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RESEARCH

Comparative study of dust acoustic solitons in two-temperature ion homogeneous and inhomogeneous plasmas

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Abstract

The dust acoustic solitary waves are theoretically investigated in dusty plasmas for different cases of with and without density gradients. These low-frequency solitary waves are studied using appropriate Korteweg–de Vries equations obtained using relevant stretched coordinates. The soliton solutions in homogeneous plasma, weakly inhomogeneous plasma and strongly inhomogeneous plasma, are thoroughly investigated for studying the effect of different parameters like dust charge and density of all the plasma species on the soliton profiles. The combination of the dust charge with its number density changes the dynamics of the solitons and that is further affected by the number density of the hot ion with respect to the cold ions.

Keywords Dusty plasma · Two-temperature ion · Soliton

Introduction

The presence of dust in plasma contributes to unusual behaviour which also generates low frequency instabilities and modes or even dominates the wave propagation in the plasma. The micrometeoroids, space debris, etc., are the source of dust in our solar system and these dust particles can be dielectrics or metallic in nature [1]. Since the introduction of dust acoustic wave, there have been lot of researches on the linear and nonlinear properties of such waves [2–10]. Lakshmi and Bharuthram investigated the impact of particle densities and temperature on large amplitude solitary waves in homogeneous cold dusty plasma consisting of electrons and two species of ions [11]; the calculations did not include the dust temperature. Asgari et al. [12] studied the solitary waves in homogeneous plasma present in only some places like moon's atmosphere, and they have used Cairns distributions for the ions. Simultaneously lot of researches were going on for the study of inhomogeneous plasma. For example, Singh and Rao [13] studied plasma with electrons, ions and negative dust with weak density gradient. Singh et al. [14] contributed through the investigation of rarefactive soliton in weakly and strongly inhomogeneous plasmas under the influence of electron inertia; they discussed about the types of modes and their evolution as solitons in these two types of inhomogeneous plasmas. Aziz and Stroth explored the plasma with density gradient having two-temperature nonisothermal electrons [15]. The investigations that have been carried out are the detailed study of solitons' behaviour and the impact of several parameters of the plasma on the solitons' propagation, which clearly indicates that the inhomogeneity in the plasma alters the characteristics of the solitons.

The investigation in the present paper shows theoretical comparable variation in soliton structures from homogeneous plasma to inhomogeneous plasma. It is interesting to study homogeneous and inhomogeneous plasmas with similar components and parameters together. Study corresponding to weak and strong inhomogeneities in plasma further adds to the fascinating properties of solitons in the dusty plasmas. In view of the result of interaction of the flux of the ions and the electrons to the dust grain, the dust charging process depletes the electron density. This signifies that the number density of the electrons is much lesser than the number density of the ions. Saturn's F ring is a good example of this type of condition [16]. Hence, finally we also explore the electron depleted plasmas for the soliton evolution.

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Basic equations

Our unmagnetised plasma model consists of inertialess electrons, two-temperature ions (hot and cold) and massive dust particles. The investigation of this plasma model will be in three parts in total, one for homogeneous plasma, second for weakly inhomogeneous plasma and third for strongly inhomogeneous plasma. The nonlinear behaviour of the soliton study is carried out using reductive perturbation technique. The normalised form of continuity equation and momentum equation for the dust particles are used, and Boltzmann distribution is used for the ions and electrons in view of the heavy mass of dust particles. Poisson's equation is used to relate the charge with the field associated with the wave generation. Here the notations used are as follows: T represents the dust temperature, and n symbolises the dust number density, m is the mass of dust, n with subscript represents the number densities, and T with subscript represents temperature of plasma species where subscript e is for electrons, c for cold ions, and h for hot ions.μ represents nature of dust charge, $\mu = +1$ for positive dust and $\mu = -1$ for negative dust, and Z is the charge on the dust particles. The normalisation factors used for the basic equations are: for number densities $Z\ n_0$, dust acoustic speed $C_s \left(= \sqrt{T_{eff}/m}\right)$ for velocities, inverse of dust plasma oscillation frequency $\omega_{PD}^{-1}\left(= \sqrt{\frac{n_0\mu^2 Z^2 e^2}{\varepsilon_0 m}}\right)$ for time, Debye length λ_D $(=C_s/\omega_{PD})$ for spatial length, and T_{eff}/Ze for potential. $T_{eff} = Z^2 n_0 / \left(\frac{n_{e0}}{T_e} + \frac{n_{c0}}{T_c} + \frac{n_{h0}}{h}\right)$ [17] where n_{e0}, n_{c0} and n_{h0} are unperturbed number densities of the electrons, cold ions and hot ions, respectively, σ is T/T_{eff} [11].

