J Theor Appl Phys (2016) 10:47–52
DOI 10.1007/s40094-015-0200-x

RESEARCH

Charged particles in curved space-time

K. Mehdizadeh[1] · O. Jalili[2]

Received: 3 October 2015 / Accepted: 30 October 2015 / Published online: 28 November 2015

Abstract Considering the dynamics of geometry and the matter fields, dynamical equations of geometry and the matter fields are re-derived. The solutions of these equations are studied. We focus on a charged particle and explain the axiomatic approach to drive the electromagnetic self-force on its motion, then the energy conservation is considered. A new mathematical concept, which is introduced in axiomatic approach in general, is discussed.

Keywords Charged particles motion · Curved space · Radiation

Introduction

In considering the plasma around a neutron star, we encounter a charged gas in the curved space. In statistical formalism of such problem usually we just need the first velocity moment of Vlasov or Boltzmann equation. In the equation of first velocity moment we study the average motion of particles in all elements. It is clear that every particle's trajectory is not in full adjustment to the element's trajectory. Some physical phenomena come back to this mismatching. For example, the first velocity moment cannot explain the Landau damping. Apart from this, the very charged particle motion is an important problem that contains some mathematical complexity. Radiation field of a charged particle motion in curved space, in general, can be divided into two parts: electromagnetic field and gravitational field. In this paper we only consider the first one. The solution of Maxwell's equation depends on several parameters, e.g., on the presence of other charged particles and the spacetime geometry. We focus on a particle that, even small, changes the spacetime geometry. One of the known ways to solve the differential equations is the use of Green functions. The Green functions that is appropriate for our problem are bitensors. Bitensor is a mathematical object that is defined in two distinct points of a manifold. The method we specially focus on it using another mathematical object: difference of two tensors belonging to two distinct tangent spaces. In this manner we come to a new mathematical concept in the formulation of the problem. These new mathematical concepts need further research and studies. We discuss some aspects of these new concepts in this paper. Our paper contains following aspects. First we describe the charge particle motion in the curved spacetime and discuss some of its complexities then in Sect. 3, the electromagnetic self-force is discussed by axiomatic approach. In Sect. 4, the Conservation of energy is considered that is the extension of energy conservation theorem to the curved space. Finally, in Sect. 5 we have discussed the new mathematical concepts that are introduced in the problem.

✉ O. Jalili
omid_jalili@yahoo.com

K. Mehdizadeh
kazem_mehdizadeh93@yahoo.com

1 Department of physics, Ayatollah Amoli Branch, Islamic Azad University, Amol, Mazandaran, Iran

2 Department of physics, North Tehran Branch, Islamic Azad University, Tehran, Iran

Particles motion in the curved space

To avoid the complexity of extended charged matters, we turned to the point particles. We can principally obtain the extended matter's fields from point's field (even it is not the case for gravitational waves, because the Einstein equations are not linear). Choice of the point assumption causes a problem: the point matters are not compatible with general relativity, because it creates black hole (a spherical matter distribution with the fixed mass and charge when its radius vanishes becomes a black hole). On the other hand, we have point particle in the world (leptons). Thus the general relativity is not a complete theory by itself, it needs to join with the quantum theory, i.e., we need quantum gravity. So far we have two candidates for quantum gravity: string theory and loop quantum gravity. In the first one, we exclude the point concept and introduce the extended object (string). In the second one, we exclude the continuum structure of spacetime and introduce the spacetime network instead of manifold structure. In the present paper, we study some small distribution of matter to simplify our considerations.

In the absence of the interaction between matter field, particle and radiation, the action is given by:

$$\begin{aligned} S &= S_{\text{Gravity}} + S_{\text{Matter}} + S_{\text{Radiation}} + S_{\text{Single Particle}} \\ &= -\frac{1}{16\pi G}\int \sqrt{-g}R\mathrm{d}x + \int \mathscr{L}\sqrt{-g}\mathrm{d}x \\ &\quad + \int \frac{-1}{4}F_{\alpha\beta}F^{\alpha\beta}\sqrt{-g}\mathrm{d}x + \int \mathrm{d}s. \end{aligned} \tag{1}$$

