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An Integrated Inventory Model With Agile Production Rate, Defective Items, Inflation, and Green Technology Investments for Deteriorating Products

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Abstract

The paper analyses an inventory model for make-to-order policy from the customer to the dealer. The production rate is variable to meet customers' demands in time and to control the emission of carbon units produced during the manufacturing process. The manufacturing process is not perfect and produces defective items depending on the production rate. The dealer invests in green technology to reduce the number of carbon units produced during different stages of manufacturing and storage. Products are deteriorating in nature and their demand is influenced by the selling price of the product and green technology investments. The effect of inflation is also considered in various costs to carry out the study. First, a mathematical model is developed with given constraints and then elaborated with a numerical example. The objective is to find out the optimum values of production time, cycle time, green technology cost, and product selling price to maximize the dealer's total profit. The model is further analyzed to check the effect of marginal changes in inventory parameters on the decision variables and the results are used to study managerial insights.

Keywords: Carbon emission, Imperfect production system, Deterioration, Green Efforts

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Introduction

Global warming is a matter of major concern and it has attracted the attention of the entire world. Un-compromising and unrestricted pollution is the cause of global warming and climate change is one of the major results of it. Everyday our industrial units release tons of harmful gases into the air and contribute a big amount in increase of carbon emissions. Aljazzar *et al*. [3] carried out a study to check the effect of delay in payments to reduce the units of carbon emission during the manufacturing process. Aliabadi *et al*. [2] considered sustainability issues and risk analysis for a system of non-instantaneous deteriorating products. It is necessary to control pollution to save the environment. Several counties have adopted the concept of renewable energy and green technology to reduce pollution. Mukhopadhyay and Goswami [9] developed the model by considering the pollution amount, pollution cost, and pollution reduction cost. Ghosh and Shah [7] were motivated by the firms taking initiatives in the reduction of carbon footprints and developed a model to explore supply chain coordination including green initiatives on the cost-sharing-based contract. Datta [5] investigated an inventory model with a flexible production rate. The study was carried out with consideration of carbon tax policy and green technology investments to obtain the decision policies.

Product quality depends on the manufacturing process and the quality of machines involved in the process. A higher production rate may be one of the reasons for defective production. However, a very low production rate is also not advisable to complete the order in-time. Production rate is also a deciding factor to control carbon emissions during the manufacturing process. Datta [4] worked out an inventory model with defective production. Sarkar *et al*. [12] developed an inventory model for defective products with an inspection and rework process. They considered variable transportation costs and obtained optimal strategies for the inventory system. Shah *et al*. [13] investigated an EPQ model considering production rate as a decision variable for returned/reworked inventory management systems. They considered an imperfect production system with a return-rework policy in the development of the model and obtained optimum strategies for the retailer. Sanjai and Periyasami [10] and Hasan *et al.* [8] also worked on a defective production system. They assumed that all the defective items are re-workable and considered possible shortages.

Inflation is also a factor that affects managerial decisions. Sarkar *et al*. [11] carried out a study on the economic manufacturing quantity model for an imperfect production system. Yadav *et al*. [17] considered the cost of holding the inventory under the effect of inflation and gave a genetic algorithm for a two-storage model for deteriorating items. Adak and Mahapatra [1] studied an inventory model for deteriorating items with consideration of delay in payments and the effect of inflation. Esmaeili and Nasrabadi [6] gave a single vendor multi-retailer supply chain model considering inflationary conditions and trade credit. Vandana *et al.* [16] investigated the impact of energy and carbon emissions in supply chain management. They also considered two-level trade credit policies in the development of the model. Shah *et al*. [14] investigated greening efforts on deteriorating inventory with price and stock-dependent product demand. Shah *et al*. [15] gave an inventory model for deteriorating products with reliability and inflation.

