \text{Subject to} \quad & \sum_{i=1}^m \varphi_i x_{ij} - \sum_{f=1}^b \eta_f z_{fj} \geq 0 \\ & j = 1, \dots, n \end{aligned} \tag{1}$$

$$\sum_{f=1}^b \eta_f z_{fj} - \sum_{r=1}^b v_r y_{rj} \geq 0 \quad j = 1, \dots, n$$

$$\varphi_i \geq 0; \eta_f \geq 0; v_r \geq 0$$

$$i = 1, \dots, m; f = 1, \dots, b; r = 1, \dots, s$$

A model that evaluates the efficiency of the DMU<sub>o</sub> in second stage is formulated in a similar way:

$$\text{Minimize} \quad e_2 = \frac{\sum_{f=1}^b \eta_f z_{fo}}{\sum_{r=1}^b v_r y_{ro}}$$

$$\text{Subject to} \quad \sum_{i=1}^m \varphi_i x_{ij} - \sum_{f=1}^b \eta_f z_{fj} \geq 0$$

$$j = 1, \dots, n \tag{2}$$

$$\sum_{f=1}^b \eta_f z_{fj} - \sum_{r=1}^b v_r y_{rj} \geq 0 \quad j = 1, \dots, n$$

$$\varphi_i \geq 0; \eta_f \geq 0; v_r \geq 0$$

$$i = 1, \dots, m; f = 1, \dots, b; r = 1, \dots, s$$

Both models (1) and (2) have the same set of constraints. The weights  $\varphi_i$ ,  $\eta_f$  and  $v_r$  are assigned to inputs, intermediate characteristics and final outputs respectively. They are variables of models (1) and (2). Symbols  $e_1$  and  $e_2$  stand for efficiency scores in stages one and two, respectively. Based on the efficiency decomposition approach, overall efficiency is defined as the weighted average of stage efficiencies:

$$e_0 = \alpha e_1 + (1 - \alpha) e_2,$$

$$0 \leq \alpha \leq 1 \tag{3}$$

The following model is solved in order to obtain the overall efficiency score (3):

$$\text{Minimize} \quad h = \left\{ \frac{\sum_{i=1}^m \varphi_i x_{io}}{\sum_{f=1}^b \eta_f z_{fo}}, \frac{\sum_{f=1}^b \eta_f z_{fo}}{\sum_{r=1}^s v_r y_{ro}} \right\}$$

$$\text{Subject to} \quad \sum_{i=1}^m \varphi_i x_{ij} - \sum_{f=1}^b \eta_f z_{fj} \geq 0$$

$$j = 1, \dots, n \tag{4}$$

$$\sum_{f=1}^b \eta_f z_{fj} - \sum_{r=1}^s v_r y_{rj} \geq 0 \quad j = 1, \dots, n$$

$$\varphi_i \geq 0; \eta_f \geq 0; v_r \geq 0$$

$$i = 1, \dots, m; f = 1, \dots, b; r = 1, \dots, s$$

Model (4) is a bi-objective fractional programming problem. It can be solved using various multiple objective techniques - weighted sum of particular objective functions, lexicographic approach, or bi-level method.

### 2.2 Basic concepts of DEA-R models

In this section, we present the relationship between DEA and DEA-R models under the assumption of constant returns to scale technology based on the ideas of Despic et al. (2007). Let us suppose the following formulation of DEA-R model:

$$\text{Maximize} \quad (u, v)$$

$$e_o = \min_j \frac{\sum_{i=1}^m v_i \left( \frac{x_{ij}}{x_{io}} \right)}{\sum_{r=1}^s u_r \left( \frac{y_{rj}}{y_{ro}} \right)}$$

$$\text{Subject to} \quad \sum_{r=1}^s u_r = 1 \tag{5}$$

$$\sum_{i=1}^m v_i = 1$$

$$u_r \geq 0; v_i \geq 0 \quad r = 1, \dots, s; i = 1, \dots, m$$

Of course the assumption of positive data is essential and all ratios  $\frac{x_{ij}}{x_{io}}$  and  $\frac{y_{rj}}{y_{ro}}$  must be defined. Despic et al. (2007) presented their DEA-R efficiency model for evaluation of DMU<sub>o</sub> in constant returns to scale technology as follows:

$$\begin{aligned} \text{Maximize} \quad & \hat{e}_o = \min_j \sum_{i=1}^m \sum_{r=1}^s w_{ir} \begin{pmatrix} \frac{x_{ij}}{x_{io}} \\ \frac{y_{rj}}{y_{ro}} \end{pmatrix} \\ \text{Subject to} \quad & \sum_{i=1}^m \sum_{r=1}^s w_{ir} = 1 \quad (6) \\ & w_{ir} \geq 0 \quad i = 1, \dots, m; r = 1, \dots, s \end{aligned}$$

Where  $w_{ir}$  is the weight of the ratio in the objective function (6). Wei et al. (2011a) proved the relationship between models (5) and (6) as  $e_o \leq \hat{e}_o$ .

### 3. Two-stage network DEA-R models

Evaluating DMUs which possess ratio data such as  $\frac{Z_j}{X_j}$  and  $\frac{Y_j}{Z_j}$  requires models

which firstly, possess the requirements of the respective production possibility set, and secondly, are able to calculate efficiency value of the units. In this section, we first suggest two-stage DEA-R models and then introduce the production possibility set in each stage. In the second part, we will propose two-stage DEA-R models using MOLP and present numerical expressions in the end. (Fig 1)

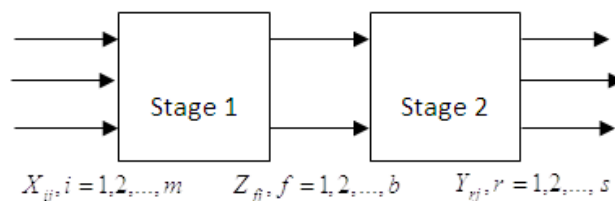


Fig 1. A two-stage process.