$\mathscr{L}$ is the Lagrangian density of the matter field. The dynamical variable in the S_{Gravity} is the metric $g_{\mu\nu}$, in the S_{Matter} is the $\phi(x)$ (matter field), in the $S_{\text{Radiation}}$ is $A_\mu(x)$ (Maxwell field) and in the last term is $x(\tau)$ (particle's path). Minimizing the action (1) with respect to $g_{\mu\nu}$ gives the Einstein's equations:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}|_{\text{Matter}} + 8\pi T_{\mu\nu}|_{\text{Radiation}} + 8\pi T_{\mu\nu}|_{\text{Single Particle}} \tag{2}$$

$G_{\mu\nu}$ is Einstein's tensor. It is obvious that both matter and the radiation, and of course the presence of the particle, contribute in the forming of the metric. Minimizing the action (1) with respect to $\phi(x)$ give the Euler–Lagrange Eq. for ϕ;

$$\delta S|_{\text{with respect to }\phi} = 0 \Rightarrow \nabla_\alpha \frac{\partial \mathscr{L}_m}{\partial \phi_{,\alpha}} - \frac{\partial \mathscr{L}_m}{\partial \phi_\alpha} = 0. \tag{3}$$

Minimizing the action (1) relative to A_μ give the Euler equation for the Maxwell field:

$$\begin{aligned} \delta S|_{\text{with respect to A}} = 0 &\Rightarrow \nabla_\alpha \frac{\partial \mathscr{L}_r}{\partial A_{,\alpha}} - \frac{\partial \mathscr{L}_r}{\partial A_\alpha} = 0 \\ &\Rightarrow \Box A^\alpha - R^\alpha_\beta A^\beta = 0, \end{aligned} \tag{4}$$

where $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$ is the covariant wave operator and R^α_β is the Ricci tensor. Note the presence of metric in both of Eqs. (3) and (4). Note also that the matter contributes in the formation of the metric (Eq. 2). Then even the non-charged matter creates electromagnetic fields, this was the point that motivated Einstein to join electromagnetism with the gravity because one of them creates the other. Minimizing the action (1) with respect to $x(\tau)$ gives the geodesic of the $g_{\mu\nu}$ manifold:

$$\nabla_U U = 0 \quad \Rightarrow \quad \ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma = 0 \quad (\text{U is the four-velocity}). \tag{5}$$

It is seen the path of particle is the geodesic of the manifold, namely the manifold that the particle itself contributes in its metric formation (Eq. 2)! The action (1) is a non-interacting action, namely the fields are free but, as we have seen, the nature of gravity is such that even the non-interacting action has the interacting behaviors: non-gravitational fields contribute in the creation of metric and this metric contribute in the other fields and this is the interaction between the fields and the particle with the metric! However, it is not the only possible interaction. Actually it is the possible minimum interaction. If the interaction terms are added, we have:

$$\begin{aligned} S &= S_{\text{Gravity}} + S_{\text{Matter}} + S_{\text{Radiation}} + S_{\text{Single Particle}} \\ &\quad + S_{\text{A and x interaction}} + S_{\text{A and }\phi\text{ interaction}}, \end{aligned} \tag{6}$$

where $S_{\text{A and x interaction}}$ is the action term of interaction of electromagnetic and one charged particle and $S_{\text{A and }\phi\text{ interaction}}$ is the action term of interaction of electromagnetic and matter field. What about the gravity–gravity interaction term namely $S_{\text{G and G interaction}}$? Let us first continue without this term. With the interaction (6), Eqs. (3), (4) and (5) are modified. Note that Eq. (2) is not changed but Eqs. (3) and (4) will have the right term, for example Eq. (4) will get the new form

$$\Box A^\alpha - R^\alpha_\beta A^\beta = -4\pi J^\alpha|_{\phi\text{ field}} - 4\pi J^\alpha|_{\text{Single particle}}, \tag{7}$$

namely the charged matter enters as currents to create the electromagnetic fields. Finally, Eq. (5) became

$$\ddot{x}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma = \frac{e}{m}F^\alpha_\beta U^\beta. \tag{8}$$

This means that the particle's path is no longer a geodesic. It is seen that dynamical structure is more fundamental than the geometrical concepts! Here the path of particle is not a geodesic. The path of particle is obtained by the minimizing the appropriate action. Then the action approach is more general than the geometrical approach. We can design a suitable Lagrangian for an electromagnetic field and represent the whole theory in a mechanical manner.

