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Comparison of Stochastic Sampling and Application in Financial Mathematics

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Abstract
The primary purpose of this research is to investigate stochastic sampling and application in financial mathematics. Describing Monte Carlo and quasi-Monte Carlo methods in high-dimensional integrals and comparing them with each other. Finally, an example of an application Brownian bridge with quasi-Monte Carlo for financial mathematics we display. In the first part, we have shown that sequences Quasi-Monte Carlo method is a better method of Monte Carlo. Concerning the calculations on dimension reduction techniques under the Sobol sequence (Quasi-Monte Carlo Method), the ratio of variables taken from the two first variables under the Standard Method decreases when d is increased.BB and PCA are almost equal. As a result, most of the variance in these two methods are dedicated to the tiny initial dimensions. Hence, BB and PCS can significantly decrease the dimension and use the low discrepancy chains' main components. Since PCA performs better than BB, the PCA is a more efficient method than SM and BB.

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1. Introduction

Solving a scenario-based dynamic or multistage stochastic program scenario generation plays a critical role, as it forms the input specification to the optimization process. Computational bottlenecks in this process place a limit on the number of scenarios employable in approximating the probability distribution of the underlying uncertainty paths. Traditional scenario generation approaches have been used to find a sampling method that best approximates the path distribution in terms of some probability metrics such as minimization of moment deviations or Wasserstein distance (Dempster et al., 2011). one of the most essential methods for simulation is the Mont Carlo method. The history of Mont Carlo methods as a computational method for calculating expectations on potentially high-dimensional domains starts in the mid-1940s with the arrival of the first programmable computers. Monte Carlo (MC) methods are essential tools for approximating high dimensional integrals in financial computation. Which is independent of the dimensional? The basic idea of QMC is to use more uniformly distributed points instead of random points. To speed up QMC, one may use variance reduction techniques borrowed from MC and dimension reduction techniques specially designed for QMC. Some dimension reduction strategies are proposed, including constructions of Brownian motion: standard construction, Brownian Bridge. Stochastic integrals are playing an important role in finance methods. Finance mathematics is one set of economic, probability theory, measure theory, stochastic process. In Monte Carlo methods, high dimension integral $I(f) = \int_{[0,1]^d} f(x)dx$An estimator typically approximates, $\sum_{k=1}^{n} f(U)^{(k)}$ $Q(f) = \frac{1}{n}$, where U(k), k=1,2,… are the uniform random points in the hypercube [0,1]d. The estimator Q (f) tends to I (f) In probability. In the quasi-Monte Carlo methods, chains with low discrepancy are used instead of random points. Various numerical experiments show that quasi-Monte Carlo (QMC) method for high-dimensional integrals in financial issues performs better than the MC, particularly when the QMC is used for the dimension reduction techniques.

In this paper, we first introduce the chain of Sobol and Halton's sequences used in the quasi-Monte Carlo method and explain how they are produced. In the last section, we present the dimension reduction techniques. Using a numerical example, compare the dimension reduction techniques under chain Sobol sequence by the indicator of truncated variance ratio of the two initial variables (Talay., 1996).

2. Literature Review and Back Ground Research

2.1. Monte Carlo Makov Chain (MCMC) Method:

For a given population L, a parameter, such as θ, is estimated. In the Monte Carlo method, an estimate detector S(x) is first determined, in which x is a random variable with density function $f_x(x)$.

The estimate detector should have the following two conditions:

A: The estimate detector should be unbiased.

$$E[S(x)] = 0$$

B: The estimate detector should have a definite variance.

$$var\big(S(x)\big) = \sigma^2$$

Regarding the random samples $X_1 \dots . N_N$ of the function, density of $f_x(x)$ is used.

$$\hat{\theta}_N(X_1 \dots . N_N) = \frac{1}{N}\sum_{n=1}^{N} S(X_n) \qquad (1)$$

As it is known:

$$var(\hat{\theta}_N) = \frac{\sigma^2}{N} < \infty \ , \ E(\hat{\theta}_N) = \theta$$

The above estimate detector is introduced as the Monte Carlo estimator.