Though considerable research work has been carried out with different aspects, this study is distinguished from the other literature by considering all the aspects together to make the model more realistic. The present model intends to answer the following research questions:

1. What is the role of agile production rate in the inventory system?

2. How do green technology investments influence carbon emission levels and total profit?

3. What is the optimum production cycle time because of inflation?

With these focused questions, the present study is carried out for deteriorating products. The effect of inflation on various costs has been taken into account. The production system is not perfect and it produces defective items. The number of defective items increases with an increase in the production rate. The production rate is flexible and can be set at any value between the lower and the upper boundaries of the production rate. Product demand is sensitive to green investments and the selling price of the product. During the different stages of the production process and storage of inventory the system release carbon gases. This study considers the carbon-cap-trade policy. The dealer uses green technology investments to reduce the units of carbon emission. The objective of the study is to maximize the total profit of the dealer. A comparison of green investments versus without green investments has been worked out to check the fruitfulness of green technology investments.

2. Abbreviations and Assumptions

2.1 Abbreviations

The symbols used to develop the mathematical model are given in **Table-1.**

Table-1

R	Price-sensitive and green investments-dependent demand rate	δ	Rate of defective items during the production process
P	Effective production rate	λ	Production regulation factor $(0 \le \lambda \le 1)$
θ	Deterioration rate $(0 \le \theta \le 1)$	ψ	Inflation rate $(0 \le \psi \le 1)$
ξ	Greening investment cost per unit time	C	Production cost of product per unit
S	Set-up cost per order	Q	Order quantity
C_0	Total amount of carbon units emitted without green investments	Cg	Total amount of carbon units emitted with green investments
Cc	Carbon Capital	Ct	Carbon tax per unit of carbon emission
d	Fixed discount on defective products	h	Inventory storage cost/ unit per unit time
t_1	Production cycle time	T	Total cycle time (production and selling)
$I_1(t)$	Inventory level at any time t $(0 \le t \le t_1)$	p	Selling price (in \$)per unit
$I_2(t)$	Inventory level at any time t $(t_1 \le t \le T)$	TP	Average total profit (in \$) of dealer

2.2 Assumptions

(1) The product deteriorates at a constant rate of deterioration.

(2) The product demand is $R = a - bp + \eta\xi$. Here a is constant demand, $b(>0)$ is price elasticity factor, and $\eta(>0)$ is sensitivity factor associated with green investments.

(3) The production rate is flexible and can be set at any level within upper and lower machine capacities. It is given by $P = P_{\min} + (P_{\max} - P_{\min})(1-\lambda)$.

(4) Here $0 \le \lambda \le 1$ is the production regulation factor, $P_{\min}$ and $P_{\max}$ are the lower and the upper limit of machine production level respectively. Thus, $P_{\min} \le P \le P_{\max}$.

(5) The production process is not perfect. It produces defective items at the rate $\delta = \lambda_1 + \lambda_2 \frac{P - P_{\min}}{P_{\max} - P_{\min}}$ (Datta-2017). It is an increasing function of production rate. λ_1 and $\lambda_1 + \lambda_2$ are lower and upper bounds of defective products respectively.

(6) The system release carbon gases at three different stages. (i) U_0 units of carbon gases are emitted per cycle due to the set-up process. (ii) U_1 units are released per item during the production phase. Hence total emission in this phase is $U_1 \cdot P \cdot t_1$. (iii) U_2 units are released per unit time during storage of product. Hence total emission in this phase is $U_2 \cdot T$. Thus total carbon emission during all three phases is $C_0 = U_0 + U_1 P t_1 + U_2 T$.

(7) A fixed amount of carbon tax is levied for each unit of carbon gas. The dealer adopts a carbon-cap-trade policy to generate revenue by selling supplementary carbon units.

(8) The dealer uses green technology investments to reduce carbon emissions. Total greening investments per unit cycle is $\int_0^T \int_0^\xi m \cdot \xi \, d\xi \, dT = \frac{m \cdot \xi^2 \cdot T}{2}$. (Ghosh and Shah-[7])

(9) Effective carbon emission units under green investments are $CU = C_1 + (C_0 - C_1) e^{-m \cdot \xi}$. Here, $C_1 = k \cdot C_0$ is the minimum threshold value of carbon units under green investments with $0 \le k \le 1$.

(10) The dealer offers a fixed amount of discount on defective items and sells them at a lower selling price.

3 Research Methods

In this section, first we develop the mathematical model for the inventory system then we discuss the solution methodology to obtain the optimal solution.

