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Closed formulas for the price and sensitivities of European options under a double exponential jump diffusion model

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Abstract

We derive closed formulas for the prices of European options and their sensitivities when the underlying asset follows a double-exponential jump diffusion model, as considered by S. Kou in 2002. This author has derived the option price by making use of double series where each term requires the computation of a sequence of special functions, such that the implementation remains difficult for a large part of financial users. Our present result provides an alternative to the Kou's formula easily to implement, even for the Excel/VBA environment.

Keywords: Options, Sensitivities, Jump diffusion models.

Introduction

The Gaussian hypothesis for financial returns is well-known for a long time to be a convenient assumption when the period considered for computation is large enough. However such a hypothesis should be rejected for high or medium frequencies. To better capture and exploit asset return patterns (as tail thicker than a normal distribution and time dependence), academic searchers and some pioneering practitioners model now the asset returns as following Levy processes. The classical arithmetic Brownian motion belongs to the large class of Levy processes which also includes the jump diffusion and Generalized Hyperbolic models (Variance Gamma, CGMY, . . .).

Merton [Me; 1976] has introduced for the first-time the jump diffusion in order to extend the Black and Scholes option pricing framework to include jumps occurrences. He has obtained explicit solution for option prices in term of series expansion when the jumps follow a Gaussian distribution. Later Kou [Ko; 2002] has also derived the European option price under a jump diffusion model associated to a double exponential distribution. The solution found by this last author is however little-bit difficult to implement for various users, as it is given in term of tail distribution which requires to value a double-series, where each term involves the computation of a sequence of some special functions.

Despite the complexities around the option pricing, as seen from [Me; 1976] and [Ko; 2002], the class of jump diffusion has been considered extensively now in theoretical and practical finance, probably for its simplicity and flexibility. As reported in a recent paper of Quittard-Pinon and Randrianarivony [Qu-Ra;2007], the solutions to pricing and hedging problems related to geometric Levy processes are rarely given in closed form. Very often the results are obtained through Monte-Carlo simulations or by numerical solutions to Partial-Integro-Differential-Equation (PIDE) or by the use of Fourier Analysis. These authors in [Qu-Ra;2007] has obtained an efficient method d through the Fast Fourier Transform (FFT) to implement the pricing of European options when the underlying asset is modeled by the process as considered by Kou and its extension. Recently Dia [Di; 2007] has been obtained European option price and the corresponding sensitivities in term of explicit Fourier series. In contrast to the Fourier transform based option pricing as seen in [Qu-Ra;2007], the formulas found by Dia involve only working with real numbers and are model independent in the sense that their structure remains unchanged for any payoff and any pricing model unlike existing pricing formulas.

Our first contribution in this paper is to derive a closed formula for a vanilla European option when the underlying asset follows a double-exponential jump diffusion models as considered first by Kou [Ko; 2002] and also recently by Quittard-Pinon& Randrianarivony [Qu-Ra;2007]. The option price we find here is given in term of a (real) Fourier integral which may be easily computed by any elementary quadrature integration method d, since the integrand is a smooth function (non singularity at zero and very fast decrease at infinity). The explicit formula obtained by Dia [Di; 2007], given in term of Fourier series, requires some finite truncation for numerical computation. However, the right order of truncation for a given error approximation is not well carried. With the integral approach, as is presented in this paper, the question of integral truncation arises. Here we are able to accurately estimate the error approximation when replacing the whole integral by an integral over a finite interval, as it is required in practical calculation. Since no Fourier Transform or FFT is needed, then the computation can be just implemented on VBA/Excel for which many practitioners in finance are well-familiarized. The execution time is noticeable, because the pricing of options with 5 00 different strikes requires around 2 seconds with a machine equipped with Intel Core, processor running at 2 Ghz and 2 Go of memory.

Our second contribution here is to provide explicit formulas for the option sensitivities which are also easy to implement. Each Greek parameter is obtained by performing a derivative of the option price formula with respect to the variable corresponding to the sensitivity under consideration.

Our price formula is obtained by exploring an idea by A. Lewis [Le;2001] related to option pricing via the generalized Fourier transform. The main point of our approach relies on making a fine analysis of the characteristic function associated to the log-asset process. The closed formulas, either for the option price or its sensitivities, found in this work may be extended for general jump diffusion (as we will perform in a further work). The finding in the present paper would give the incentive to make use of double exponential jump diffusion models as an interesting alternative to the (limited) Black and Scholes pricing used by a large part of academics and practitioners since the seventies.

The well-known pioneering option pricing results by Merton [Me; 1976] and Kou [Ko; 2002] are recalled in Section 2. We put here the emphasis on the implementation of these results.

For shortness the results related to Fourier transform as those found in [Di] and [Qu-Ra] are not reported here. Then we present, in Section 3, our main results about the option price and sensitivities in the case of a double exponential jump-diffusion mo del as considered by Kou. It appears here that our formulas are easy to perform in comparison of the other existing results [Me; 1976], [Ko; 2002], [Qu-Ra; 2007] and [Di; 2007]. In section 4, we will examine and solve the issue linked to the practical implementation of our formulas. Then we move in section 5 to some numerical examples, whose the corresponding tables will be displayed in the annex part. Our conclusion is given in section 6. Finally in section 8, which is the annex part, we present the ideas about the proofs of our results. For shortness the details of these last are not presented here but the willing reader may consult them, together with the MatLab co des, in the web site: www.ssrn.com.

2 Known results and notations

2.1 The Merton and Kou option pricing formulas

Under the objective probability measure p, the financial asset price St is assumed to follow the jump diffusion process defined by the stochastic differential equation

$$\frac{dS_t(\cdot)}{S_{t},-(\cdot)} = \mu dt + \sigma \quad dW_t(\cdot) + d\left[\sum_{i=1}^{N_t(\cdot)} \left\{\exp\left(J_i(\cdot)\right) - 1\right\}\right] \tag{1}$$

Here μ and σ are real constants, with 0 < σ, which represent respectively the drift and the volatility. Here W = $(W_t(\cdot))_t$ represents a standard Brownian motion, that is W_0 = 0 and $t^{-\frac{1}{2}} W_t(\cdot)$ follows the standard gaussian law with mean 0 and variance 1. Also N= $(N_t(\cdot))_t$ defines a Poisson process with rate λ, with rate 0 < λ, that is p $\left[\mathrm{N_t}(\cdot) = k\right] = \exp\left[-\lambda t\right] \frac{1}{k!} (\lambda t)^{k^t}$. The process $J = (J_1(\cdot), ..., J_i(\cdot), ..., ...)$ represents the asset price jump. The sources of randomness W, N and J are assumed to be independent. The convention $\sum_{i=1}^{0} = 0$ is used.