2. 2. Quasi-random sequences

The simplest example for the quasi-random sequences is the van der Corput sequence at the dimension (d=1). To produce this Sequence, we write n in binary decimal. The nth point of x_n is obtained reversing the digits of the decimal point in the opposite n. Halton and Sobol's sequences are other quasi-random sequences, whose algorithms are mentioned below (M. B. Giles, B, J. Waterhouse, 2009).

2. 2. 1. Halton sequence

The Halton sequence is the most basic low discrepancy with multiple dimensions. This Sequence is the expansion of the van der Corput sequence in dimension d. The nth number of Halton sequence in one dimension for a prime base p_d can be achieved through the following algorithm.

1. Write n as a number in base p_d:

$$n=\sum_{i=0}^{l} a_i(n)p_d^i = a_0 p_d^0 + a_1 p_d^1 + \ldots + a_l p_d^l \qquad (2)$$

2. Reverse the digits in the decimal point. Write the number in base 10:

$$\phi_{p_d}(n)=\sum_{i=0}^{l} \frac{a_i(n)}{p^{i+1}} \qquad (3)$$

Generally, the Halton sequence in dimension d is:

$$x_n = (\varphi_2(n), \varphi_3(n), \ldots, \varphi_{p_d}(n))$$

3.2. Sobol Sequences

S-dimensional Sobol sequence for all dimensions uses prime number 2. The first dimension of the Sobol' Sequence is the van der Corput sequence in base 2, and the higher dimensions are variations of the Sequence of the first dimension. To generate the j-th component of Sobol sequence, an initial polynomial of the degree n in Z is required.

$$x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1}x + 1 \qquad (4)$$

Where the coefficients $a_1, \ldots, a_{n-1}$ are all either 0 or 1. The Sequence of positive integers are defined with the following recursive equation:

$$m_k = 2a_1 m_{k-1} \oplus 2^2 a_2 m_{k-2} \oplus \ldots \oplus 2^{n-1} a_{n-1} m_{k-n+1} \oplus 2^s m_{k-n} \oplus m_{k-n}$$

Where $\oplus$ denotes a bit-by-bit addition. Initial values $\{m_1, m_2, \ldots\}$ must be chosen so that each m_k , $1<k<n$ is an odd number smaller than 2^k. Positive numbers $\{v_1, v_2, \ldots\}$ are defined as $v_k = \frac{m_k}{2^k}$. Then the j-th components of the point i-th or $x_{i,j}$ in a Sobol sequence is (tJ. Spanier,1995):

$$x_{i,j} = i_1 v_{1,j} \oplus i_2 v_{2,j} \oplus \ldots$$

3. The Methodology and Models

In this research, after the review of the literature review and background, the methodology and models of research was determined writhed as below:

3.1. Stochastic differential equations and Monte Carlo approximation:

In this simulation method, the expected values of $E[g(X(T))]$ For one answer, X, is presented from a given stochastic differential equation with a given function g. Generally, the approximation error consists of two parts, random error, and time-discretization error. The statistical error was estimated using the central limit theorem. The error estimation for time-discretization of Euler method directly with an additional remaining term that measures the ½ order accuracy for strong approximation.

Consider the following stochastic differential equation:**[9]**

$$dX(t) = a\big(t, X(t)\big) + b\big(t, X(t)\big)dW(t) \qquad (5)$$

The value of $E[g(X(T))]$ can be calculated using Monte Carlo method for $t_0 \le t \le T$. Based on the MCMC method:

$$E[g(X(T))] \cong \sum_{j=1}^{N} \frac{g(\bar{X}(T;\omega_j))}{N}, \qquad (6)$$

Where, $\bar{\boldsymbol{X}}$ is an approximation of X. based on the Euler method, the error in Monte Carlo method is as follows (Dagpunar, 2007):

$$E[g(X(T))] - \sum_{j=1}^{N} \frac{g(\bar{X}(T;\omega_j))}{N},$$
$$= E[g(X(T)) - g(\bar{X}(T))]$$
$$-\sum_{j=1}^{N} \frac{g\left(\bar{X}(T;\omega_j)\right) - E[g(\bar{X}(T))]}{N}$$

3.3. Black-Scholes Model and Asian option pricing

Consider a problem of Asian option pricing with optional arithmetic mean, the final discounts for a European-Asian style will be:

Max (Save – K, 0)

Where K is the price when T is due and$S_{ave} = \frac{1}{d}\sum_{j=1}^{d} S_{t_j}$ is the arithmetic average of the basic asset at intervals equal to Δt, j=1,…,d,

$\Delta t = T / d$ and $t0 = 0$.