3.1 Mathematical Model

Phase-I:

Production and sales phase $(0 \le t \le t_1)$

The production rate is higher than the demand and not perfect hence during this time interval,the inventory level is decreaseddue to demand and deterioration as shown below in equation (1)

$$\frac{d}{dt} I_1(t) = (1-\delta)P - R - \theta \cdot I_1(t) \, ; \; 0 \le t \le t_1 \qquad (1).$$

Equation (1) is a first-order linear ordinary differential equation with an initial condition $I_1(0) = 0$. Solving it we get inventory level at any time *t* is:

$$I_1(t) = \frac{(1-\delta)P - R}{\theta} \left[1 - e^{-\theta \cdot t} \right] \qquad (2)$$

Phase-II: Production and sales phase $(t_1 \le t \le T)$

In this phase, the inventory level decreases due to product demand and deterioration.It is governedby the differential equation as shown in equation (3).

$$\frac{d}{dt} I_2(t) = -R - \theta \cdot I_1(t) \, ; \; t_1 \le t \le T \qquad (3)$$

Equation (3) is a first-order linear ordinary differential equation with boundary condition $I_2(T) = 0$. Solving it we get inventory level at any time *t* is:

$$I_2(t) = \frac{R}{\theta} \left[e^{\theta(T-t)} - 1 \right] \qquad (4)$$

Using continuity of inventory level in the unit cycle we have, $I_1(t_1) = I_2(t_1)$. Using the relation and simplifying for *T* we get,

$$T = t_1 + \frac{\ln\left[\left\{(1-\delta)P - R\right\}\left\{1-e^{-\theta\cdot t_1}\right\} + R\right]}{\theta} \tag{5}$$

The total order quantity is $Q = RT$.
Various costs associated with the model are shown below.

(a) Set-up cost: $S\cdot e^{-\psi T}$

(b) Manufacturing cost: $\dfrac{C\cdot P\left(1-e^{-\psi\cdot t_1}\right)}{\psi}$

(c) Carbon tax cost: $Ct\cdot CU$

(d) Storage cost: $h\left[\int_0^{t_1} e^{-\psi t} I_1(t)\,dt + \int_{t_1}^{T} e^{-\psi t} I_2(t)\,dt\right]$

(e) Green investment cost: $\dfrac{m\cdot\xi^2\cdot T}{2}$

Hence the total cost is as shown in equation (6).

$$TC = S\cdot e^{-\psi T} + \frac{C\cdot P\left(1-e^{-\psi t_1}\right)}{\psi} + h\left[\int_0^{t_1} e^{-\psi t} I_1(t)\,dt + \int_{t_1}^{T} e^{-\psi t} I_2(t)\,dt\right] + \frac{m\cdot\xi^2\cdot T}{2} + Ct\cdot CU \tag{6}$$

Revenue generated through different sources is as shown below:

(a) From sales of perfect items: $p\cdot(1-\delta)\cdot Q$

(b) From sales of defective items: $(p-d)\cdot\delta\cdot Q$

(c) From sales of excess units of carbon gas: : $Ct\cdot(Cc-CU)$

Hence total revenue of the dealer is as shown in equation (7).

$$TR = p\cdot(1-\delta)\cdot Q + (p-d)\cdot\delta\cdot Q + Ct\cdot(Cc-CU)$$

Thus, the total profit of the dealer per unit cycle is as shown in equation (8).

$$TP = \frac{1}{T}\left[\begin{array}{l}\left\{p\cdot(1-\delta)\cdot Q + (p-d)\cdot\delta\cdot Q + Ct\cdot(Cc-CU)\right\} - \\ \left\{S\cdot e^{-\psi T} + \dfrac{C\cdot P\left(1-e^{-\psi\cdot t_1}\right)}{\psi} + h\left[\displaystyle\int_0^{t_1} e^{-\psi t} I_1(t)\,dt + \int_{t_1}^{T} e^{-\psi t} I_2(t)\,dt\right] + \dfrac{m\cdot\xi^2\cdot T}{2} + Ct\cdot CU\right\}\end{array}\right]$$

3.2 Solution Methodology

Step 1: Assign values to all inventory parameters other than decision variables.