The sequence $J_1(\cdot), ..., J_i(\cdot), ..., ...$ is made by independent identically distributed random variables, which implies that each $J_i(\cdot)$ follows the same law as $J_1(\cdot)$. By the Ito Lemma, for diffusion processes with jump, and choosing a risk neutral probability Q (equivalent to p) under which the process exp[-rT]$S_t(\cdot)$ becomes a martingale, then it may be seen that necessarily $S_t(\cdot)$ takes the form

$$S_t(\cdot) = S \exp\left[\left(r - \frac{1}{2}\sigma^2 - \lambda k\right)t + \sigma W_t(\cdot) + \sum_{i=1}^{N_t(\cdot)} J_i(\cdot)\right] \tag{2}$$

Where

$$k \equiv E\left[\exp\left(J_t(\cdot)\right) - 1\right]$$

Merton [Me] has considered the case where $J_1(\cdot)$ follows a normal law with mean μ_J and variance σ_J^2, with $0 < \sigma_J$. In the sequel we refer this case as the MJD (Merton-Jump- Diffusion) model. A quasi explicit solution for options prices in a series expansion, due to Merton [Me; 1976], is available and states that

$$\textbf{Price\_Call\_MJD} = \exp\left[-(1+\kappa)\lambda\right] \times \tag{3}$$

$$\sum_{n=0}^{\infty} \frac{((1+\kappa)\lambda)^n}{n!} C_{bs}\left(S, \tau, \sqrt{\sigma^2 + \frac{n}{\tau}\sigma_J^2}, K, r - \lambda k + \frac{n}{\tau}\ln(1+k) \right)$$

Where $1 + k = \exp\left[\mu_J + \frac{1}{2}\sigma_J^2\right]$ and $C_{bs}(S, \tau, \sigma, K, r)$ denotes the well-known Black-Scholes call-price given by

$$C_{bs}(S, \tau, \sigma, K, r) = S\Phi(d) - K\exp\{-r\tau\}\Phi(d - \sigma\sqrt{\tau} \tag{4}$$

With $d = \frac{1}{\sigma\sqrt{\tau}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)\tau\right]$ and Φ denotes the CDF (cumulative distribution function) of the normal standard Gaussian law defined by

$$\Phi(x) = \int_x^{\infty} \exp[-\frac{1}{2}y^2]\frac{1}{\sqrt{2\pi}}dy \tag{5}$$

The Black-Scholes price (4) is recognized as given by closed-formula since it is just defined from the Gaussian CDF (5) whose an approximated value can be obtained by standard numerical integration (as the quadrature approach for instance) and implemented in usual softwares as Excel and MatLab. The Merton call-price **Price_Call_MJD**, as defined in (3), is more involved than the Black-Scholes price since it requires the valuation of a series involving infinite terms of Black-Scholes prices. In practical computation, we are just limited to consider a finite number of terms which implies for us to take care of the precision related to the approximated price calculated. This point seems not clarified in various literature making reference to the Merton price (3).

The call delta and gamma may be obtained by computing formally the derivatives from the price **Price_Call_MJD**, as defined in (3), however the differentiation deserves some justification, particularly for the vega, vanna and volga sensitivities. Moreover, in practical calculations, the term orders required for the truncations of the derivatives expressions are not so clear.

S. Kou [Kou; 2002] has considered also the option pricing when the underlying asset follows the jump diffusion process (2) in the case where the jump $J_1(\cdot)$ follows an asymmetric double exponential distribution with the density

$$f(y) = p\lambda_1 \exp[-\lambda_1 y] \quad for \;\; 0 < y \quad and \quad f(y) = (1-p)\lambda_2 \exp[\lambda_2 y] \quad for \;\; y < 0$$

It is assumed from now that

$$1 < \lambda_1, \qquad 0 < \lambda_2 \qquad and \qquad 0 \le p \le 1$$

such that p (resp. 1 - p) represents the probability of an upward (resp. downward) jump. S. Kou [Ko; 2002] have proved that the price of a European call option, with the time-to-maturity, exercise price K and underlying value S is

Price_Call_KJD =

$$S\gamma\left(r + \frac{1}{2}\sigma^2 - \lambda k, \sigma, (1+k)\lambda, \frac{p\lambda_1}{(1+k)(\lambda_1 - 1)}, \lambda_1 - 1, \lambda_2 + 1; \ln\left(\frac{K}{S}\right), \tau \right)$$

$$- K\exp[-r\tau]\gamma\left(r - \frac{1}{2}\sigma^2 - \lambda k, \sigma, \lambda, p, \lambda_1, \lambda_2; \ln\left(\frac{K}{S}\right), \tau \right) \qquad (6)$$

With

$$k = \frac{p\lambda_1}{\lambda_1 - 1} + \frac{(1-p)\lambda_2}{\lambda_2 + 1} - 1$$

And

$$\gamma(\mu,\sigma,\lambda,p,\lambda_1,\lambda_2;x,\tau)=P[\{x\le Z_\tau(\cdot)\}]$$

where $Z_\tau(\cdot)=\mu\tau+W_\tau(\cdot)+\sum_{i=1}^{N_t(\cdot)}J_i(\cdot)$. The full expression of the tail distribution is displayed *in* the annex part. It is written in term of double series where each term requires the computation of a sequence of special function, such that the code implementation is a little bit complicated in comparison with the Merton price **Price_Call_MJD**.

Computations of the call-option sensitivities from the price **Price_Call_KJD**, as written in (6), are not obvious. So the delta and gamma are derived in S. Kou, G. Petrella and H. Wang [Ko-Pe-Wa; 2005] by making use of the Laplace transform.