We assume that a Black-Scholes model for the basic assessment is as follows:

$$dS_t = \mu S_t d_t + \sigma S_t dB_t \qquad (7)$$

Where μ is the basic average return, and σ is volatility and B_t is the standard Brownian motion.

Based on the natural risk of pricing principle, the asset value at time zero is given as:

$$C_A = E_Q[e^{-rT} \max(S_{ave} - K, 0)]$$

Where EQ[.] is the hope of the normal risk Q.

Let μ = r (r is the relative risk-benefit), so an analytical solution for formula (1) is as follows (Oksendal, B,.1998):

$$S_t = S_0 \exp\left(\left(r - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right) \qquad (8)$$

Simulating the Asian option pricing, we need to simulate Brownian motion.

We have shown that one standard method to generate Brownian motion with successive periods is as follows (Johannes, M., Poulson, N., 2010):

$$B_{t_j} = B_{t_{j-1}} + \sqrt{\Delta t}\, Z_j \qquad j=1,\ldots,d$$

Where Z1,…, Zd are random variables independent on a standard normal distribution. According to the above function, the Asian option pricing can be written as:

$$C_A = \int_{C^d} e^{-rt} \max\left(0, \frac{1}{d}\sum_{j=1}^{d} S_0 \exp[\left(r - \frac{\sigma^2}{2}\right)t_j + \sigma\sqrt{\Delta t}\sum_{i=1}^{j} \phi^{-1}(x_i)\,] - K\right) dx \qquad (9)$$

Where $\phi(.)$ is the standard normal distribution, the above integral is compared and simulated using the three MCMC sampling, Sobol and Halton sequences (Talay,D 1986; Talay,D., 1996; Johannes, M., Poulson, N., 2010).

4. Result of Finding

This research's main purpose was to investigate a Comparison of two stochastic sampling methods and applications in financial mathematics. Hence, the analysis of the data presented in this section.

4.1. The Comparison of three methods for Asian option pricing

In the above calculations, the Sobol sequence's simulation method is more accurate than the Halton sequences and MCMC methods for determining the Asian option pricing.

Table 1: The Comparison of three methods for Asian option pricing

Type of Simulation	Value of Option	Error	Std
Sobol Sequence	31.5581	0.1388	0.7082
Halton Sequence	31.6421	0.2887	1.4731
MC MC	28.5803	3.6231	1.8485

Note: Table (1) The comparison of three methods for Asian option pricing under the Sobol, Halton, MCMC methods for: S0 = 100, σ = 0.3, r = 0.05, T = 1, and K = 70.
Source: Finding of Research.

The European call option allows the owner to buy the asset at the given time T (expiration or effective date) for the determined price K (effective price). Therefore, the efficiency of the European call is max $(0, S(T) - K)$.

The Asian option is the option that the efficiency depending on the mean asset.

The efficiency for mean purchase price with the execution price K is the $(\bar{S}(T) - K)^+$ in which $\bar{S}(T)$ is the mean price in $[0, T]$.

If S(t) indicates the price at the time t, a widely used model in financial mathematics is as follows:

$$\frac{dS}{S} = \mu dt + \sigma dB$$

DB is the standard Brownian motion, μ is the annual expected growth rate, and σ denotes changes (standard deviation of return). Using the Ito lemma, the above stochastic differential equation is solved as follows:

$$S(t) = S_0\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma dB\right)$$

Or

$$S(t) = S_0\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}.z\right)$$

Therefore, the model follows the geometric Brownian motion, and the final price is obtained within the framework of the Black-Scholes model under the risk-neutral measure as follows: (Kloeden and Platen, E., (1995):

$$C(S_t, t) = e^{-r(T-t)} E_t[payoff(S_T)]$$

Thus, for the European call option:

$$C = e^{-r(T-t)}\, E_\emptyset\left(S(0)e^{\left(r-\sigma^2/_2\right)T + Z\sigma\sqrt{T}} - K\right)^+$$