### 3.1 Efficiency in two-stage DEA-R model

Assume that we have  $(X_j, Z_j, Y_j)$  for DMU<sub>j</sub> where  $\frac{Z_j}{X_j}$  and  $\frac{Y_j}{Z_j}$  ratios are available. Our aim is to evaluate the DMUs in a two-stage process using the defined ratios. Our proposed output-oriented CRS model for the first stage is as follows:

$$\begin{aligned} \text{Minimize} \quad & E_1 = \beta_1 \\ \text{Subject to} \quad & \sum_{i=1}^m \sum_{f=1}^b w_{if} \frac{\frac{z_{fj}}{x_{io}}}{\frac{x_{ij}}{z_{fo}}} \leq \beta_1 \\ & j = 1, \dots, n \quad (7) \\ & \sum_{i=1}^m \sum_{f=1}^b w_{if} = 1, \\ & w_{if} \geq 0 \quad i = 1, \dots, m; f = 1, \dots, b. \end{aligned}$$

The output-oriented DEA-R envelopment model for evaluation of the DMU<sub>o</sub> can be formulated as follows:

$$\begin{aligned} \text{Maximize} \quad & \alpha_1 \\ \text{Subject to} \quad & \sum_{j=1}^n \lambda_j^1 \left( \frac{Z_j}{X_j} \right) \geq \alpha_1 \left( \frac{Z_o}{X_o} \right) \quad (8) \\ & \sum_{j=1}^n \lambda_j^1 = 1, \quad \lambda_j^1 \geq 0, \quad j = 1, \dots, n. \end{aligned}$$

**Definition 1.**  $DMU_o$  is DEA-R efficient (output-oriented DEA-R efficient) if and only if the optimal objective function value of model (8)  $\alpha_1^* = 1$ .

In the second stage, our proposed output-oriented DEA-R model is as follows:

$$\begin{aligned} &\text{Minimize} && E_2 = \beta_2 \\ &\text{Subject to.} && \sum_{r=1}^s \sum_{f=1}^b w_{if} \frac{\frac{y_{rj}}{z_{fj}}}{\frac{y_{ro}}{z_{fo}}} \leq \beta_2 \\ &&& j = 1, \dots, n \tag{9} \\ &&& \sum_{r=1}^s \sum_{f=1}^b v_{rf} = 1 \\ &&& v_{rf} \geq 0 \quad r = 1, \dots, s; f = 1, \dots, b \end{aligned}$$

The output-oriented envelopment model under CRS technology in stage 2 for evaluation of  $DMU_o$  can be written as follows:

$$\begin{aligned} &\text{Maximize} && \alpha_2 \\ &\text{Subject to} && \sum_{j=1}^n \lambda_j^2 \left( \frac{Y_j}{Z_j} \right) \geq \alpha_2 \left( \frac{Y_o}{Z_o} \right) \\ &&& \sum_{j=1}^n \lambda_j^2 = 1, \\ &&& \lambda_j^2 \geq 0, \quad j = 1, \dots, n. \tag{10} \end{aligned}$$

**Definition 2.**  $DMU_o$  is DEA-R efficient (output-oriented DEA-R efficient) if and only if the optimal objective function value of model (10)  $\alpha_2^* = 1$ .

**3.2 Two-stage Network DEA-R models based on MOLP**

In this section, we will propose two-stage network DEA models which use MOLP. First, a bi-objective linear programming model is suggested for measuring overall efficiency of  $DMU_o$  with ratio data defined as  $\frac{Z_j}{X_j}$  and  $\frac{Y_j}{Z_j}$  (CRS, output-

oriented), And finally, the bi-objective linear programming model is solved using the lexicographic and adaptive weighted sum approaches. Combining the restrictions of models (7) and (9), we suggest the bi-objective linear model (11) for measuring overall efficiency of the two-stage DEA-R process. Model (11) is proposed for output-oriented evaluation of  $DMU_o$  under the assumption of constant returns to scale technology based on MOLP.

$$\begin{aligned} &\text{Minimize} && \{\gamma_1, \gamma_2\} \\ &\text{Subject to.} && \sum_{i=1}^m \sum_{f=1}^b w_{if} \frac{\frac{z_{fj}}{x_{ij}}}{\frac{z_{fo}}{x_{io}}} \leq \gamma_1 \\ &&& j = 1, \dots, n \tag{11} \end{aligned}$$

$$\begin{aligned} &&& \sum_{r=1}^s \sum_{f=1}^b v_{rf} \frac{\frac{y_{rj}}{z_{fj}}}{\frac{y_{ro}}{z_{fo}}} \leq \gamma_2 \quad j = 1, \dots, n \\ &&& \sum_{r=1}^s \sum_{f=1}^b v_{rf} = 1 \quad \sum_{i=1}^m \sum_{f=1}^b w_{if} = 1 \\ &&& v_{rf} \geq 0; w_{if} \geq 0 \\ &&& r = 1, \dots, s; f = 1, \dots, b; i = 1, \dots, m \end{aligned}$$

We can solve the bi-objective model (11) using the weighted sum method. For this purpose, we need to define coefficients  $p_1$  and  $p_2$  for the first and second objective functions, respectively, where  $p_1 + p_2 = 1$ ,  $p_1, p_2 > 0$ . It is clear that solving the optimization with the aggregated objective function given as weighted sum of two particular objective functions with the same set of constraints results to the Pareto solution of the MOLP problem. Another possibility how to solve the model (11) is to use lexicographic approach. At first the model with the first objective function ( $\gamma_1$ ) is solved – the optimal objective function value is  $\gamma_1^*$ .

The second step, the set of constraints is extended by  $\gamma_1 = \gamma_1^*$  and the second objective function is optimized. Using this approach a Pareto efficient solution of the MOLP problem (11) is given. We propose an envelopment model for evaluating the overall efficiency of a two-stage network DEA-R as follows:

$$\begin{aligned}
 & \text{Maximize} && (\alpha_1 + \alpha_2) \\
 & \text{Subject to} && \sum_{j=1}^n \lambda_j^1 \left( \frac{Z_j}{X_j} \right) \geq \alpha_1 \left( \frac{Z_o}{X_o} \right) \\
 & && \sum_{j=1}^n \lambda_j^2 \left( \frac{Y_j}{Z_j} \right) \geq \alpha_2 \left( \frac{Y_o}{Z_o} \right) \quad (12) \\
 & && \sum_{j=1}^n \lambda_j^1 = P_1, \quad \sum_{j=1}^n \lambda_j^2 = P_2, \\
 & && \lambda_j^1 \geq 0, \lambda_j^2 \geq 0, \quad j = 1, \dots, n.
 \end{aligned}$$

Model (12) is a linear programming problem, in which  $p_1$  and  $p_2$  are parameters determining overall efficiency of the two-stage process. The variable  $\lambda_j^1$  and  $\lambda_j^2$  correspond to stage 1 and stage 2

respectively. If  $\sum_{j=1}^n \lambda_j^2 = 0$  then only stage 1 process is considered. Similarly, if  $\sum_{j=1}^n \lambda_j^1 = 0$ , then we consider the process in stage 2 only. However, if we consider  $\sum_{j=1}^n \lambda_j^1 = p_1$  and  $\sum_{j=1}^n \lambda_j^2 = p_2$ , where  $p_1 + p_2 = 1$ ,  $p_1, p_2 > 0$ , the Pareto optimal solution of model (11) defines the overall efficiency of DMU<sub>o</sub> of the two-stage models with ratio data.