From now it is assumed that the current time is 0 and we consider a call-option with the strike price K, time-to-maturity T and written on asset modeled by a sto chastic process as given in (2). Let us denote by **Price_Call_KJD** the call price under the parameter Θ, with $\Theta=(\lambda,\lambda_1,\lambda_2,p)$. Such a price also depends on the values of T , S, K, σ , r, q and Θ. To reject such dependence we will write

Price_Call_KJD $\equiv$ **Price_Call_KJD(T , S, K ,** σ **, r, q,** Θ**).**

The right notation would be **Price_Call_KJD** (0, T, S, K, σ, r, q, Θ) if we would like to emphasize that it represents the present time 0 price of such a call. By the fundamental asset pricing theory, one has

(7)

Price_Call_KJD(T, S, K, , r, q,) = $\exp[-rT]E_Q\left[\max\{S_T(\cdot)-K;0\}\right]$

For a given call-option, the determining variables are the spot level S and the volatility σ.

Therefore we can also assume the existence of some regular two-variables function C such that

C(S, σ)=**Price_Call_KJD**(T, S, K, σ, r, q, Θ **)**

Recall that the sensitivities of the call-price with respect to these variables S and σ are given by the following expressions:

Delta_Call_KJD(T, S, K, σ , r, q, Θ)

$$\equiv C_S^{(1)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C(S+\varepsilon,\sigma) - C(S,\sigma)\}$$

Gamma_Call_KJD(T,S,K,σ ,r,q,Θ)

$$\equiv C_{SS}^{(2)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C_S^{(1)}(S+\varepsilon,\sigma) - C_S^{(1)}(S,\sigma)\}$$

Speed_Call_KJD(T,S,K,σ ,r,q,Θ)

$$\equiv C_{SSS}^{(3)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C_{SS}^{(2)}(S+\varepsilon,\sigma) - C_{SS}^{(2)}(S,\sigma)\}$$

Vega_Call_KJD(T,S,K,σ ,r,q,Θ)

$$\equiv C_{\sigma}^{(1)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C(S+\sigma,\varepsilon) - C(S,\sigma)\}$$

Venna_Call_KJD(T,S,K,σ ,r,q,Θ)

$$\equiv C_{S\sigma}^{(2)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C_{\sigma}^{(1)}(S+\varepsilon,\sigma) - C_{\sigma}^{(1)}(S,\sigma)\}$$

and

Volga_Call_KJD(T,S,K,σ ,r,q,Θ)

$$\equiv C_{\sigma\sigma}^{(2)}(S,\sigma) = \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \{C_{\sigma}^{(1)}(S,\sigma+\varepsilon) - C_{\sigma}^{(2)}(S,\sigma)\}$$

It may be noted here that we have not introduced the call-price sensitivity with respect to the passage of time, as is often the case in financial literature. Indeed as is seen in [Ra; 2009], the effect of a time-passage s, with $0 < s < T$ is exactly given by

Price_Call_KJD(T-s, S, K, σ , r, q, Θ) **- Price_Call_KJD**(T, S, K, σ , r, q, Θ),

So that for the derivative hedging viewpoint the main issue is to determine the sensitivities with respect to the spot and volatility valued at the point (T - s, S, K, σ , r, q, Θ). Except for the Speed case, here we limit our computations to the case of second order sensitivities. As considered in [Ra; 2009], when considering a constant volatility, the knowledge of the speed may be useful in controlling the hedging-error.

3 Main Results

In this section, we consider the Kou's framework Jump diffusion and provide the alternative pricing and sensitivities formulas for the results obtained in [Ko; 2002], [Qu-Ra; 2007] and [Di; 2007].

Proposition 1 (call price) *The price of the cal l is given by*

$$\textbf{Price\_Call\_KJD} \equiv \textbf{Price\_Call\_KJD}(T, S, K, \sigma, r, q, \lambda, \lambda_1, \lambda_2, p)$$

$$= A_1 S - A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_0(\ln S, \sigma) \quad (8)$$

where

$$A_1 \equiv A_1(T, q) = \exp[-qT] \quad (9)$$

(10)

$$A_2 \equiv A_2(T, K, r, q, \lambda, \lambda_1, \lambda_2, p) = \frac{1}{\pi} K^{\frac{1}{2}} \exp[-\frac{1}{2}(r + q + \lambda\{b-1\}T]$$

$$b \equiv b(\lambda_1, \lambda_2, p) = \frac{p\lambda_1}{\lambda_1 - 1} + \frac{(1-p)\lambda_2}{\lambda_2 + 1} \quad (11)$$

(12)

$$I_0(x, \sigma) \equiv I_0(x, \sigma; T, K, r, q, \lambda, \lambda_1, \lambda_2, p) = \int_0^\infty f_0(v, x) g_0(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

with

$$g_0(v) \equiv g_0(v; T, \lambda, \lambda_1, \lambda_2, p) \quad (13)$$

$$= \exp[\lambda T\{\beta_1(v) - 1\}] \frac{1}{\left(v^2 + \frac{1}{4}\right)}$$

$$\beta_1(v) \equiv \beta_1(v; \lambda_1, \lambda_2, p) \quad (14)$$

$$= \frac{p\lambda_1\left(\lambda_1 - \frac{1}{2}\right)}{\left(\lambda_1 - \frac{1}{2}\right)^2 + v^2} + \frac{(1-p)\lambda_2\left(\lambda_2 - \frac{1}{2}\right)}{\left(\lambda_2 - \frac{1}{2}\right)^2 + v^2}$$

(15)

$$f_0(v,x) \equiv f_0(v,x;T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \cos[h_0(v,x)]$$

$$h_0(v,x) \equiv h_0(v,x;T,K,r,q,\lambda,\lambda_1,\lambda_2,p)$$

(16)

$$= \{x + (r-q)T - \ln(K) - \lambda T(b-1) + \lambda T\beta_2(v)\}v$$

and

$$\beta_2(v) \equiv \beta_2(v;\lambda_1,\lambda_2,p)$$

$$= \frac{p\lambda_1}{\left(\lambda_1 - \frac{1}{2}\right)^2 + v^2} - \frac{(1-p)\lambda_2}{\left(\lambda_2 - \frac{1}{2}\right)^2 + v^2} \quad (17)$$

the call-put parity relation asserts that the difference between the price of a call and the corresponding put with the same strike K and time-to-maturity T is given by exp[-qT]S- Kexp[-rT]. As a consequence the price of the put is given by

Price_Call_KJD $\equiv$ **Price_Call_KJD**(T, S, K, $\sigma, r, q, \lambda, \lambda_1, \lambda_2, p$)