For the Asian purchase option:

$$C = e^{-rT} E_{Z\sim N(0,1)}\left[\left(\frac{1}{n}\sum_{j=1}^{n} X_j\right) - K\right]^+$$

We assume the evaluation of option pricing through Monte Carlo, quasi-Monte Carlo, and hybrid methods in practice:

$$S_0 = 100\,, K = 100\,, T = 1\,, r = 5\%\,, \sigma = 0.10\,, N = 1000$$

The following tables show the evaluation of European and Asian call options through different methods and compare them with the precise answer (meanwhile, the Sobol sequences have been used for generation of uniform samples in the QMC method). The table for the evaluation of European options through different methods.

An Asian option with the following parameters is assumed to be evaluated:

$$S_0 = 100\ ,\ \mathrm{K} = 100\ ,\ \mathrm{T} = 1\ ,\ \mathrm{r} = 5\%\ ,\ \sigma = 0.10$$

The table for the evaluation of Asian options through different methods.

4.2. Dimension reduction techniques

4.2.1. Techniques for generating Brownian motion paths

To generate Brownian motion from normal variables, a proper analysis of covariance matrix V=LLT is required. A comparison on Standard method (SM), Brownian

Table 2: the evaluation of European option through different methods

Title	Price	Standard deviation	Time of computation
Answer of Black-Scholes model	5.4713	-	-
MC MC	5.6539	0.0334	6
Halton Sequence	5.3367	0.0246	8
Sobol Sequence	5.4485	0.0042	5

Table3: the evaluation of Asian option through different methods

Title	Price	Standard deviation	Time of computation
Answer of Black-Scholes model	3.8352	-	-
MCMC	3.3182	0.1348	7
Halton Sequence	3.1675	0.1295	6
Sobol Sequence	3.1778	0.1291	7

The table for the evaluation of Asian option through different methods, N = 1000

Bridge (BB), and principal component analysis (PCA) will follow:

In the SM, Brownian motion (B(t1),…,B(td))T, is generated according to using the following equation:

$$B(t_j) = B(t_{j-1}) + \sqrt{\Delta t}\, Z_j$$

Where{Zj , j=1,…,d} are standard normal variables. And correspond to the following Cholesky decomposition of the matrix V=LLT:

$$L = \sqrt{\Delta t}\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

In the BB, it is assumed that dimension d is a power of 2, i.e., d=2m (m is a non-negative integer). The BB methods can be written as the following sequences:

$$B(T) = \sqrt{T}\, Z_1$$

$$B\left(\frac{T}{2}\right) = \frac{1}{2}[B(0) + B(T)] + \sqrt{\frac{T}{4}}\, Z_2$$

$$= \frac{\sqrt{T}}{2} Z_1 + \frac{\sqrt{T}}{2} Z_2$$

$$B\left(\frac{T}{4}\right) = \frac{1}{2}\left[B(0) + B\left(\frac{T}{2}\right)\right] + \sqrt{\frac{T}{8}}\, Z_3$$

$$= \frac{\sqrt{T}}{2} Z_1 + \frac{\sqrt{T}}{2} Z_2 + \sqrt{\frac{T}{8}}\, Z_3$$

$$B\left(\frac{3T}{4}\right) = \frac{1}{2}\left[B\left(\frac{T}{2}\right) + B(T)\right] + \sqrt{\frac{T}{8}}\, Z_4$$

$$= \frac{3\sqrt{T}}{4} Z_1 + \frac{\sqrt{T}}{4} Z_2 + \sqrt{\frac{T}{8}}\, Z_4$$

$$B\left(\frac{(2^m-1)T}{2^m}\right) = \frac{1}{2}\left[B\left(\frac{(2^m-2)T}{d}\right) + B(T)\right] + \sqrt{\frac{T}{2^{m+1}}}\, Z_{2^m}$$

$$= \frac{2^m-1}{2^m} Z_1 + \dots + \sqrt{\frac{T}{2^{m+1}}}\, Z_{2^m}$$

Where Zj and j=1,…,d are standard normal variables.