### 3.3 Illustration

The illustration presented in this section is given from (Kao and Hwang, 2008) where 24 non-life insurance companies have been analyzed. Despotis et al. (2016) and Guo et al. (2016) have previously used this data set in order to measure efficiency of two-stage processes and overall efficiency. We will use the same data set in order to evaluate the two-stage DEA-R process using presented MOLP techniques. The original data set is presented in Table 1.

**Table 1. Inputs, intermediate variables, and outputs of 24 DMUs**

DMU	x <sub>1</sub>	x <sub>2</sub>	z <sub>1</sub>	z <sub>2</sub>	y <sub>1</sub>	y <sub>2</sub>
1	1178744	673512	7451757	856735	984143	681687
2	1381822	1352755	10020274	1812894	1228502	834754
3	1177494	592790	4776548	560244	293613	658428
4	601320	594259	3174851	371863	248709	177331
5	6699063	3531614	37392862	1753794	7851229	3925272
6	2627707	668363	9747908	952326	1713598	415058
7	1942833	1443100	10685457	643412	2239593	439039
8	3789001	1873530	17267266	1134600	3899530	622868
9	1567746	950432	11473162	546337	1043778	264098
10	1303249	1298470	8210389	504528	1697941	554806
11	1962448	672414	7222378	643178	1486014	18259
12	2592790	650952	9434406	1118489	1574191	909295

13	2609941	1368802	13921464	811343	3609236	223047
14	1396002	988888	7396396	465509	1401200	332283
15	2184944	651063	10422297	749893	3355197	555482
16	1211716	415071	5606013	402881	854054	197947
17	1453797	1085019	7695461	342489	3144484	371984
18	757515	547997	3631484	995620	692731	163927
19	159422	182338	1141951	483291	519121	46857
20	145442	53518	316829	131920	355624	26537
21	84171	26224	225888	40542	51950	6491
22	15993	10502	52063	14574	82141	4181
23	54693	28408	245910	49864	0.1	18980
24	163297	235094	476419	644816	142370	16976

**Table 2. Efficiency scores obtained by models (1), (2), (7) and (9)**

DMU	$e_1$	$E_1$	Difference	$e_2$	$E_2$	Difference
1	0.9926	0.9617	0.0309	0.7134	0.7152	-0.0018
2	0.9985	0.9987	-0.0002	0.6449	0.6311	0.0138
3	0.69	0.6618	0.0282	<b>1</b>	<b>1</b>	0
4	0.7243	0.7243	0	0.4323	0.4347	-0.0024
5	0.8375	0.8218	0.0157	<b>1</b>	<b>1</b>	0
6	0.9637	0.9637	0	0.4057	0.4119	-0.0062
7	0.7521	0.7521	0	0.5378	0.525	0.0128
8	0.7256	0.7038	0.0218	0.5113	0.4911	0.0202
9	<b>1</b>	<b>1</b>	0	0.292	0.2899	0.0021
10	0.8615	0.8615	0	0.6736	0.6829	-0.0093
11	0.7405	0.7246	0.0159	0.3267	0.2816	0.0451
12	<b>1</b>	<b>1</b>	0	0.7596	0.7666	-0.007
13	0.8107	0.7918	0.0189	0.5435	0.5023	0.0412
14	0.7246	0.7246	0	0.5178	0.5135	0.0043
15	<b>1</b>	<b>1</b>	0	0.7047	0.6806	0.0241
16	0.9072	0.8881	0.0191	0.3847	0.3857	-0.001
17	0.7233	0.7233	0	<b>1</b>	<b>1</b>	0
18	0.7935	0.7685	0.025	0.3976	0.3737	0.0239
19	<b>1</b>	<b>1</b>	0	0.4158	0.4158	0
20	0.9332	0.9332	0	0.9014	0.9014	0
21	0.7505	0.7505	0	0.2795	0.2906	-0.0111
22	0.5895	0.5802	0.0093	<b>1</b>	<b>1</b>	0
23	0.8501	0.8217	0.0284	0.5599	0.5599	0
24	<b>1</b>	<b>1</b>	0	0.3351	0.3351	0

This example consists of two inputs ( $x_1, x_2$ ), two intermediate measures ( $z_1, z_2$ ) and two outputs ( $y_1, y_2$ ).

All ratios  $\frac{z_1}{x_1}, \frac{z_1}{x_2}, \frac{z_2}{x_1}, \frac{z_2}{x_2}, \frac{y_1}{z_1}, \frac{y_1}{z_2}, \frac{y_2}{z_1}$ ,

$\frac{y_2}{z_2}$  are known. That is why we can

analyze the efficiency of the two-stage system using DEA and DEA-R models (1), (2), (7) and (9). (Table 2)