= **Price_Call_KJD** $- A_1 S + K\exp[-rT]$

$$= K\exp[-rT] - A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_0(\ln S, \sigma) \quad (18)$$

Proposition 2 (delta call) *the delta of the cal l is given by*

Delta_Call_KJD $\equiv$ **Delta_Call_KJD**(T, S, K, $\sigma, r, q, \lambda, \lambda_1, \lambda_2, p$)

$$= A_1 - A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{1}{2}} I_1(\ln S, \sigma) \quad (19)$$

where

(20)

$$I_1(x,\sigma) \equiv I_1(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^\infty f_1(v,x) g_1(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

with

$$f_1(v,x) \equiv f_1(v,x;T,K,r,q,\lambda,\lambda_1,\lambda_2,p)$$
$$= \frac{1}{2}\cos[h_0(v,x)] - v\sin[h_0(v,x)] \quad (21)$$

$$g_1(v) \equiv g_0(v;T,\lambda,\lambda_1,\lambda_2,p) \quad (22)$$

with $g_0(v;T,\lambda,\lambda_1,\lambda_2,p)$ *and* $h_0(v,x)$ *are defined respectively as in (13) and (16).*

Using this result and identity (18), it appears that the delta of the put is given by

Delta_Call_KJD $\equiv$ **Delta_Call_KJD**(T, S, K, $\sigma,r,q,\lambda,\lambda_1,\lambda_2,p$)

= Delta_Call_KJD $-A_1$

$$= A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{1}{2}} I_1(\ln S,\sigma) \quad (23)$$

Proposition 3 (gamma call) *the gamma of the call is given by*

Gamma_Call_KJD $\equiv$ **Gamma_Call_KJD**(T, S, K, $\sigma,r,q,\lambda,\lambda_1,\lambda_2,p$)

$$= A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{3}{2}} I_2(\ln S,\sigma) \quad (24)$$

where

(25)

$$I_2(x,\sigma) \equiv I_2(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^\infty f_2(v,x) g_2(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

with

(26)

$$f_2(v,x) \equiv f_0(v,x;T,K,r,q,\lambda,\lambda_1,\lambda_2,p)$$

(27)

$$g_2(v) \equiv g_2(v;T,\lambda,\lambda_1,\lambda_2,p) = \exp[\lambda T\{\beta_1(v)-1\}]$$

with $f_0(v,x;T,K,\tau,q,\lambda,\lambda_1,\lambda_2,p)$ *is defined respectively as in (15).*

It may be noted that

Gamma_Call_KJD (28) ≡ **Gamma_Call_KJD**

Proposition 4 (speed call) *the speed of the call is given by*

$$\textbf{Speed\_Call\_KJD} \equiv \textbf{Speed\_Call\_KJD}(\mathrm{T, S, K}, \sigma, r, q, \lambda, \lambda_1, \lambda_2, p)$$

$$= -A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{5}{2}} I_3(\ln S, \sigma) \qquad (29)$$

where

$$I_3(x,\sigma) \equiv I_3(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^\infty f_3(v,x) g_3(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv \qquad (30)$$

$$f_3(v,x) \equiv f_3(v,x;T,K,r,q,\lambda,\lambda_1,\lambda_2,p)$$

$$= \frac{3}{2}\cos[h_0(v,x)] + v\sin[h_0(v,x)] \qquad (31)$$

and

$$g_3(v) = g_2(v;T,\lambda,\lambda_1,\lambda_2,p) \qquad (32)$$

with $g_2(v;T,\lambda,\lambda_1,\lambda_2,p)$ *is defined respectively as in (27).*

It may be noted that

Speed_Put_KJD (33) ≡ **Speed_Put_KJD**

Proposition 5 (vega call) *The vega of the call is given by*

$$\textbf{Vega\_Call\_KJD} \equiv \textbf{Vega\_Call\_KJD}(\mathrm{T, S, K}, \sigma, r, q, \lambda, \lambda_1, \lambda_2, p)$$

$$= A_2 \sigma T \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_4(\ln S, \sigma) \qquad (34)$$

where

$$I_4(x,\sigma) \equiv I_4(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^\infty f_0(v,x) g_4(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv \qquad (35)$$

and

$$g_4(v) = g_2(v; T, \lambda, \lambda_1, \lambda_2, p) \quad (36)$$

with $f_0(v, x)$ *and* $g_2(v; T, \lambda, \lambda_1, \lambda_2, p)$ *are defined as in (15). And (27) respectively.*
Clearly it appears that

Vega_Call_KJD = Vega_Call_KJD (37)

Proposition 6 (Volga call) the Volga of the call is given by

Vega_Call_KJD $\equiv$ **Vega_Call_KJD**(T, S, K, $\sigma, r, q, \lambda, \lambda_1, \lambda_2, p$)

$$= A_2 T \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_5(\ln S, \sigma) \quad (38)$$

where

$$I_5(x,\sigma) \equiv I_5(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^\infty f_0(v,x) g_5(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv \quad (39)$$

and

$$g_5(v) \equiv g_2(v; T, \sigma, \lambda, \lambda_1, \lambda_2, p)\left\{1 - \sigma^2 T\left(\frac{1}{4} + v^2\right)\right\} \quad (40)$$

with $f_0(v, x)$ *and* $g_2(v; T, \lambda, \lambda_1, \lambda_2, p)$ *are defined as in (15). And (27) respectively.*
It may be noted that

Vega_Call_KJD = Vega_Call_KJD (41)

Proposition 7 (Vanna call) *The vanna of the call is given by*

Venna_Call_KJD $\equiv$ **Venna_Call_KJD** (T, S, K, $\sigma, r, q, \lambda, \lambda_1, \lambda_2, p$)

$$= A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{1}{2}} I_6(\ln S, \sigma) \quad (42)$$

where

(43)

$$I_6(x,\sigma) \equiv I_6(x,\sigma,T,K,r,q,\lambda,\lambda_1,\lambda_2,p) = \int_0^{\infty} f_1(v,x) g_6(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

and

$$g_6(v) \equiv g_1(v;T,\lambda,\lambda_1,\lambda_2,p)\left(\frac{1}{4}+v^2\right) \quad (44)$$

with $f_1(v,x)$ *and* $g_1(v;T,\lambda,\lambda_1,\lambda_2,p)$ *are defined as in (15). And (27) respectively.*
It is clear that

Vanna_Call_KJD = Vanna_Call_KJD (45)

All of these Propositions may be seen as particular cases of the general results we are now presenting in the next Section.