PCA is a dimension reduction technique based on the decomposition of an eigenvalue of the covariance matrix V. A decomposition is as:

$$V = P\ D\ PT$$

Where D=diag(λ1,…,λd) is a diagonal matrix of eigenvalues of V arranged according to an non-additive directive and columns P, are the eigenvectors with equal length.

Brownian motion is obtained based on the selection of

$$L = P\ D1/2$$

where D1/2 $=\mathrm{diag}(\sqrt{\lambda_1}, \dots, \sqrt{\lambda_d})$

Among these techniques, the BB and PCA are extensively used in QMC.

4.3. ANOVA decomposition, truncated variance ratio

4.3.1. ANOVA decomposition

Suppose f(x) is a integrable function on [0,1]d. Let $\mathcal{v}$=1,2,…,d} For any subset $\mathcal{v}$ u ⊆, let │u│denote the cardinality of u and $\mathcal{v} - u$ denote its complementary set in $\mathcal{v}$. Let xu be │u│- dimensional vector containing the coordinates of x with the index u. Then f(x) may be written as the sum of its 2d ANOVA terms f .x/ D $f(x) = \sum_{u \subseteq \mathcal{v}} f_u(x)$

The ANOVA term fu(x) is defined by

$$f_u(x) = \int_{[0,1]^{|\mathcal{v}-u|}} f(x)\, dx_{\mathcal{v}-u} - \sum_{v \subset u} f_u(x)$$

With $f_0 = \int_{[0,1]^d} f(x)\, dx = I(f)$.

The ANOVA term $f_u(x)$ is part of the function which only depends on the variables x_j ، j∈ u

The ANOVA terms have other following properties:

1. $\int_0^1 f_u(x) dx_j = 0$ for any j∈ u.
2. $\int_{[0,1]^d} f_u(x) f_v(x) dx = 0$ ، for any subset of u ≠ v.
3. $\sigma^2(f) = \int_{[0,1]^d} f^2(x) dx - [I(f)]^2$ is the variance of f so

$$\sigma^2(f) = \sum_{u \subseteq \mathcal{v}} \sigma_u^2(f)$$

where

$$\sigma_u^2(f) = \int_{[0,1]^d} [f_u(x)]^2 dx$$

for $|u| > 0$ is the variance of f_u and $\sigma_0^2(f) = 0$.

4-3-2. Effective dimension

Definition: A brief definition of effective dimension is that it is the smallest dt, such that

$$\sum_{u \subseteq \{1,2,\ldots,d_t\}} \sigma_u^2(f) \geq p\sigma^2(f)$$

Where p is a ratio with ۰<p<1.

4.3.3. Truncated variance ratio

The k- dimensional variance ratio of f is defined as:

$$\tau_k = \frac{1}{\sigma^2(f)} \sum_{u \subseteq \{1,2,\ldots,k\}} \sigma_u^2(f)$$

The k- dimensional variance ratio of τ_k for the first k variable is significant.

4.4. The effective dimension of the problem of Asian option pricing under standard, BB and PCA methods:

The Brownian motion is generated based on the time series …,3T/4 , T/4 , T/2 and as

$$B_T = \sqrt{T}\, \phi^{-1}(x_{i,1})$$

$$B_{\frac{T}{2}} = \frac{1}{2}(B_0 + B_T) + \sqrt{\frac{T}{4}}\, \phi^{-1}(x_{i,2})$$

$$B_{(d-1)T/_d} = \frac{1}{2}\left(B_{(d-2)T/_d} + B_T\right) + \sqrt{T/_{2d}}\, \phi^{-1}(x_{i,d})$$

Where (xi,1, …, xi,d) is the i-th point of the low discrepancy chains.

Another method to reduce dimension is PCA. To define the method of PCA, first, we introduce a standard procedure:

$$\begin{pmatrix} B_{t_1} \\ \vdots \\ B_{t_d} \end{pmatrix} = L \begin{pmatrix} Z_1 \\ \vdots \\ Z_d \end{pmatrix} \qquad (10)$$

Where L is a lower triangular matrix with a nonzero coefficient $\sqrt{\Delta t}$.

