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New Methods Based on Stability Versus Changes for Efficient Decision Making Units in Data Envelopment Analysis

S. Jafarigorzin*[1], A. Mirzanezhad[2], I. Asadi Talooki[3]

(1) The "khaje Nasir Toosi" University of Technology, Faculty of Electrical Engineering, Tehran, Iran

(2) The "Sharif" University of Technology, Faculty of Mechanical Engineering, Tehran, Iran

(3) Islamic Azad University, Faculty of Engineering, Sari Branch, Mazandaran, Iran

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Abstract

One of the drawbacks of Data Envelopment Analysis (DEA) is the problem of lack of discrimination among efficient Decision Making Units (DMU) and hence yielding many numbers of DMUs as efficient. The main purpose of this paper is to overcome this inability. In this paper, we explain new methods based on stability ranges for weights of inputs and outputs, these weights are the weights, that DMUs remain efficient. For illustration numerical example is given.

Keywords: Data Envelopment Analysis (DEA), Ranking Stability ranges

* Corresponding author: Email: Sorena.Jafari75@gmail.com

1. Introduction

Data Envelopment Analysis DEA is a fractional programming technique that has been developed by [1]. It is used to measure the productive efficiency their relative efficiencies. This analysis determines the productivities of DMUs, specified as the ratio of the weighted sum of outputs to the weighted sum of inputs, compares them to each other and determines the most efficient DMUs. One of the models for ranking efficient DMUs is cross efficiency evaluation [2]. The Cross Evaluation method was developed as a DEA extension that can be utilized to identify best performing DMUs and to rank DMUs using Cross Efficiency score [3]. The main idea of Cross Evaluation is to use DEA in a peer evaluation instead of a self-evaluation model. There are two principal advantages of Cross Evaluation: (1) It provides a unique ordering of the DMUs, and (2) IT eliminates unrealistic weight restrictions from application area experts [4]. The Cross Weight evaluation is one of the methods for ranking DMUs that was introduced for the first time [5].

2. Data Envelopment Analysis

Assuming that there are n DMUs each with m inputs and s outputs, the relative efficiency of a particular DMU_O $(o \in \{1,2,..,n\})$ is obtained by solving the following fractional programming problem:

$$
\begin{aligned}
&\theta_o = \max \frac{\sum_{r=1}^{s} u_r y_{or}}{\sum_{i=1}^{m} v_i x_{oi}} \\
&s.t. \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \qquad j = 1,2,\dots,n \qquad (1) \\
&u_r \ge 0 \qquad r = 1,2,3,\dots,s \\
&v_i \ge 0 \qquad i = 1,2,3,\dots,m
\end{aligned}
$$

Where j is the DMU index $j = 1,2,\dots,n$, r the output index, $r = 1,2,\dots,s$ and i the input index i=1,2,..., ,m, y_{rj} the value of the rth output for the jth DMU, x_{ij} the value of the i input for the jth DMU, u_r the weight given to the rth output, v_i the weight given to the i input. DMU_o is efficient if and only if $\theta_o = 0$ [6].

DMU_o selects weights that maximize its output to input ratio, subject to the constraints. A relative efficiency score of 1 indicates that the DMU under consideration is efficient, whereas a score less than 1 imply that it is inefficient. This fractional program can be converted into a linear programming problem where the optimal value of the objective function indicates the relative efficiency of DMU_O. The reformulated linear programming problem, also known as the Linear CCR model, is as follows [6]:

$$
\begin{aligned}
&\theta_o^* = \theta_o = \max \sum_{r=1}^{s} u_r y_{or} \\
&s.t. \sum_{i=1}^{m} v_i x_{oi} = 1 \qquad (2) \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\dots,n \\
&u_r \ge 0 \qquad r = 1,\dots,s \\
&v_i \ge 0 \qquad i = 1,\dots,m
\end{aligned}
$$

3. New Methods for Ranking Efficient Decision Making Units based on nonlinear and linear programming

First we solve model (2) for measuring efficiency of DMU_S. Let $N_0 = \{j | \theta_i^* = 1\}$. Now we consider the model (3) for evaluating of DMU_k, $(k \in N_o)$:

$$w_k = min(max\left\{\frac{z_{1k}}{g_{1k}}, \frac{z_{2k}}{g_{2k}}\right\}$$

subject to:

$$
\begin{aligned}
&\sum_{r=1}^{s} u_r y_k = 1 \qquad k \in N_o \\
&\sum_{r=1}^{s} v_i x_k = 1 \qquad k \in N_o \qquad (3) \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1,\dots,n, \; j \ne k \\
&V \le g_{1k} \le u_r \le z_{1k}, r = 1,\dots,s \\
&V \le g_{2k} \le v_i \le z_{2k}, \; i = 1,\dots,m
\end{aligned}
$$