4 On numerical computations
4.1 Approximated formulas

To perform the call-price or its sensitivities, from the formulas stated in the previous section, we are reduced to compute (improper integrals) as

$$I(x,\sigma) = \int_0^{\infty} f(v,x) g(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

Actually, for the practical viewpoint, we can only do numerical computation of proper integral over finite interval as

$$I(x,\sigma,M) = \int_0^{M} f(v,x) g(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$$

for some nonnegative real number M sufficiently large in order that the remaining integral term $\int_M^{\infty} f(v,x) g(v) \exp[-\frac{1}{2}\sigma^2 T v^2] dv$ is small enough and can be neglected.

The approximated values of **Price_Call, Delta_Call, Gamma_Call, Speed_Call, Vega_Call, Volga_Call and Vanna_Call are** de_ned respectively by

(46)

$$\textbf{Price\_Call}_{\mathrm{M}} = A_1 S - A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_0\left(\ln S, \sigma, M\right)$$

(47)

$$\textbf{Delta\_Call}_{\mathrm{M}} = A_1 - A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{1}{2}} I_1\left(\ln S, \sigma, M\right)$$

(48)

$$\textbf{Gamma\_Call}_{\mathrm{M}} = A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{3}{2}} I_2\left(\ln S, \sigma, M\right)$$

$$\textbf{Speed\_Call}_{\mathrm{M}} = -A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{-\frac{5}{2}} I_3\left(\ln S, \sigma, M\right) \quad (49)$$

(50)

$$\textbf{Vega\_Call}_{\mathrm{M}} = (\sigma T) A_2 \exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} I_4\left(\ln S, \sigma, M\right)$$

(51)

$$\textbf{Volga\_Call}_{\mathrm{M}} = (TA_2)\exp[-\frac{1}{8}\sigma^2 T] S^{\frac{1}{2}} \left\{I_4\left(\ln S, \sigma, M\right) - (\sigma^2 T) I_5\left(\ln S, \sigma, M\right)\right\}$$

and

(52)

$$\textbf{Vanna\_Call}_{\mathrm{M}} = (\sigma \quad T) A_2 \exp[-\frac{1}{8}\sigma^2 T] \sigma T S^{-\frac{1}{2}} I_6\left(\ln S, \sigma, M\right)$$

We introduce the approximation errors

$$\textbf{error\_Price\_Call}_{\mathrm{M}} = \left|\textbf{Price\_Call\_Price\_Call}_{\mathbf{M}}\right|$$

$$\textbf{error\_Delta\_Call}_{\mathrm{M}} = \left|\textbf{Delta\_Call\_Delta\_Call}_{\mathbf{M}}\right|$$

$$\textbf{error\_Gamma\_Call}_{\mathrm{M}} = \left|\textbf{Gamma\_Call\_Gamma\_Call}_{\mathbf{M}}\right|$$

$$\textbf{error\_Speed\_Call}_{\mathrm{M}} = \left|\textbf{Speed\_Call\_Speed\_Call}_{\mathbf{M}}\right|$$

$$\mathbf{error\_Vega\_Call}_{\mathrm{M}} = \left|\mathbf{Vega\_Call\_Vega\_Call}_{\mathbf{M}}\right|$$

$$\mathbf{error\_Volga\_Call}_{\mathrm{M}} = \left|\mathbf{Volga\_Call\_Volga\_Call}_{\mathbf{M}}\right|$$

and

$$\mathbf{error\_Vanna\_Call}_{\mathrm{M}} = \left|\mathbf{Vanna\_Call\_Vanna\_Call}_{\mathbf{M}}\right|$$

The question, when making use of approximations $\mathbf{Price\_Call}_{\mathrm{M}}$, $\mathbf{Delta\_Call}_{\mathrm{M}}, \mathbf{Gamma\_Call}_{\mathrm{M}}, \ldots$ and so on, is reduced to determine the nonnegative real M such that the errors $\mathbf{error\_Price\_Call}_{\mathrm{M}}, \mathbf{error\_Delta\_Call}_{\mathrm{M}}, \mathbf{error\_Gamma\_Call}_{\mathrm{M}}, \ldots$ do not exceed a given size the user can tolerate. Of course we should be aware that the computation of each definite integral $I(x,\sigma,M) = \int_0^M f(v,x)g(v)\exp[-\frac{1}{2}\sigma^2 T v^2]dv$ itself suffers from usual numerical error. For shortness, let us introduce the expression

$$\|A\|_\infty = \lambda\left\{\frac{p\lambda_1}{\left(\lambda_1 - \frac{1}{2}\right)} + \frac{(1-p)\lambda_2}{\left(\lambda_2 - \frac{1}{2}\right)} + 1\right\} \tag{53}$$

Proposition 8 *For the* $\mathbf{error\_Price\_Call}_{\mathrm{M}}, \mathbf{error\_Delta\_Call}_{\mathrm{M}}$, $\mathbf{error\_Gamma\_Call}_{\mathrm{M}}$, and $\|A\|_\infty$ *as defined above then we have the following estimates*

$$\mathbf{error\_Price\_Call}_{\mathbf{M}}$$

$$\le A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{\frac{1}{2}} \int_M^\infty \frac{1}{\left(\frac{1}{4} + v^2\right)} \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \tag{54}$$

$$\mathbf{error\_Delta\_Call}_{\mathbf{M}}$$

(55)

$$\leq A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{-\frac{1}{2}} \int_M^\infty \frac{\left(\frac{1}{2}+v\right)}{\left(\frac{1}{4}+v^2\right)} \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv$$

error_Gamma_Call$_\mathbf{M}$

(56)

$$\leq A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{-\frac{3}{2}} \int_M^\infty \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv$$

error_Speed_Call$_\mathbf{M}$

(57)

$$\leq A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{-\frac{5}{2}} \int_M^\infty \left(\frac{3}{2}+v\right) \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv$$

error_Vega_Call$_\mathbf{M}$

$$\leq (\sigma T) A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{\frac{1}{2}} \int_M^\infty \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \qquad (58)$$

error_Volga_Call$_\mathbf{M}$

(59)

$$\leq T A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{\frac{1}{2}} \int_M^\infty \left\{1 + (\sigma^2 T)\left(\frac{\frac{1}{4}+v^2}{\frac{1}{4}}\right)\right\} \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv$$

And

error_Vanna_Call$_\mathbf{M}$

(60)

$$\leq (\sigma T) A_2 \exp\left[\left(-\frac{1}{8}\sigma^2 + \|A\|_\infty\right)T\right] S^{-\frac{1}{2}} \int_M^\infty \left(\frac{1}{2}+v\right) \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv$$

To benefit from this result, we have to fine (easily computable) high bounds for these infinite integrals involved from (54) to (60). To this end, the following result is available.