**Table 3. Results obtained by MOLP models (11) and (12) with different parameters**

DMU	$(p_1, p_2)$ (1,0)	$(p_1, p_2)$ (0,1)	$(p_1, p_2)$ (0.5,0.5)	$(p_1, p_2)$ (0.75,0.25)	$(p_1, p_2)$ (0.25,0.75)	$(p_1, p_2)$ (0.1,0.9)	$(p_1, p_2)$ (0.9,0.1)
1	0.9617	0.7152	0.8203	0.8854	0.7642	0.734	0.9296
2	0.9987	0.6311	0.7734	0.8718	0.6951	0.6552	0.9437
3	0.6618	1	0.7965	0.7229	0.8867	0.9514	0.6849
4	0.7243	0.4347	0.5433	0.6209	0.483	0.4528	0.6791
5	0.8218	1	0.9022	0.8601	0.9486	0.9788	0.8367
6	0.9637	0.4119	0.5771	0.7219	0.4807	0.4369	0.8498
7	0.7521	0.525	0.6184	0.6787	0.5679	0.5414	0.7209
8	0.7038	0.4911	0.5785	0.6351	0.5312	0.5064	0.6746
9	1	0.2899	0.4495	0.6202	0.3525	0.3120	0.8032
10	0.8615	0.6829	0.7619	0.8087	0.7203	0.6974	0.8396
11	0.7246	0.2816	0.4056	0.5201	0.3324	0.2999	0.6261
12	1	0.7666	0.8679	0.9293	0.8141	0.7849	0.9705
13	0.7918	0.5023	0.6147	0.6921	0.5529	0.5214	0.7486
14	0.7246	0.5135	0.6011	0.6571	0.5539	0.5289	0.6960
15	1	0.6806	0.8099	0.8950	0.7397	0.7031	0.9552
16	0.8881	0.3857	0.5378	0.6699	0.4492	0.4088	0.7857
17	0.7233	1	0.8394	0.7771	0.9127	0.9632	0.7439
18	0.7685	0.3737	0.5028	0.6079	0.4287	0.3939	0.6951
19	1	0.4158	0.5874	0.7400	0.4869	0.4416	0.8768
20	0.9332	0.9014	0.9170	0.9250	0.9091	0.9045	0.9299
21	0.7505	0.2906	0.4189	0.5377	0.3431	0.3095	0.6479
22	0.5802	1	0.7343	0.6482	0.8468	0.9325	0.6056
23	0.8217	0.5599	0.6660	0.7357	0.6084	0.5783	0.7850
24	1	0.3351	0.5020	0.6684	0.4019	0.3590	0.8344

The second and third column of Table 2 presents the efficiency score obtained by models (1) and (7), i.e. models measuring the efficiency in the first stage of the production process using the output-oriented DEA model (1) and DEA-R model under CRS technology (7) respectively. There is a very small difference between efficiency values of the first stage in DEA and DEA-R – in many cases the efficiency scores are identical. The units 9, 12, 15, 19 and 24 are efficient at this stage. The last two columns of Table 2 present similar results for the second stage of the production process. They are calculated using models (2) and (9). The units 3, 5, 17 and 22 are efficient in the second stage measuring by both models.

Table 3 contains the results obtained by MOLP models (11) and (12). The

adaptive weighted sum method was used to solve the multi-objective model (11). By setting different values for parameters  $p_1$  and  $p_2, p_1 + p_2 = 1, p_1, p_2 > 0$ , the model returns Pareto optimal solution of model (11). In the first two columns of Table 3, parameters  $p_1$  and  $p_2$  are considered as (1,0) and (0,1). In this case the results of the model (12) lead to the same conclusions as models (7) and (9) that measure efficiencies of both stages independently, i.e. the units 9, 12, 15, 19 and 24 are efficient for  $(p_1, p_2) = (1, 0)$  and the units 3, 5, 17 and 22 are efficient for  $(p_1, p_2) = (0, 1)$ . This corresponds to conclusions presented in Table 2. The last five columns of Table 3 contain efficiency scores computed using (12) with setting of different values of both parameters (0.5, 0.5), (0.75, 0.25), (0.25, 0.75), (0.9, 0.1) and (0.1, 0.9). This

allows to decision makers analyzing the two-stage production process in more detail.

**4. Three-stage network DEA-R processes**

This section presents a three-stage network production process with inputs  $X^1, X^2$  and  $X^3$ , and final outputs  $Y^1, Y^2$  and  $Y^3$  for all three stages.  $Z^1$  are intermediate measures - the outputs of the first stage and the inputs of the second stage.  $Z^2$  are the outputs of the second stage entering the third stage as its inputs.  $Z^3$  are the outputs of the third stage.

We propose a model for evaluation three-stage network DEA-R processes under the following assumptions:

a. The proposed model is output-oriented DEA-R envelopment model under the assumption of CRS technology.

b. Our proposed model is a parametric linear model in a three-stage network process; this model aims to increase the outputs in every stage in order to evaluate units with ratio data.

In all network stages, we consider the constraint  $\sum_{j=1}^n \lambda_j^t = P_t$  corresponding to each stage  $t$ , on the condition that  $P_1+P_2+P_3 = 1$ . Therefore, since  $\lambda_j^t \geq 0$ , if  $\sum_{j=1}^n \lambda_j^t = 0$ , then  $\lambda_j^t = 0$  for every  $j$ .

In general, we consider two scenarios for the parameters  $P_t$ :

i) If  $P_t \in \{0,1\}$  and  $P_1+P_2+P_3 = 1$ , then the proposed model can calculate the efficiency of every stage.

ii) If  $P_t \in [0,1]$  and  $P_1+P_2+P_3 = 1$ , then the proposed model can calculate the overall efficiency of our network.

c. Let  $I_1, I_2$  and  $I_3$  are the sets of indices of inputs in all three stages. Similarly  $F_1, F_2$  and  $F_3$ , and  $R_1, R_2$  and  $R_3$  are the sets of indices of intermediate measures and final outputs respectively.

d. In the suggested model, parameters  $P_1, P_2$  and  $P_3$  correspond to variables  $\lambda_j^1, \lambda_j^2$  and  $\lambda_j^3$ , respectively. Now, since the three stages of our network as well correspond

to  $\lambda_j^1, \lambda_j^2$  and  $\lambda_j^3$ , therefore the parameters  $P_1, P_2$  and  $P_3$  have a very significant role in calculation of stage and overall efficiencies.

e. Variables  $\lambda_j^1, \lambda_j^2, \lambda_j^3$  correspond to stages 1, 2 and 3, respectively.

f. Since the three-stage network process is output-oriented, the proposed model aims to increase the output-to-input ratios, which are increased radially. Variables  $\varphi_1, \varphi_2$  and  $\varphi_3$  are used to increase the outputs in stages one, two and three of a network with ratio data, respectively.