Let us continue. General relativity has introduced the notions of geodesic and metric instead of acceleration and force, but we can continue the same classical notions. For this, we rewrite the geodesic equation in the following manner:

$$U^\mu \nabla_\mu U = 0 \Longrightarrow \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\nabla_\mu U = 0 \Longrightarrow \quad \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\left(\frac{\mathrm{d}U^\alpha}{\mathrm{d}x^\mu} + \Gamma^\alpha_{\mu\nu}U^\nu\right) = 0$$
$$\Longrightarrow \quad \frac{\mathrm{d}^2x^\alpha}{\mathrm{d}\tau^2} + \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\Gamma^\alpha_{\mu\nu}U^\nu = 0 \quad \Longrightarrow \quad m_0\frac{\mathrm{d}^2x^\alpha}{\mathrm{d}\tau^2} = -m_0\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\Gamma^\alpha_{\mu\nu}U^\nu. \tag{9}$$

Thus, we can interpret the right hand side as gravitational force. Note that the radiation, matter and the single particle, all of these contribute in formation of this force because all of them contribute in making the $\Gamma^\alpha_{\mu\nu}$ tensor. In the presence of the interaction, from Eq. (8), the particle's path can be written as:

$$\nabla_U P^\alpha = eF^\alpha_\beta P^\beta \quad \Longrightarrow \quad m_0\ddot{x}^\alpha = -m_0\Gamma^\alpha_{\beta\gamma}\dot{x}^\beta\dot{x}^\gamma + eF^\alpha_\beta U^\beta \quad (\mathrm{Pisthefour-momentum}). \tag{10}$$

It is seen that electromagnetic force has appeared twice: once, implicitly in determining the Γ (the first term in the right) and the second times as the explicit term (second term in the right). However, in the literature usually when speaking about the force, it means only the explicit force, namely:

$$f = eF^\alpha_\beta U^\beta = \nabla_U P^\alpha. \tag{11}$$

Now we come back to our question about the $S_{\mathrm{G\ and\ G\ interaction}}$ term. It is known that the gravity has energy [1], then it can create gravity. It is the one of the nonlinear features of gravity (other nonlinearity aspect of gravity is the very nonlinearity of Einstein's tensor). But there is no such thing as energy tensor for gravity. This is because according to equivalence principle, we can choose a locally flat spacetime such that the gravitational energy vanishes. But if a tensor is zero in one frame it is so in all frames. Thus we must take into account the gravitational energy in some different sort. Such consideration is important in some cosmological large-scale structure [2]. Here, we work in some smaller scales such that this nonlinearity aspect is absent.

Electromagnetic self-force in axiomatic approach

To obtain first the radiation field of a point charged particle we must solve Eq. (7) for one particle source, namely:

$$\Box A^\alpha - R^\alpha_\beta A^\beta = -4\pi J^\alpha|_{\mathrm{Single\ particle}}, \tag{12}$$

where the source is singular at the particle position and vanishes elsewhere. The singular behavior of the source term enters into the solution and the field will be singular at the position of particle. We can regularize the infinity at the particle's mass and obtain normal value for the self-force. Such procedure is equivalent to using the regular potential rather than the usual retardation potential [3]:

$$AR = \frac{1}{2}(A_{\mathrm{ret}} - A_{\mathrm{adv}}), \tag{13}$$

where A_{ret} and A_{adv} are the retarded and advanced potential, respectively. Both of these fields are singular at the particle's position, but the A_{R} field is not (because it is the difference of two singular fields). Actually the regular potential satisfies the homogenous equation, hence its regular behavior is not surprising:

$$\Box A^\alpha_R - R^\alpha_\beta A^\beta_R = 0. \tag{14}$$

Historically Dewitt and Brehme [5] extended the Dirac' flat space result for the radiation of charged point particle [4]. Their result in normal coordinate is (corrected by Hobbs several years after that [6]):