Proposition 9 Let 0<M. *Then we have*

$$\int_M^\infty \frac{1}{\left(\frac{1}{4}+v^2\right)} \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \le \pi \exp\left[-\frac{1}{2}\sigma^2 T M^2\right] \tag{61}$$

For $\frac{\sqrt{2}}{\sigma\sqrt{T}} \le M$ *one has*

$$\int_M^\infty \frac{\left(\frac{1}{2}+v\right)}{\left(\frac{1}{4}+v^2\right)} \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \le c_1 \pi \exp\left[-\frac{1}{4}\sigma^2 T M^2\right]$$

with $c_1 = \left\{\frac{1}{2} + \frac{\sqrt{2}}{\sigma\sqrt{T}}\right\} \exp\left[-\frac{1}{2}\right]$ (62)

For $\frac{2}{\sigma\sqrt{T}} \le M$ *one has*

$$\int_M^\infty \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \le c_2 \pi \exp\left[-\frac{1}{4}\sigma^2 T M^2\right]$$

with $c_2 = \left\{\frac{1}{4} + \frac{4}{\sigma^2 T}\right\} \exp[-1]$ (63)

For $\frac{\sqrt{6}}{\sigma\sqrt{T}} \le M$ *one has*

$$\int_M^\infty \left(\frac{3}{2}+v\right) \exp\left[-\frac{1}{2}\sigma^2 T v^2\right] dv \le c_3 \pi \exp\left[-\frac{1}{4}\sigma^2 T M^2\right] \tag{64}$$

with $c_3 = \left\{ \frac{3}{8} + \frac{6\sqrt{6}}{(\sigma\sqrt{T})^3} \right\} \exp\left[-\frac{3}{2} \right] + \frac{\sqrt{2}}{4\sigma\sqrt{T}} \exp\left[-\frac{1}{2} \right] + \frac{6}{\sigma^2 T} \exp[-1]$

For $\frac{2\sqrt{2}}{\sigma\sqrt{T}} \le M$ *one has*

$$\int_M^\infty \left(\frac{1}{4} + v^2 \right) \exp\left[-\frac{1}{2}\sigma^2 T v^2 \right] dv \le c_4 \pi \exp\left[-\frac{1}{4}\sigma^2 T M^2 \right]$$

with $c_4 = \left\{ \frac{1}{16} + \frac{64}{(\sigma\sqrt{T})^4} \right\} \exp[-2] + \frac{2}{\sigma^2 T} \exp[-1]$ (65)

For $\frac{\sqrt{6}}{\sigma\sqrt{T}} \le M$ *one has*

$$\int_M^\infty \left(\frac{1}{2} + v \right) \exp\left[-\frac{1}{2}\sigma^2 T v^2 \right] dv \le c_5 \pi \exp\left[-\frac{1}{4}\sigma^2 T M^2 \right]$$

(66)

with $c_5 = \left\{ \frac{1}{8} + \frac{6\sqrt{6}}{(\sigma\sqrt{T})^3} \right\} \exp\left[-\frac{3}{2} \right] + \frac{\sqrt{2}}{4\sigma\sqrt{T}} \exp\left[-\frac{1}{2} \right] + \frac{6}{\sigma^2 T} \exp[-1]$

Each estimates in this Last Proposition is not sharp, as we may see in its pro of, in the sense that the constant π may be replaced by $\frac{1}{M}$. However, for large values of M, the most important term is $\exp\left[-\frac{1}{4}\sigma^2 T M^2 \right]$ which decreases very fast to 0 rather than $\frac{1}{M}$. We keep the constant π to ease the determination of explicit values of M in the sequel.

Proposition 8 and 9 are useful tools for practical computations of the call price and its sensitivities. For instance to get **Price_Call** from its approximate value **Price_Call**$_M$ for M such that the absolute error **error_Price_Call**$_M$ remains under some given nonnegative $\varepsilon(ax \quad \varepsilon = 10^{-10}$ for example), then is sufficient to choose the real Ma such that $c\exp\left[-\frac{1}{2}\sigma^2TM^2\right]<\varepsilon$ with $c=\pi A_2\exp\left[\left(-\frac{1}{8}\sigma^2+\|A\|_\infty\right)T\right]S^{\frac{1}{2}}$. Any nonnegative M is suitable when $c\le\varepsilon$. For $1<\frac{c}{\varepsilon}$ then we can take M such $\sqrt{\frac{2}{\sigma^2T}\ln\left[\frac{c}{\varepsilon}\right]}<M$.

Similarly to perform **Delta_Call** ≈ **Delta_Call**$_M$ with **error_Price_Call**$_M$ < ε, for some given nonnegative ε, then we have to take M, with $\frac{\sqrt{2}}{\sigma\sqrt{T}}\le M$, and such that $c_1'\exp\left[-\frac{1}{4}\sigma^2TM^2\right]<\varepsilon$ where $c_1'=\pi\quad c_1A_2\exp\left[\left(-\frac{1}{8}\sigma^2+\|A\|_\infty\right)T\right]S^{-\frac{1}{2}}<\varepsilon$. Any nonnegative M, with $\frac{\sqrt{2}}{\sigma\sqrt{T}}\le M$, is suitable for $c_1'\le\varepsilon$. For $1<\frac{c_1'}{\varepsilon}$ then we can take M such $\max\left\{\frac{\sqrt{2}}{\sigma\sqrt{T}},\sqrt{\frac{4}{\sigma\sqrt{T}}\ln\left[\frac{c_1'}{\varepsilon}\right]}\right\}<M$.