The DEA-R model we propose for overall evaluation of three-stage serial production process is as follows:

Maximize  $(\varphi_1 + \varphi_2 + \varphi_3)$

Subject to  $\sum_{j=1}^n \lambda_j^1 \begin{pmatrix} z_{fj}^{(1)} \\ x_{ij}^{(1)} \end{pmatrix} \geq \varphi_1 \begin{pmatrix} z_{fo}^{(1)} \\ x_{io}^{(1)} \end{pmatrix}$

$i \in I_1; f \in F_1$  (13)

$\sum_{j=1}^n \lambda_j^1 \begin{pmatrix} y_{rj}^{(1)} \\ x_{ij}^{(1)} \end{pmatrix} \geq \varphi_1 \begin{pmatrix} y_{ro}^{(1)} \\ x_{io}^{(1)} \end{pmatrix} \quad i \in I_1; r \in R_1$

$\sum_{j=1}^n \lambda_j^2 \begin{pmatrix} z_{lj}^{(2)} \\ x_{ij}^{(2)} \end{pmatrix} \geq \varphi_2 \begin{pmatrix} z_{lo}^{(2)} \\ x_{io}^{(2)} \end{pmatrix} \quad i \in I_1; l \in F_1$

$\sum_{j=1}^n \lambda_j^2 \begin{pmatrix} y_{rj}^{(2)} \\ x_{ij}^{(2)} \end{pmatrix} \geq \varphi_2 \begin{pmatrix} y_{ro}^{(2)} \\ x_{io}^{(2)} \end{pmatrix} \quad i \in I_2; r \in R_2$

$\sum_{j=1}^n \lambda_j^3 \begin{pmatrix} z_{tj}^{(3)} \\ x_{ij}^{(3)} \end{pmatrix} \geq \varphi_3 \begin{pmatrix} z_{to}^{(3)} \\ x_{io}^{(3)} \end{pmatrix} \quad i \in I_3; t \in F_3$

$\sum_{j=1}^n \lambda_j^3 \begin{pmatrix} y_{rj}^{(3)} \\ x_{ij}^{(3)} \end{pmatrix} \geq \varphi_3 \begin{pmatrix} y_{ro}^{(3)} \\ x_{io}^{(3)} \end{pmatrix} \quad i \in I_3; r \in R_3$

$\sum_{j=1}^n \lambda_j^3 \begin{pmatrix} y_{rj}^{(3)} \\ z_{tj}^{(3)} \end{pmatrix} \geq \varphi_3 \begin{pmatrix} y_{ro}^{(3)} \\ z_{to}^{(3)} \end{pmatrix} \quad i \in I_3; t \in F_3$

$\sum_{j=1}^n \lambda_j^2 \begin{pmatrix} z_{lj}^{(2)} \\ z_{fj}^{(1)} \end{pmatrix} \geq \frac{z_{lo}^{(2)}}{z_{fo}^{(1)}} \quad f \in F_1; l \in F_2$

$\sum_{j=1}^n \lambda_j^3 \begin{pmatrix} z_{tj}^{(3)} \\ z_{lj}^{(2)} \end{pmatrix} \geq \frac{z_{to}^{(3)}}{z_{lo}^{(2)}} \quad t \in F_3; l \in F_2$

$$\sum_{j=1}^n \lambda_j^3 \left( \frac{y_{ij}^{(3)}}{z_{ij}^{(2)}} \right) \geq \frac{y_{ro}^{(3)}}{z_{lo}^{(2)}} \quad r \in R_3; l \in F_2$$

$$\sum_{j=1}^n \lambda_j^1 = p_1, \quad \sum_{j=1}^n \lambda_j^2 = p_2, \quad \sum_{j=1}^n \lambda_j^3 = p_3$$

$$p_1 + p_2 + p_3 = 1,$$

$$\lambda_j^1 \geq 0; \lambda_j^2 \geq 0; \lambda_j^3 \geq 0 \quad j = 1, \dots, n.$$

Model (13) is a linear programming problem, in which  $p_1$ ,  $p_2$  and  $p_3$  are parameters determining overall efficiency for three-stage network processes. Generally speaking, model (13) can be a suitable alternative for model (8) in stage one and model (10) in stage two; in this regard, if we consider  $P_1 = 1$  and  $P_2 = P_3 = 0$ , we can only calculate the efficiency of stage one, as  $\sum \lambda_j^1 = 1$  and  $\sum \lambda_j^2 = \sum \lambda_j^3 = 0$ . Similarly, we can calculate the efficiency of later stages by changing the parameters  $P_1$ ,  $P_2$  and  $P_3$  to zeros or ones. However, if the parameters  $P_1$ ,  $P_2$  and  $P_3$  were strictly greater than zero, then all constraints used in each network stage would influence the overall efficiency.

### 5. A case study

Let us consider 22 Iranian medical centers

which provide necessary services to special patients with tumors. The data set available is taken from summer months 2016. The treatment process of patients with cancer diseases can generally be divided into three stages:

**Stage 1** includes patients needing surgery. Patients at this stage generally have special diseases and their tumors are either benign of types A or B, or they are malignant. All three groups of patients need surgery according to conclusions by an expert physician.

**Stage 2** includes patients who need chemotherapy based on doctor's orders. This group generally consists of patients with special diseases who either have had a surgery in previous years and currently need chemotherapy, or did not undergo the surgery because of their age or other specific factors and now chemotherapy is needed.

**Stage 3** includes radiotherapy. Generally, this group of patients either has undergone surgery, chemotherapy and radiotherapy during past years and according to doctor's diagnosis need radiotherapy, or has finished the treatment process and need radiotherapy in order to destroy the cancer cells after surgery or chemotherapy.

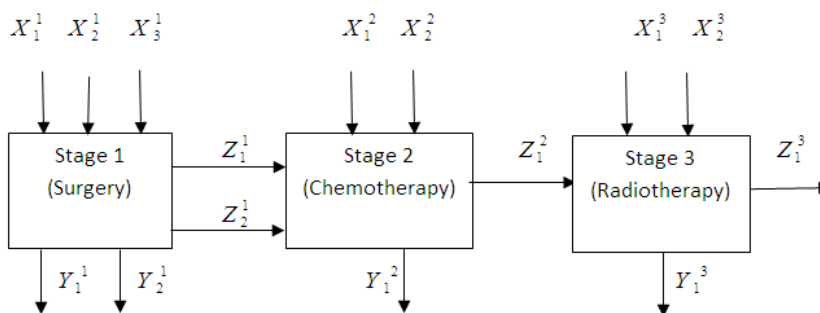


Figure 2. The three-stage process

Generally, in treatment of patients with tumors, medical centers provide the following treatment processes after the radiology and sampling stages.

- i) If the tumor was malignant, the stages of surgery, chemotherapy and radiotherapy are indeed essential.
- ii) If the tumor was malignant but the patient was not able to undergo surgery due to age limits, chemotherapy and radiotherapy are performed.
- iii) If the tumor was benign, there could only be a need for surgery based on doctor's diagnosis.