$$\begin{aligned}
F^{\pm}_{\alpha'\beta'} = 2e[&r^{-2}u_{[\alpha'}\Omega_{\beta']} - \frac{1}{2}r^{-1}(a^\alpha\Omega_\alpha)u_{[\alpha'}\Omega_{\beta']} \\
&+ \frac{1}{2}r^{-1}a_{[\alpha'}u_{\beta']} + \frac{3}{8}(a^\alpha\Omega_\alpha)^2u_{[\alpha'}\Omega_{\beta']} \\
&- \frac{1}{6}u^\alpha u^\beta\Omega^\gamma\Omega^\delta R_{\alpha\gamma\beta\delta}u_{[\alpha'}\Omega_{\beta']} - \frac{3}{4}(a^\alpha\Omega_\alpha)a_{[\alpha'}u_{\beta']} \\
&+ \frac{1}{6}\Omega_{[\beta'}R_{\alpha']\sigma\alpha\tau}u^\alpha\Omega^\sigma\Omega^\tau + \frac{1}{8}u_{[\alpha'}\Omega_{\beta']}a^2 - \frac{1}{2}\dot{a}_{[\alpha'}\Omega_{\beta']} \\
&\pm \frac{2}{3}\dot{a}_{[\alpha'}u_{\beta']} + \frac{1}{12}u_{[\alpha'}\Omega_{\beta']}R - \frac{1}{12}u_{[\alpha'}R_{\beta']\gamma}\Omega^\gamma \\
&+ \frac{1}{2}\Omega_{[\alpha'}R_{\beta']\gamma}u^\gamma + \frac{1}{12}u_{[\alpha'}\Omega_{\beta']}R_{\gamma\delta}\Omega^\gamma\Omega^\delta \\
&+ \frac{1}{2}R_{[\alpha'|\gamma|\beta']\delta}u^\gamma\Omega^\delta - \frac{1}{12}u_{[\alpha'}\Omega_{\beta']}R_{\gamma\delta}u^\gamma u^\delta \\
&+ \frac{1}{6}u_{[\alpha'}R_{\beta']\gamma\delta\epsilon}u^\gamma u^\delta\Omega^\epsilon \mp \frac{1}{3}u_{[\alpha'}R_{\beta']\gamma}u^\gamma] \\
&\pm e\int_{\tau^\pm}^{\pm\infty}\nabla_{[\beta'}G^{\pm}_{\alpha']\alpha''}u^{\alpha''}(\tau'')\mathrm{d}\tau'' + O(r)
\end{aligned} \tag{15}$$

The last term, tail term, represents the failure of Huygens principle. In the this equation e is the particle's charge, r the distance from particle position (affine parameter of space-like curve from the particle position to observation point), a the particle's acceleration, $\dot{a}$ the time derivative of acceleration and the others are some geometrical quantities. At the particle's position the first three terms of above equation are singular and the rest are regular. It is seen that these singular terms depend only on four-velocity and four-acceleration and nothing else such as curvature or derivative of acceleration. Thus with the equal velocity and acceleration, in the difference of two such expression the singular terms can be eliminated. Actually the force exerted on these two particles have the same infinity structure such that in the difference they will be canceled. Thus in our particle size limit we can postulate the following axioms [7]:

Electromagnetic Axiom 1 (Comparison axiom) Consider two points, P and $\widetilde{P}$, each lying on time-like world lines in possibly different spacetimes which contain Maxwell fields F^{ab} and $\widetilde{F}^{ab}$ sourced by particles of charge e on the world lines. If the four-accelerations of the world lines at P and $\widetilde{P}$ have the same magnitude, and if we identify the neighborhoods of P and $\widetilde{P}$ via Riemann normal coordinates such that the four-velocities and four-accelerations are identified, then the difference in the electromagnetic forces f^a_{EM} and $\widetilde{f}^a_{\mathrm{EM}}$ is given by the limit as $r \to 0$ of the Lorentz force associated with the difference of the two fields averaged over a sphere at geodesic distance r from the world line at P.

$$f^a_{\mathrm{EM}} - \widetilde{f}^a_{\mathrm{EM}} = \mathrm{lim}_{r\to 0}(\mathrm{e}\langle \mathrm{F}^{ab} - \widetilde{\mathrm{F}}^{ab}\rangle)\mathrm{u_b} \tag{16}$$

As an application of above axiom, we can compare the force applied on a charged particle in an arbitrary spacetime to Minkowski spacetime. It is a good choice to consider field which its force approaches to zero. The symmetry consideration shows that the force of the symmetric field, $F_{ab} = \frac{1}{2}(F^-_{ab} + F^+_{ab})$, may vanish. Thus we accept the following axiom [7]:

Electromagnetic Axiom 2 (flat spacetime axiom) If (M, g_{ab}) is Minkowski spacetime, the world line is uniformly accelerating, and F_{ab} is the half-advanced, half-retarded solution, $F_{ab} = \frac{1}{2}(F^-_{ab} + F^+_{ab})$ then $f^a = 0$ at every point on the world line.