4.2 Numerical Value of M

We consider the price error approximation in the Kou's setting where S = 100, K = 100, r = 0.05, q = 0, σ = 0.16, λ = 1, λ_1 = 10, λ_2 = 5, p = 0.4 and ε = 10 12. Therefore one has

$$\mathbf{Price\_Call}_M\approx A_1S-A_2\exp[-\frac{1}{8}\sigma^2T]S^{\frac{1}{2}}I_0(\ln S,\sigma,M)$$

and the value of M depends on the time-to-maturity T as we can observe from the following table

T	0.25	0.50	0.75	1
M	102	73	60	52

4.3 Computation execution time

The Fast Fourier Transform pricing, as considered by various authors as in [Qu-Ra] is seen to be computationally fast when willing to generates a matrix of prices with different strikes.
However this approach suffers from accuracy. Here our formulas involve the computation of Fourier integral, and we are able to have a good control on the calculation error, because of our Propositions 8 and 9. Moreover the computation execution time is also very fast if we just have to deal with few options together. As an illustration, we consider the price error approximation in the Kou's setting with S = 100, r = 0.05, q = 0, σ = 0.16, λ = 1, λ_1= 10, λ_2=5, p=0.4 and $\varepsilon = 10^{-12}$. Making calculation with a machine equipped with Intel (R) Core (TM) and processor running at 2.00 GHZ and 2.00 Go of memory, the following result has been obtained

nb of strikes	Execution time (second)
2	0.0198
40	0.1878
100	0.4605
500	2.0135
5000	22.0080

5 Numerical examples

As an illustration of our results, we will focus on the setting of Kou with S = 100, r = 0.05, q = 0, σ = 0.16, λ = 1, λ_1= 10, λ_2=5 and p = 0.4. The results obtained are summarized over table 8.3.1 to table 8.3.6 presented in the Annex part.

6 Conclusion

1. In this paper we have derived a closed formula for a vanilla European option when the underlying asset follows a double exponential jump diffusion models, as considered first by Kou [Ko; 2002] and also recently by Quittard-Pinon & Randrianarivony [Qu-Ra;2007]. The option price we obtain is given in term of a (real) Fourier integral which may be easily computed by any elementary quadrature integration method, since the integrand is a smooth function (non singularity at zero and very fast decrease at infinity). Moreover we are able to accurately estimate the error approximation resulting from replacing the whole integral by an

integral over a finite interval. Since no Fourier Transform or FFT is needed, then the computation can be just implemented on VBA/Excel for which many practitioners in finance are well-familiarized. Our other contribution is about providing explicit formulas for the option sensitivities which are also easy to implement by a large part of practitioners. The closed formulas, either for the option price or its sensitivities, found in this work would be a new impetus on the use of double exponential jump diffusion model as an interesting alternative to the (limited) Black and Scholes pricing used by a large part of academics and practitioners in finance since the seventies.

2. Our present work may be extended to the pricing of European option when the underlying asset belongs a general class of jump diffusion models. We hope to perform such an investigation for a near future investigation.

3. The work in this paper is focused only on the price and sensitivities for vanilla European option under jump diffusion models. However it should be noted that in exchange markets most of available options are of American-style. Not only the results obtained in this paper are interesting for their own-sake but they may be very useful since, the pricing of an American-style option may be reduced to the pricing of suitable European call options as is shown and performed in [La-Fu,Ma,Li,Zh;2005].

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8 Annex

8.1 The Kou's pricing formulas

The full expression of $\gamma(\mu,\sigma,\lambda,\lambda_1,\lambda_2;x,\tau)$ on which the Kou's formula is as follows:

$$\gamma(\mu,\sigma,\lambda,\lambda_1,\lambda_2;x,\tau) =$$

$$\frac{\exp\left[\frac{1}{2}(\sigma\lambda_1)^2\tau\right]}{\sigma\sqrt{2\pi\tau}}\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}(\sigma\sqrt{\tau}\lambda_1)^k I_{k-1}\left(x-\mu\tau;-\lambda_1,-\frac{1}{\sigma\sqrt{\tau}},-\sigma\lambda_1\sqrt{\tau}\right)P_{n,k}$$

$$+\frac{\exp\left[\frac{1}{2}(\sigma\lambda_1)^2\tau\right]}{\sigma\sqrt{2\pi\tau}}\sum_{n=1}^{\infty}\pi_n\sum_{k=1}^{n}(\sigma\sqrt{\tau}\lambda_2)^k I_{k-1}\left(x-\mu\tau;\lambda_2,-\frac{1}{\sigma\sqrt{\tau}},-\sigma\lambda_2\sqrt{\tau}\right)Q_{n,k}$$

(67)

$$+\pi_0\Phi\left(-\frac{x-\mu\tau}{\sigma\sqrt{\tau}}\right)$$

$$\pi_n = \exp[-\lambda\tau]\frac{1}{n!}(\lambda\tau)^n$$

Here $P_{n,n} = p^n, Q_{n,n} = (1-p)^n$ and for $k \in \{1, n-1\}$

$$P_{n,k} = \sum_{i=k}^{n-1} C_{n-k-1}^{i-k} C_n^i \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^{i-k}\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-i} p^i(1-p)^{n-i}$$

$$Q_{n,k} = \sum_{i=k}^{n-1} C_{n-k-1}^{i-k} C_n^i \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right)^{n-i} \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right)^{i-k} p^{n-i} (1-p)^i$$

The sequence of function I_m is defined, for $0 < \beta, \alpha \neq 0$ and for all $-1 \leq m$

$$I_m(c;\alpha,\beta,\delta) = -\frac{\exp\begin{bmatrix} \alpha & c \end{bmatrix}}{\alpha} \sum_{i=0}^{m} (\frac{\beta}{\alpha})^{m-i} Hh_i(\beta c - \delta)$$
$$+\left(\frac{\beta}{\alpha}\right)^{m+1} \left(\frac{\sqrt{2\pi}}{\beta}\right) \exp\left(\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}\right) \Phi\left(-\beta c + \delta + \frac{\alpha}{\beta}\right)$$

And for $\beta < 0, \alpha < 0$ and for all $-1 \leq m$

$$I_m(c;\alpha,\beta,\delta) = -\frac{\exp\begin{bmatrix} \alpha & c \end{bmatrix}}{\alpha} \sum_{i=0}^{m} (\frac{\beta}{\alpha})^{m-i} Hh_i(\beta c - \delta)$$

$$-\left(\frac{\beta}{\alpha}\right)^{m+1} \left(\frac{\sqrt{2\pi}}{\beta}\right) \exp\left(\frac{\alpha\delta}{\beta} + \frac{\alpha^2}{2\beta^2}\right) \Phi\left(\beta c - \delta - \frac{\alpha}{\beta}\right)$$

Here Hh is defined

$$Hh_{-1}(x) = \exp[-0.5x^2]$$
$$Hh_0(x) = \sqrt{2\pi}\Phi(-x)$$

And for $1 \leq n$ then

$$Hh_n(x) = \frac{1}{n}\{Hh_{n-2}(x) - xHh_{n-1}(x)\}$$

8.2 Some words about Proofs

For shortness we do give here the proof details, but promise to post them together with the MatLab codes in the web site www.ssrn.com.