If the covariance matrix is $B_{t_1}, \ldots, B_{t_d}$ then the ij factor in matrix V is equal to

Vij = min (ti, tj) and i,j=1,…,d The PCA method is obtained by replacing the following with matrix the matrix L in (3):

$$M = \left(\sqrt{\lambda_1}\, v_1, \ldots, \sqrt{\lambda_d}\, v_d\right)$$

Where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d$ are the eigenvalues of the matrix V and vd,…,v1 are its eigenvectors.

In our calculations, the parameters of (2) are used as follows (S.Heston,1993):

The above table and figure show that the Standard Method's effective dimension is increased as the effective dimension increases. Still, the effect is nearly constant for different dimensions in the BB method and the PCA method and equals 2. The truncated variance ratio for the first two variables is shown in the following table and figure

Table 3: The effective dimension of the problem of Asian option pricing under standard, BB, and PCA methods

d	SM	BB	PCA
8	7	5	2
16	14	7	2
32	27	7	2
64	53	8	2
S0 =100 and σ = 0.2 و r = 0.1 and T=1 year and K=100			

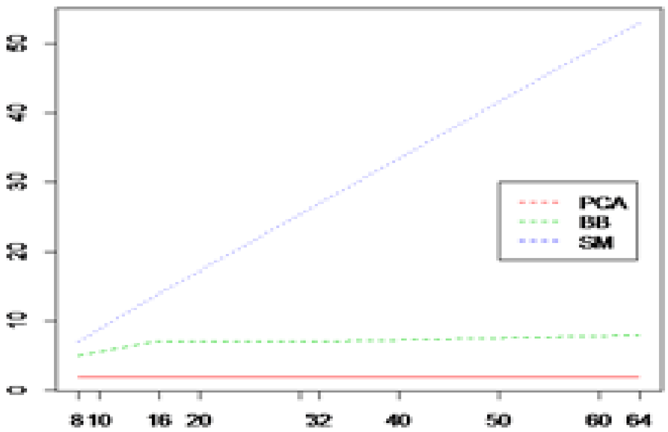

Figure 1. The affective dimension of the problem of Asian option pricing under standard, BB, and PCA methods

Source: Finding of Research.

Table 4: Total variance for the first two variables in the problem of Asian option pricing under the standard, BB, and PCA methods

d	SM	BB	PCA
8	0.4931	0.9457	0.9981
16	0.2591	0.9349	0.9979
32	0.1070	0.9276	0.9975
64	0.405	0.9266	0.9978

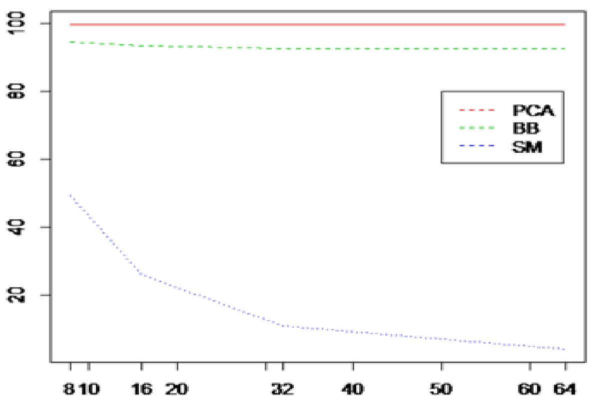

Figure 2. Total variance for the first two variables in the problem of Asian option pricing under the standard, BB, and PCA methods

Source: Finding of Research.

5. Conclusion

This research investigates the main purpose of comparing two stochastic sampling methods and applications in financial mathematics. Therefore, In the first part, we have shown that sequences Quasi-Monte Carlo method is a better method of Monte Carlo. Concerning the calculations on dimension reduction techniques under the Sobol sequence (Quasi-Monte Carlo Method), the ratio of variables taken from the two first variables under the Standard Method decreases when d is increased. While the BB and PCA are almost equal. As a result, most of the variance in these two methods are dedicated to the tiny initial dimensions. Hence, BB and PCS can significantly decrease the dimension and use the low discrepancy chains' main components. Since PCA performs better than BB, the PCA is a more efficient method than SM and BB.

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