The treatment process is graphically illustrated on Figure 2. It is three-stage process where the following input, intermediate and output variables are used:

$x_1^1$  the number of patients with type A benign tumors needing a surgery,

$x_2^1$  the number of patients with type B benign tumors needing a surgery,.

$x_3^1$  the number of patients with malignant tumors needing a surgery.

$y_1^1$  patients who only require surgery according to the doctor's diagnosis and their surgery is only successful in the first stage; based on the post-surgery pathology report, these patients don't need to continue treatment and only make annual visits to the physician for follow-ups.

$y_2^1$  : Patients who undergo surgery, but cannot continue the treatment process due to age limits. Meaning the patients are not in a suitable condition to continue treatment after surgery and it is in their best interest to stop the process.

$z_1^1$  : Number of special patients who have benign tumors, but for whom the need for chemotherapy becomes apparent in the radiological tests at the start of surgery.

$z_2^1$  : Number of patients with malignant tumors who need chemotherapy following surgery.

$x_1^2$  : Number of special patients who start the treatment process with chemotherapy because of their age or problems with other diseases such as heart or liver disease.

$x_2^2$  : Number of patients who have had surgery in previous years and their respective physician has prescribed chemotherapy in the current annual screening.

$y_1^2$  : Number of special patients who have finished chemotherapy but don't need radiotherapy because of special conditions or other diseases.

$z_1^2$  : Number of patients who have undergone chemotherapy in stage 2 and do not require radiotherapy

$x_1^3$  : Number of patients who have undergone chemo and radiotherapy in previous years but currently require further radiotherapy based on doctor's orders.

$x_2^3$  : Number of patients who only need radiotherapy based on their tumor type and age.

$z_1^3$  : Number of patients who recover following radiotherapy (relatively satisfied with treatment).

$y_1^3$  : Number of patients who recover after undergoing all treatment stages (completely satisfied with treatment).

Regarding patients visiting the 22 medical centers, due to the special conditions of patients and uncertainty of the treatment process in each stage (since there is a chance of cell proliferation and dependency of patient's immune system), it's often impossible to obtain accurate data in terms of inputs and outputs for each stage. Consequently, we usually only

have access to a ratio of  $\frac{z_{ij}^{(1)}}{x_{ij}^{(1)}}$ ,  $\frac{y_{ij}^{(1)}}{x_{ij}^{(1)}}$ ,  $\frac{z_{ij}^{(2)}}{x_{ij}^{(2)}}$

and  $\frac{z_{ij}^{(3)}}{x_{ij}^{(3)}}$ .

**Table 4. Data related to the 22 Iranian medical centers in summer 2016 (stage 1)**

DMU	$\frac{z_1^1}{x_1^1}$	$\frac{z_2^1}{x_1^1}$	$\frac{z_1^1}{x_2^1}$	$\frac{z_2^1}{x_2^1}$	$\frac{z_1^1}{x_3^1}$	$\frac{z_2^1}{x_3^1}$	$\frac{y_1^1}{x_1^1}$	$\frac{y_1^1}{x_2^1}$	$\frac{y_1^1}{x_3^1}$	$\frac{y_2^1}{x_1^1}$	$\frac{y_2^1}{x_2^1}$	$\frac{y_2^1}{x_3^1}$
1	0.10	0.14	0.15	0.22	0.43	0.61	0.93	1.40	4.00	0.58	0.88	2.50
2	0.22	0.06	0.08	0.02	1.56	0.44	0.72	0.25	5.00	1.07	0.37	7.41
3	0.01	0.51	0.01	0.30	0.02	0.66	0.98	0.59	1.27	1.66	1.00	2.16
4	0.38	0.07	0.50	0.09	1.88	0.34	0.81	1.08	4.06	0.39	0.52	1.96
5	0.10	0.19	0.04	0.07	0.27	0.52	0.73	0.29	2.03	2.26	0.89	6.24
6	0.17	0.23	0.19	0.26	0.56	0.76	0.70	0.81	2.36	0.62	0.71	2.08
7	0.12	0.07	0.06	0.04	0.05	0.03	0.93	0.48	0.42	1.88	0.98	0.85
8	0.43	0.22	0.48	0.25	1.00	0.51	0.57	0.64	1.34	0.72	0.81	1.69
9	0.77	0.37	0.64	0.31	0.33	0.16	0.51	0.43	0.22	0.34	0.29	0.15
10	0.01	0.02	0.05	0.15	0.01	0.02	0.04	0.37	0.04	0.03	0.30	0.03
11	0.54	0.04	5.26	0.39	0.59	0.04	0.94	9.18	1.02	0.09	0.89	0.10
12	0.04	0.07	0.03	0.05	0.03	0.05	0.98	0.70	0.62	1.04	0.75	0.66
13	0.08	0.07	0.11	0.09	0.37	0.32	0.97	1.28	4.37	0.70	0.92	3.16
14	0.59	0.10	0.30	0.05	1.10	0.18	0.95	0.48	1.77	1.93	0.97	3.62
15	0.13	0.11	0.19	0.16	0.50	0.42	0.91	1.34	3.58	0.53	0.78	2.08
16	0.31	0.51	0.24	0.40	0.61	1.00	0.83	0.64	1.61	1.09	0.84	2.11
17	0.33	0.23	0.45	0.32	0.37	0.26	0.82	1.12	0.93	0.69	0.95	0.79
18	1.00	0.07	1.40	0.10	1.25	0.09	0.93	1.31	1.17	0.58	0.81	0.73
19	0.12	0.03	0.15	0.04	0.23	0.06	0.90	1.16	1.75	0.62	0.80	1.21
20	0.33	0.08	0.42	0.10	0.58	0.13	0.14	0.18	0.25	0.07	0.08	0.11
21	0.06	0.23	0.03	0.11	0.04	0.16	0.97	0.47	0.69	2.06	0.99	1.47
22	0.30	0.21	0.39	0.27	1.31	0.91	0.30	0.39	1.31	0.21	0.27	0.91