To obtain the F_{ab}, we can simply use Eq. (15) with the tail term and delete all terms that contain the curvature. Then we obtain the following expression for a charged particle self-force in an arbitrary spacetime:

$$\begin{aligned} f^a_{\mathrm{EM}} = {} & e(F^{\mathrm{in}})^{ab}u_b + \frac{2}{3}e^2(\dot{a}^a - a^2u^a) \\ & + \frac{1}{3}e^2(R^a_b u^b + u^a R_{bc}u^b u^c) \\ & + e^2 u_b \int_{-\infty}^{\tau^-} \nabla^{[b}(G^-)^{a]c'} u_{c'}(\tau')\mathrm{d}\tau' \end{aligned} \tag{17}$$

where $(F^{\mathrm{in}})^{ab} = F^{ab} - (F^-)^{ab}$. Equation (17) is the same as the result that Dewitt and Brehme [5] gave before, but here we derive it with the less and easy manipulation.

With the given force we can simply drive an expression for acceleration:

$$a = \frac{f^a_{\mathrm{EM}}}{m} \tag{18}$$

But the right hand side depends on acceleration too! There is a known recipe to solve it. In the same manner as Born series in scattering, we can substitute the acceleration in the right side from the left side and iterate this work. Then we obtain the following expression for the first several terms in that series:

$$\begin{aligned} a^a = {} & \frac{e}{m}(F^{\mathrm{in}})^{ab}u_b + \frac{2}{3}\frac{e^2}{m}\left(\frac{e}{m}u^c\nabla_c(F^{\mathrm{in}})^{ab}u_b\right. \\ & + \frac{e^2}{m^2}(F^{\mathrm{in}})^{ab}F^{\mathrm{in}}_{bc}u^c - \left.\frac{e^2}{m^2}u^a(F^{\mathrm{in}})^{bc}u_c F^{\mathrm{in}}_{bd}u^d\right) \\ & + \frac{1}{3}\frac{e^2}{m}(R^a_b u^b + u^a R_{bc}u^b u^c) \\ & + \frac{e^2}{m}u_b \int_{-\infty}^{\tau^-} \nabla^{[b}(G^-)^{a]c} u_c(\tau')d\tau' \end{aligned} \tag{19}$$

Conservation of energy

In our problem the total energy of the system is:

$$\begin{aligned} E_{\mathrm{Total}} = {} & E_{\mathrm{Particle}} + E_{\mathrm{Gravity}} + E_{\mathrm{Electromagnetism}} \\ & + E_{\mathrm{Interaction\ of\ one\ particle\ to\ EM\ field}}. \end{aligned} \tag{20}$$

For the energy concept to be a reasonable concept, the spacetime must have the time-like Killing vector [8]. Even for such spacetime it is practically hard to write explicitly an energy expression for every term in the right had side of Eq. (20). For example, for E_{Particle} we need to exactly know spacetime metric, one metric that the particle has some role in creation of it! However, in some asymptotical case, there exists a simple relation. For example, for asymptotically flat spacetime in the comparison of the initial and final energy we have:

$$\Delta E_{\mathrm{Particle}} + \Delta E_{\mathrm{Electromagnetism}} + \Delta E_{\mathrm{Particle-Electromagnetism\ Interaction}} = 0. \tag{21}$$

It have been proved that [9]:

$$\int_{\mathcal{I}^+} t^a T_{ab}\varepsilon^b_{cde} - \int_{\mathcal{I}^-} t^a T_{ab}\varepsilon^b_{cde} = \int_{\mathcal{M}} t^a f_a \mathrm{d}\tau, \tag{22}$$

where t^a is time-like Killing vector, $\mathcal{I}^+(\mathcal{I}^-)$is space-like hyper-surface at $t \to +\infty (t \to -\infty)$, T_{ab} the particle's energy momentum tensor and f_a the electromagnetic self-force, i.e., Eq. (17). The left hand side of this equation is the change in particle's energy and the right hand side is the work done on it. It simply means:

$$\Delta E_{\mathrm{Particle}} + \Delta E_{\mathrm{Particle-Electromagnetism\ Interaction}} = 0. \tag{23}$$

There is no electromagnetic energy change and no gravitational energy change: $\triangle E_G = 0, \triangle E_{\mathrm{EM}} = 0$.