Step 2: Obtain first-order partial derivatives of profit function and equate them to zero and obtain optimum values of decision variables p, t_1 and ξ respectively.

Step 3: Check the concavity of the profit function through the Hessian matrix H. The function is concave if $D_1 < 0, D_2 > 0$ and $D_3 < 0$. where,

$$H = \begin{bmatrix} \frac{\partial^2 TP}{\partial p^2} & \frac{\partial^2 TP}{\partial p\partial\xi} & \frac{\partial^2 TP}{\partial p\partial t_1} \\ \frac{\partial^2 TP}{\partial\xi\partial p} & \frac{\partial^2 TP}{\partial\xi^2} & \frac{\partial^2 TP}{\partial\xi\partial t_1} \\ \frac{\partial^2 TP}{\partial t_1\partial p} & \frac{\partial^2 TP}{\partial t_1\partial\xi} & \frac{\partial^2 TP}{\partial t_1^2} \end{bmatrix}, D_1 = \left|\frac{\partial^2 TP}{\partial p^2}\right|, D_2 = \begin{vmatrix} \frac{\partial^2 TP}{\partial p^2} & \frac{\partial^2 TP}{\partial p\partial\xi} \\ \frac{\partial^2 TP}{\partial\xi\partial p} & \frac{\partial^2 TP}{\partial\xi^2} \end{vmatrix} \text{ and } D_3 = \begin{vmatrix} \frac{\partial^2 TP}{\partial p^2} & \frac{\partial^2 TP}{\partial p\partial\xi} & \frac{\partial^2 TP}{\partial p\partial t_1} \\ \frac{\partial^2 TP}{\partial\xi\partial p} & \frac{\partial^2 TP}{\partial\xi^2} & \frac{\partial^2 TP}{\partial\xi\partial t_1} \\ \frac{\partial^2 TP}{\partial t_1\partial p} & \frac{\partial^2 TP}{\partial t_1\partial\xi} & \frac{\partial^2 TP}{\partial t_1^2} \end{vmatrix}.$$

Step 4: Substitute values of decision variables obtained above in equation (5) and equation (8) to get optimum values of cycle time and total profit respectively.

4. Numerical Example and Sensitivity Analyses

In this section, we illustrate with a numerical example and then we perform the sensitivity analyses.

4.1 Numerical Example

We provide a numerical example to explain and analyze the model numerically.

Example: Consider $S = \$\,100$ per order, $a = 400, b = 2.5, C = \$30$ per unit, $d = \$10$ per unit, $\psi = 0.1$ $h = \$0.05$ /unit/day, $\lambda_1 = 0.1, \lambda_2 = 0.2, P_{\max} = 800$, $P_{\min} = 500$, $U_0 = 10$, $U_1 = 0.02$, $U_2 = 1$, $Ct = 2$, $Cc = 200, \theta = 0.2$, $\eta = 0.1, \lambda = 1$, $k = 0.5$, $m = 0.5$.

We follow the solution methodology and use Maple-18 software to obtain the solution. The results obtained are: $p = \$82.29$, $t_1 = 10.67$ days, $GI = \$2508.9$

and $TP = \$12875.04$ Further, using the hessian matrix we get $D_1 = -5 < 0$, $D_2 = 2.49 > 0$ and $D_3 = -22.56 < 0$ that ensures the concavity.

4.2 Sensitivity Analyses

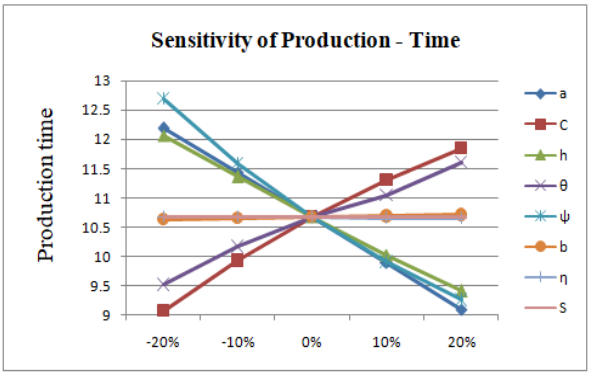
In this section first, we check the effect of agile production rate on different parameters by varying the value of the production regulation factor (λ). **Table-2** represents the effect of change in (λ) on production rate, defective production rate, total green investment cost, and total profit.