**Table 5. Data related to the 22 Iranian medical centers in summer 2016 (stage 2)**

DMU	$\frac{z_1^2}{z_1^1}$	$\frac{z_1^2}{z_2^1}$	$\frac{z_1^2}{x_2^1}$	$\frac{z_1^2}{x_2^2}$	$\frac{y_1^2}{x_1^2}$	$\frac{y_1^2}{x_2^2}$	$\frac{z_1^3}{x_1^3}$	$\frac{z_1^3}{x_2^3}$
1	0.43	0.30	0.25	0.18	0.09	0.07	0.22	0.13
2	0.24	0.83	0.24	0.42	0.17	0.29	0.19	0.09
3	1.80	0.04	0.26	0.12	0.26	0.12	0.34	0.15
4	0.05	0.30	0.15	0.11	0.25	0.18	0.44	0.34
5	0.33	0.17	0.13	0.39	0.15	0.45	0.17	0.14
6	1.07	0.79	1.25	0.83	0.58	0.39	0.30	0.17
7	1.40	2.33	0.20	0.17	0.14	0.12	1.42	0.49
8	0.09	0.17	0.17	0.18	0.33	0.35	0.82	0.15
9	0.19	0.38	0.38	0.20	0.92	0.48	0.83	0.35
10	2.25	0.75	0.24	0.41	0.49	0.82	0.68	0.53
11	0.01	0.13	0.09	0.06	0.21	0.13	1.36	1.06
12	0.60	0.33	0.04	0.06	0.07	0.10	0.69	0.12
13	1.14	1.33	0.57	0.67	0.21	0.25	0.18	0.14
14	0.26	1.57	0.61	2.20	0.06	0.20	1.17	0.82
15	2.17	2.60	0.37	0.18	0.06	0.03	1.06	1.13
16	0.91	0.56	0.11	0.09	0.05	0.04	0.85	0.61
17	0.20	0.28	0.22	0.07	0.15	0.04	0.76	0.68
18	0.03	0.44	0.20	0.44	0.34	0.78	0.70	0.56
19	0.20	0.75	0.13	0.36	0.26	0.76	0.23	0.17
20	0.08	0.33	0.24	0.42	0.05	0.08	0.04	0.17
21	6.00	1.50	0.20	0.10	0.08	0.04	0.18	0.28
22	0.12	0.17	0.31	0.25	0.06	0.05	0.27	0.42

**Table 6. Data related to the 22 Iranian medical centers in summer 2016 (stage 3)**

DMU	$\frac{z_1^3}{x_1^3}$	$\frac{z_1^3}{x_2^3}$	$\frac{y_1^3}{x_1^3}$	$\frac{y_1^3}{x_2^3}$	$\frac{y_1^3}{z_1^3}$	$\frac{z_1^3}{z_1^2}$	$\frac{y_1^3}{z_1^2}$
1	0.22	0.13	0.63	0.38	2.86	0.54	1.54
2	0.19	0.09	0.77	0.38	4.00	0.90	3.60
3	0.34	0.15	0.78	0.34	2.27	1.22	2.78
4	0.44	0.34	0.56	0.43	1.28	2.25	2.88
5	0.17	0.14	0.21	0.17	1.21	1.17	1.42
6	0.30	0.17	0.32	0.18	1.06	1.13	1.20
7	1.42	0.49	1.33	0.46	0.94	2.43	2.29
8	0.82	0.15	1.06	0.19	1.29	4.67	6.00
9	0.83	0.35	1.39	0.58	1.67	3.00	5.00
10	0.68	0.53	1.08	0.84	1.59	1.89	3.00
11	1.36	1.06	1.79	1.39	1.32	4.75	6.25
12	0.69	0.12	2.85	0.51	4.11	3.00	12.33
13	0.18	0.14	0.42	0.33	2.35	2.13	5.00
14	1.17	0.82	3.50	2.47	3.00	1.27	3.82
15	1.06	1.13	2.50	2.67	2.35	1.31	3.08
16	0.85	0.61	1.62	1.17	1.91	1.10	2.10
17	0.76	0.68	2.06	1.84	2.69	1.44	3.89
18	0.70	0.56	3.05	2.44	4.36	1.75	7.63
19	0.23	0.17	1.26	0.93	5.57	0.78	4.33
20	0.04	0.17	0.38	1.56	9.33	0.20	1.87
21	0.18	0.28	0.42	0.65	2.31	1.33	3.08
22	0.27	0.42	1.10	1.74	4.15	0.87	3.60

Therefore, we can easily evaluate the units using the suggested network DEA-R model. Using the structure of MOLP in three-stage network DEA-R has the following outcomes:

a) By considering  $P_1 = M_1$ ,  $P_2 = M_2$  and  $P_3 = M_3$  in model (17) where

$M_1 + M_2 + M_3 = 1$  and  $M_1, M_2, M_3 \in \{0,1\}$  efficiency value of each process would be calculated separately.

b) If  $M_1, M_2, M_3 \in \{0,1\}$  in model (17), we would arrive at the overall efficiency and the evaluation criterion would depend on the values of  $M_1$ ,  $M_2$  and  $M_3$ .

**Table 7. Comparison of efficiency values obtained from model (17) using different parameters**

DMU	Efficiency score (Stage 1)	Efficiency score (Stage 2)	Efficiency score (Stage 3)	$p_3 = 0.4$ $p_1 = p_2 = 0.3$	$p_1 = 0.8$ $p_2 = p_3 = 0.1$	$p_2 = 0.8$ $p_1 = p_3 = 0.1$	$p_3 = 0.8$ $p_1 = p_2 = 0.1$
	1	1	0.2556	0.2164	0.3011	0.6049	0.2709
2	1	0.6089	0.3365	0.5047	0.7928	0.5844	0.3786
3	1	0.6105	0.3283	0.4975	0.7884	0.5831	0.3702
4	1	0.4003	0.4737	0.528	0.7931	0.433	0.4905
5	1	0.61	0.2456	0.4132	0.7294	0.5499	0.284
6	0.9086	1	0.2386	0.4335	0.7145	0.7523	0.2807
7	0.9878	1	1	0.9963	0.9903	0.9988	0.9988
8	0.9974	0.5158	0.9825	0.7755	0.911	0.5705	0.9022
9	1	1	0.6876	0.8462	0.9566	0.9566	0.7335
10	0.3662	1	0.5206	0.5298	0.4038	0.7904	0.5236
11	1	0.2617	1	0.5416	0.7799	0.307	0.7799
12	1	0.3519	1	0.6441	0.8444	0.4043	0.8444
13	1	0.8564	0.5442	0.7218	0.9087	0.8211	0.5928
14	1	1	1	1	1	1	1
15	0.9689	1	1	0.9905	0.9749	0.9968	0.9968
16	1	0.3034	0.6591	0.5275	0.7805	0.3462	0.6085
17	1	0.2642	0.767	0.511	0.764	0.3069	0.6572
18	1	0.9706	1	0.991	0.997	0.9763	0.997
19	0.9343	1	0.4861	0.6925	0.8606	0.8987	0.5397
20	0.4877	0.272	0.5846	0.4162	0.4589	0.3014	0.5151
21	1	1	0.3975	0.6226	0.8684	0.8684	0.452
22	1	0.2607	0.6813	0.4907	0.7517	0.3016	0.6032

Model (17) is a linear programming problem;  $p_1$ ,  $p_2$  and  $p_3$  are determining parameters for overall efficiency of the three-stage network process.