Results and discussion

The radiation of charged particles in curved space is an important subject in general relativistic kinetic theory. In formulating it, we see the nonlinear nature of general relativity that comes through two ways. We have seen that even the non-interacting particles have some kind of interaction: the particle changes the geometry and the new geometry acts on other particles motion. The general relativity replaces geodesic with old Newtonian concept, i.e., the force. However, we can use the force concept as before. This issue is based on a fundamental fact: the algebraic methods (the mechanical approach) is more generic than geometric method. Where the Einstein's geodesic method fails, the dynamical approach works. The radiation of charged particle in curved space like any plasma problem is the simultaneous solving of several equations: the set of Maxwell equations, the particles motion equations and the geometry equations. With some conditions, like the fixed background assumption, we need to solve the two sets of these three sets. However, we can solve these sets with the usual methods such as the Green function method and so on; we confront a new mathematical concept in the Green function method, the bitensor. In general, the physical quantities are the local concepts that belong to the tangent space of every spacetime points. Even the definition of derivative that is the comparison of quantities at two different points must be changed to become a local quantity. But the bitensors are not a local quantity. A bitensor is not built simply by juxtaposing of two tangent space quantities but these quantities multiplied as the numbers, thus this juxtaposing is not the direct product or tensor product! These new manipulations of geometrical quantities enter again in the axiomatic approach. In this paper, we focused on axiomatic approach to obtain the self-electromagnetic force. We have seen in this approach we need to subtract the geometrical quantities belong to two different manifolds. In the bitensor approach, the product quantities belong to different points of one manifold, but in the axiomatic approach these two points belong to different manifolds. Thus the operations are not simply juxtaposing the quantities or anything else but the difference of these quantities! In physical point of view, when we can subtract two quantities which are the same kinds also must have equal dimension. Thus here that we are subtracting two things belong to different manifolds, we come to a new concept that needs more investigations. This is a new geometrical concept. We may ask some questions: can we make these subtracted quantities a new tensorial quantity by suitable assumptions about each quantity? If such extension is present, what is the derivative of it?... We see that there are some technical questions about these new quantities that need some further researches.

Conclusions

To obtain electromagnetic radiation of a charged particle in curved space, we must solve the differential equation of electromagnetic potential $A_\mu(x)$ with the single particle source term. The traditional ways to do it are the method of Green function and the conformal mapping method [10]. Since the self-field and the self-force become infinity, we must then regularize them by renormalized mass. After that we obtain finite values for the field and force. The energy conservation can be obtained with some subtlety. In curved space we can reobtain the energy work theorem. In a parallel and equivalent manner with some assumption about space-time manifold, we can obtain aforementioned result but with very less calculation. This parallel approach is the axiomatic approach. In this approach, we compare tensorial object belonging to two different manifolds. The comparison of tensorial object belonging to a specific manifold are well known. But comparison of quantities that belong to two different manifolds is new. This new phenomenon is very interesting and valuable for further researches.

Acknowledgments The authors would like to convey their gratitude to express their thanks to professor M.V. Takook and professor M. Saravi for their comments and useful discussions.

References

1. Arnowitt, R., Deser, S., Misner, C.W.: Dynamical structure and definition of energy in general relativity. Phys. Rev. **116**, 1322 (1959)
2. de Felice, F., Clarke, C.J.S.: Relativity on Curved Manifolds. Cambridge University Press, Cambridge (1992)
3. Poisson, E., Pound, A., Vega, I.: The motion of point particles in curved spacetime. Living Rev. Relativ. **14**. doi:10.12942/lrr-2011-7
4. Dirac, P.A.M.: Classical theory of radiating electrons. Proc. Roy. Soc. A **167**, 148 (1938)
5. DeWitt, B.S., Brehme, R.W.: Radiation damping in a gravitational field. Ann. Phys. **9**, 220–259 (1960)

6. Hobbs, J.M.: Radiation damping in conformally flat universes. Ann. Phys. **47**, 141 (1968)
7. Quinn, T.C., Wald, R.M.: Axiomatic approach to electromagnetic and gravitational radiation reaction of particles in curved spacetime. Phys. Rev. D **56**, 3381 (1997)
8. Parker, L., Toms, D.: Quantum Field Theory in Curved Spacetime. Cambridge University Press, Cambridge (2009)
9. Quinn, T.C., Wald, R.M.: Energy conservation for point particles undergoing radiation reaction. Phys. Rev. D **60**, 064009 (1999)
10. Bicák, J., Krtouš, P.: Fields of accelerated sources: born in de Sitter. J. Math. Phys. **46**, 102504 (2005)