We define
$d_k = [z_{1k} - g_{1k}], d'_k = [z_{2k} - g_{2k}], w = \left\{\frac{z_{1k}}{g_{1k}}, \frac{z_{2k}}{g_{2k}}\right\} \& \ h_k = (z_{1k} + z_{2l}) - (g_{1k} + g_{2k})$,
one of the logical criterions for ranking efficient DMUs is h_k. If $h_g > h_k$, DMU_g is better than DMU_k, because DMU_g is more stable than DMU_k versus changes. $g_{1k} \le u_r \le z_{1k}$ this range, is stable for weight of outputs and $g_{2k} \le v_i \le z_{2k}$ this range, is stable for weight of intputs. These weights are the weghts, that DMUs remain efficient. Now we solve Model (3) *l* times, each time for one DMU (efficient DMU). Model (3) is a nonlinear programming; we can convert to linear program. But, the other suggested model for ranking efficient DMUs:

$$\begin{aligned} &\mu_k = \max(u_1 d_k + u_2 d'_k) \\ &s.t. \\ &u_1 d_k + u_2 d'_k \le 1 \\ &u_1, u_2 \ge 0 \end{aligned} \tag{4}$$

This Model (Model (4)) is without input and two outputs, $\mu_g > \mu_k$, DMU_g is better than DMU_k and in the Ranking for DMUs, it has a better rank than DMU_g. You can see the result of new methods for ranking efficient DMUs in numerical example.

4. Example

Example 1: (Efficiency evaluation of six departments in a university):
The input-output variables for six departments in a university of defined as follows and the related data are given in Table (1).
X_1 number of academic staff
X_2 academic staff salaries in thousands of pounds
X_3 support staff salaries in thousands of pounds
Y_1 number of undergraduate's students
Y_2 number of postgraduate students
Y_3 number of research papers
The results of applying the models for the set of data are shown in Table (2), Table (3) and Table (4) respectively.

Table 1. Data of six departments in a university

Unit	X_1	X_2	X_3	Y_1	Y_2	Y_3
DMU_1	12	400	20	60	35	17
DMU_2	19	750	70	139	41	40
DMU_3	42	1500	70	225	68	75
DMU_4	45	2000	250	253	145	130
DMU_5	19	730	50	132	45	45
DMU_6	41	2350	600	305	159	97

Table 2. Result of the DEA model

Unit	Efficiency	V_1	V_2	V_3	u_1	u_2	u_3
DMU_1	1	0	0.0016	0.0016	0.0144	0.0039	0
DMU_2	1	0	0.0013	0.0013	0.0060	0.0039	0
DMU_3	1	0	0.0004	0.0004	0.0044	0	0
DMU_4	1	0	0.0004	0.0004	0	0.0028	0.0046
DMU_5	1	0	0.0012	0.0012	0	0.0085	0.0138
DMU_6	1	0.0073	0.0003	0.0003	0.0012	0.0033	0.0010

Table 3. Application from performance of new methods for ranking efficient DMUs=10^{-6}, At least weight for Outputs and inputs is

Unit	g_{2k}	z_{2k}	g_{1k}	z_{1k}	h_{1k}	*Ranking of DMUs*
DMU_1	0.0023	0.0023	0.0072	0.0127	0.0054	4
DMU_2	0.0000	0.0013	0.0019	0.0061	0.0055	3
DMU_3	0.0003	0.0069	0.0011	0.0031	0.0085	2
DMU_4	0.0000	0.0005	0.0008	0.0029	0.0026	5
DMU_5	0.0013	0.0013	0.0045	0.0045	0.0000	6
DMU_6	0.0000	0.0073	0.0010	0.0033	0.0096	1

5. Conclusion

The models suggest in this paper is used to rank and evaluate the efficiency of DMUs the advantage of the models over other models are on the calculation of stability ranges of inputs and outputs weights versus changes. The result seems to be logical and have economic and managerial interpretation.

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