Table-2: Changes in various parameters with respect to change in production regulator factor

λ	P	δ	CU	GI	TP
0	800	0.30	127.90	\$2636.36	\$10992.89
0.25	725	0.25	114.94	\$2613.92	\$11458.57
0.50	650	0.20	102.44	\$2586.02	\$11926.90
0.75	575	0.15	90.41	\$2551.59	\$12398.46
1.00	500	0.10	78.83	\$2508.89	\$12875.04

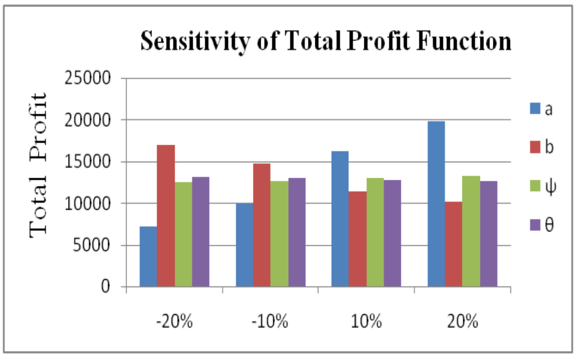
Next, we change other parameters by -20%, -10%, 10%, and 20% to check the behavior of the production time. The changes are shown in **Figure 1**.

Figure 1 Sensitivity of production time with respect to marginal changes in other parameters



The total profit function is highly sensitive to the demand parameters, deterioration rate, and inflation rate. Thus next we analyze the sensitivity of the total profit function for marginal changes in constant demand, price elasticity factor, deterioration rate, and inflation rate. It is as shown in **Figure 2**.

Figure 2 Sensitivity of total profit with respect to marginal changes in other parameters



5. Results

The following managerial aspects can be obtained from the sensitivity analyses.

- A higher production rate increases carbon emission units as well as defective products.
- Optimal production rate helps to reduce the number of defective products, carbon emission units as well as green investment costs and meet the product demand within time limit.
- Production time is very sensitive to the inflation rate. If the inflation rate is high then the production time decreases.
- The production time increases for higher rate of deterioration.
- Moreover, if the deterioration rate is higher, then the product gets spoiled rapidly. As a result, the inventory cycle time decreases.
- Cycle time shows a natural decrease with higher inventory costs.

- An increase in the constant product demand parameter results in increases in the total profit.
- Price elasticity factor, deterioration rate, and inflation rate are inversely proportional to the total profit function.
- By comparison of results in two different scenarios viz. 'with green technology investments' and 'without green technology investments', we found that it is fruitful to invest in green technology. The model has been studied for both cases and the results obtained are shown in **Table-3**.

Table-3: Comparison of results for greening investments versus without greening investments

Case	P	t_1	T	CU	GI	TP
With green technology	$82.30	10.67	40.87	78.83	$2508.90	$12875.04
Without green technology	$81.97	10.74	40.94	158.34	NA	$12806.20

6. Conclusion

This study is carried out for a deteriorating product-inventory system. The production rate is flexible and can be set at any level as per need between the machine limits. The dealer uses green technology investments to control carbon emissions. This is a defective production system where the defective production is directly proportional to the production rate. Product demand is price sensitive and affected by green investments. The study has been carried out by considering the effect of inflation. Results obtained show that a lower production rate not only reduces defective production but also helps to reduce carbon emissions and to increase the total profit of the dealer. Thus, production regulation is beneficial to control emission of carbon units. Inflation dominates the production time. Greening investments play a very important role in the inventory system as it helps to reduce carbon emission up to a certain level. Greening investments proved to be a handy tool to cut carbon tax costs and generate revenue through the sale of excess units of carbon. Finally, a comparison of the cases 'with green investments' and 'without green investments' have been worked out to elucidate the need for green technology.

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