Firstly, due to the priority of stage 1 compared to other stages,  $p_1=1$  and

$$p_2 = p_3 = 0, \text{ meaning that } \sum_{j=1}^n \lambda_j^1 = p_1 = 1.$$

Therefore, based on the categorization of  $I_1$ ,  $F_1$  &  $R_1$  for the first stage, we can

observe that units 1, 2, 3, 4, 5, 9, 11, 12, 13, 14, 16, 17, 18, 21 & 22 are all efficient; column 2 of Table 7 demonstrates this fact. If we only consider the second stage, units 6, 7, 9, 10, 14, 15, 19 & 21 are efficient according to Table 7's third column. However, as demonstrated in column 4, units 7, 11, 12, 14, 15 & 18 are efficient in stage 3. Obviously, unit 14 is efficient in all three stages. Therefore, we can conclude that

patients receive quality services in the surgery stage, but chemotherapy and radiotherapy stages lack proper service provision.

After a detailed study of stages 2 & 3, we arrived at the following issues:

a) Due to drug shortages and inability of patients to receive drugs (in terms of weak immune systems and allergic reactions to drugs), the chemotherapy stage requires special attention.

b) The services provided in the radiotherapy stage lack quality due to the high number of patients, their desire for liberation from the treatment process (pain & suffering during treatment & disease) and problems with radiotherapy devices.

Therefore, it is of great importance in this stage to fix the issues with radiotherapy machines, train the personnel to provide better services, convince the patients to stick to timetables in between radiotherapy sessions and make them aware of the dangers of radiotherapy.

Columns 5 to 8 from Table 7 present the overall efficiency obtained from model (17).

Four groups of weights were selected for the vector  $(p_1, p_2, p_3)$ :

**Group 1:** Equal priority for all three network stages with parameters  $(p_1, p_2, p_3) = (0.3, 0.3, 0.4)$ . Unit 14 is the only efficient one, however, units 7, 15 & 18 are closer to being efficient.

**Group 2:** Priority of the first stage in all three stages and finding the Pareto efficient solution using parameters  $(p_1, p_2, p_3) = (0.8, 0.1, 0.1)$ . Unit 14 is the only one efficient, however, units 7, 8, 9, 13 & 15 are closer to efficiency.

**Group 3:** Priority of the second stage in all three stages and finding the Pareto efficient solution using parameters  $(p_1, p_2, p_3) = (0.1, 0.8, 0.1)$ . But units 7, 9, 15 & 18 are closer to efficiency.

**Group 4:** Priority of the third stage in all three stages and finding the Pareto

efficient solution using parameters  $(p_1, p_2, p_3) = (0.1, 0.1, 0.8)$ . But units 7, 15 & 18 are closer to efficiency.

As can be witnessed in figures 5, 6 & 7, the first stage has similar behavior to the overall efficiency of the network, but stages 2 & 3 aren't similar, therefore the medical centers don't have an overall high efficiency. Since only unit 14 is efficient, the first stage has good efficiency but the second and third stages showed weak performances. Therefore, it's essential to revise the process of treatment in stages 2 & 3.

## 6. Conclusions

DEA-R models (a combination of DEA and ratio data) are used in data envelopment analysis for evaluation of decision-making units, when inputs and outputs are not available and we only have access to a defined ratio of data. In many DMUs, intermediate links play an important role in the network structure of DEA. Therefore, we evaluated 22 medical centers in this paper using network DEA-R models based on the structure of MOLP. Therefore, the reasons for using a three-stage network DEA-R model are as follows:

I) When  $x_j$ ,  $y_j$  and  $z_j$  are not available for  $DMU_j$  in all three-stages and we only

have access to a ratio of  $\frac{z_j}{x_j}$  and  $\frac{y_j}{z_j}$ ,

network DEA-R models can be a suitable alternative for network DEA.

II) We can obtain a ratio of the data at a very low cost; for instance, the ratio of recovered patients to all patients in each stage (surgery, chemotherapy & radiotherapy). Therefore, using the acquired ratio statistics for each stage, we can compare the centers' performances in each time period to the point which would improve their quality of services.

Thus, as can be seen in Table 7, 68% of the centers were efficient in stage 1, 31%



were efficient in stage 2 and 27% became efficient in stage 3. However, based on overall efficiency, unit 14 was the only one deemed efficient. With an optimistic view of model (17)'s optimal solutions (columns 5 to 8 of Table 7), we can say that units 6, 15 & 18 had proper performances in all three stages; on this basis, 18% of the units were approved in all stages based on overall efficiency value. Finally, our overall suggestions for the studied medical centers are as follows:

a) The first stage of treatment requires the most attentive service provision; most of the centers, except units 10 & 20, had an acceptable performance in the first stage (*Surgery*).

b) The second stage (*Chemotherapy*) requires even more attention in service provision. In this regard, units 6, 7, 9, 10, 14, 15 & 19 were efficient and units 1, 11, 12, 16, 17, 20 & 22 had weak performances. Those centers need to receive necessary training on service provision, patient guidance and drug prescription.

c) Overall, the centers had weaker performances in stage 3 (*Radiotherapy*) comparing to stages 1 & 2. Only units 7, 11, 12, 14, 15 & 18 were deemed efficient in this stage. The other units need to receive necessary training and revise their radiotherapy method as well as their methods for separating patients based on location of radiotherapy in the body, age requirements and types of disease. For future studies, we suggest determining medical centers' level of progress or regress based on the Malmquist index, rating units based on transverse efficiency and determining returns to scale based on DEA-R models.

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