Proposition 1 may be obtained from the following formula due to A. Lewis [Le;2001] (in p.14)

Price_Call = exp [-qT]S

(68)

$$-\frac{1}{\pi}K^{\frac{1}{2}}S^{\frac{1}{2}}\exp\left[-\frac{1}{2}(r+q)T\right]\int_0^{\infty}R_e\left\{\exp[kui]\phi\left(u-\frac{i}{2};X_T\right)\right\}\frac{1}{\left(\frac{1}{4}+u^2\right)}du$$

where

$$k = x-\ln[K]+T(r-q) \qquad with \quad x=\ln[S]$$

ϕ is the characteristic function associated to the process

$$X_t(\cdot)=\ln\left[\frac{S_t(\cdot)}{S}\right]-(r-q)t=\omega \quad t+\sigma W_t(\cdot)+\sum_{j=1}^{N_t(\cdot)}J_i(\cdot)$$

that is

$$\phi(u;X_t)=E\big[\exp\big[X_t(\cdot)ui\big]\big]=\exp\big[t\psi(u)\big] \tag{69}$$

with

(70)

$$\psi(u)=\omega \quad ui-\frac{1}{2}\sigma^2u^2+L(u) \qquad with \qquad L(u)=\phi(u;Y_t)=E\big[\exp\big[Y_t(\cdot)ui\big]\big]$$

Then the full computation of $\phi(u-\frac{i}{2};X_T)$ leads to the result stated in Proposition 1.

Proofs of Propositions 2 to 7 are based on derivation of the main price formula stated in Proposition 1. The formal derivations are clear but the main key and justification lie on the assumption that

$$\int_0^{\infty}\exp\big[tA(v)\big]\frac{1}{\left(\frac{1}{4}+v^2\right)}v^j\exp\left[-\frac{1}{2}\sigma^2Tv^2\right]dv<\infty \tag{71}$$

with

$$A(v) = \lambda\left\{\frac{p\lambda_1\left(\lambda_1 - \frac{1}{2}\right)}{\left(\lambda_1 - \frac{1}{2}\right)^2 + v^2} + \frac{(1-p)\lambda_2\left(\lambda_2 - \frac{1}{2}\right)}{\left(\lambda_2 - \frac{1}{2}\right)^2 + v^2} - 1\right\}$$

this assumption (71) may be established by using facts as stated in Propositions 8 and 9.

Propositions 8 and 9 are technical results which come from the fast decrease of the exponential term in the integral defined in (68).

8.3 Tables

in the sequel, we will focuse on the setting of Kou with S = 100, r = 0.05,

q = 0, σ = 0.16, λ = 1, λ_1= 10, λ_2 =5 and p = 0.4.

8.3.1 Prices of the call and put

strike	price call Kou	price call BS	price put Kou	price put BS
90	14.8119	12.8767	2.5898	0.6546
92	13.2764	11.2372	3.0049	0.9657
94	11.8140	9.6971	3.4931	1.3763
96	10.4346	8.2704	4.0644	1.9002
98	9.1473	6.9683	4.7277	2.5487
100	7.9594	5.7981	5.4904	3.3291
102	6.8761	4.7634	6.3577	4.2450
104	5.8997	3.8631	7.3320	5.2953
106	5.0304	3.0927	8.4132	6.4755
108	4.2653	2.4441	9.5988	7.7776
110	3.5996	1.9069	10.8837	9.1909

8.3.2 Delta of the call and put

strike	delta call Kou	delta call BS	delta put Kou	delta put BS
90	0.8540	0.8866	-0.1460	-0.1134
92	0.8231	0.8448	-0.1769	-0.1552
94	0.7867	0.7952	-0.2133	-0.2048
96	0.7450	0.7384	-0.2550	-0.2616

98	0.6984	0.6758	-0.3016	-0.3242
100	0.6477	0.6093	-0.3523	-0.3907
102	0.5941	0.5408	-0.4059	-0.4592
104	0.5387	0.4724	-0.4613	-0.5276
106	0.4831	0.4061	-0.5169	-0.5939
108	0.4284	0.3436	-0.5716	-0.6564
110	0.3760	0.2861	-0.6240	-0.7139

8.3.3 Gamma of the call and put

strike	Gamma Kou	Gamma BS
90	0.0127	0.0170
92	0.0155	0.0211
94	0.0184	0.0251
96	0.0212	0.0288
98	0.0239	0.0318
100	0.0262	0.0339
102	0.0279	0.0351
104	0.0290	0.0352
106	0.0293	0.0343
108	0.0290	0.0325
110	0.0281	0.0301

8.3.4 Vega of the call and put

strike	Vega Kou	Vega BS
90	10.1579	13.5862
92	12.3708	16.8612
94	14.6953	20.0816
96	16.9973	23.0095
98	19.1286	25.4227
100	20.9453	27.1437
102	22.3254	28.0617
104	23.1836	28.1422
106	23.4792	27.4251
108	23.2181	26.0124
110	22.4475	24.0493

8.3.5 Vanna and Vega of the call and put

strike	Vanna Kou	Volga BS
90	-0.8521	99.0557
92	-0.9293	96.6260
94	-0.9516	88.0844
96	-0.9060	74.7078

98	-0.7871	58.7490
100	-0.5986	42.9829
102	-0.3530	30.1370
104	-0.0696	22.3555
106	0.2281	20.8259
108	0.5167	25.6366
110	0.7750	35.8689

8.3.6 Speed of the call and put

strike	Speed BS
90	-0.0016
92	-0.0015
94	-0.0016
96	-0.0016
98	-0.0015
100	-0.0013
102	-0.0010
104	-0.0007
106	-0.0003
108	0.0001
110	0.0004