Homogeneous plasma

For homogeneous plasma, the equations thus obtained after normalisation are as follows

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial^2 \phi}{\partial x^2} - n_e + n_c + n_h + n\mu Z = 0 \tag{2}$$

$$n_e = n_{e0} e^{(\phi T_{eff}/ZT_e)} \tag{3}$$

$$n_c = n_{c0} e^{(-\phi T_{eff}/ZT_c)} \tag{4}$$

$$n_h = n_{h0} e^{(-\phi T_{eff}/ZT_h)} \tag{5}$$

$$n\frac{\partial v}{\partial t} + nv\frac{\partial v}{\partial x} + n\mu\frac{\partial \phi}{\partial x} + 3\sigma\frac{\partial n}{\partial x} = 0 \tag{6}$$

These equation is treated with stretched space–time coordinates ξ and η [18]

$$\xi = \varepsilon^{1/2}(x - \lambda_0 t), \quad \eta = \varepsilon^{3/2} t$$

ε represents the small dimensionless expansion parameter, and λ_0 is the phase velocity of the dust acoustic wave. The quantities like n, n_e, n_c, n_h, v, ϕ are expanded about the equilibrium state in terms of ε. Hence,

$$n = 1/Z + \varepsilon n_1 + \varepsilon^2 n_2 + \dots,$$
$$n_j = n_{j0} + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \dots,$$
$$f = \varepsilon f_1 + \varepsilon^2 f_2 + \dots$$

where j signifies e, c, and h and $f \equiv \phi, v$.

Now we drive Korteweg–de Vries (KdV) equation using reductive perturbation technique. For this, various equations are obtained at various orders of ε. For example,

At order $\varepsilon^{5/2}$,

$$-\lambda_0 n_{2\xi} + n_{1\eta} + 1/Z v_{2\xi} + n_1 v_{1\xi} + v_1 n_{1\xi} = 0 \tag{7}$$

$$\phi_{1\xi\xi} - n_{e2} + n_{c2} + n_{h2} + n_2 \mu Z = 0 \tag{8}$$

$$n_{e2} = n_{e0}\phi_2 T_{eff}/ZT_e \tag{9}$$

$$n_{c2} = -n_{c0}\phi_2 T_{eff}/ZT_c \tag{10}$$

$$n_{h2} = -n_{h0}\phi_2 T_{eff}/ZT_h \tag{11}$$

$$-\frac{\lambda_0}{Z}v_{2\xi} - \lambda_0 n_1 v_{1\xi} + \frac{1}{Z}v_{1\eta} + \frac{v_1}{Z}v_{1\xi} + \frac{\mu}{Z}\phi_{2\xi} + \mu n_1 \phi_{1\xi} + 3\sigma n_{2\xi} = 0 \tag{12}$$

The subscript ξ and η represents the respective differentiation.

For KdV equation, the second-order quantities will be eliminated using Eqs. (7)–(12) and substituting $v_1 = \lambda_0 n_1 Z$ and $\phi_1 = \left(\lambda_0^2 - 3\sigma\right) n_1 Z/\mu$, together with $\lambda_0 = \sqrt{\frac{\mu^2}{N_p T_{eff}} + 3\sigma}$ which is derived from the first-order equations. Here $N_p = \left(\frac{n_{e0}}{T_e} + \frac{n_{c0}}{T_c} + \frac{n_{h0}}{T_h}\right)$. The KdV equation thus obtained is

$$n_{1\eta} + \alpha n_1 n_{1\xi} + \beta n_{1\xi\xi\xi} = 0 \tag{13}$$

Here, $\alpha = \frac{3Z}{2}\left[\lambda_0 - \frac{\sigma}{\lambda_0}\right]$ is the coefficient of nonlinear term of the equation, whereas $\beta = \frac{\lambda_0^3}{2\mu^2}\left[1 - \frac{3\sigma}{\lambda_0^2}\right]^2$ is the coefficient of dispersive term. The KdV equation is further solved using the transformation $\tau = \xi - U\eta$, where U is a constant velocity [19]. On integration under the boundary conditions that $n_1(\tau) \to 0$ and its derivative $n_{1\tau} \to 0$, at $\tau \to \pm\infty$, the solution of the KdV equation is obtained as:

$$n_1(\tau) = n_m \mathrm{sech}^2(\tau/W) \tag{14}$$

where $n_m = 3U/\alpha$ represents the amplitude of the soliton and $W = 2\sqrt{\beta/U}$ represents the width of the soliton. In addition to these properties of the soliton, we can evaluate the soliton energy (E) based on the formula $E = \frac{4n_m^2 W}{3}$ [19].

Inhomogeneous plasma with weak density gradient

The basic fluid Eqs. (1)–(6) are treated with following stretched space–time coordinates τ and χ [18], appropriate for the inhomogeneous plasma

$$\tau = \varepsilon^{1/2}\left(\frac{x}{\lambda_0} - t\right), \quad \chi = \varepsilon^{3/2} x$$

The quantities $n, n_e, n_c, n_h, \phi, v$ are expanded about the equilibrium state in terms of ε in the following manner for the case of inhomogeneous plasma

$$f = f_0(x) + \varepsilon f_1(x,t) + \varepsilon^2 f_2(x,t) + \cdots \tag{15}$$

where $f \equiv \left(n, n_e, n_c, n_h, \phi, v\right)$.

As was done in part 1, here also we summarise the various equations obtained at different orders of ε. For example

At order $\varepsilon^{5/2}$,

$$\begin{aligned} &-\lambda_0 n_{2\tau} + n_0 v_{2\tau} + n_1 v_{1\tau} + v_1 n_{1\tau} + v_0 n_{2\tau} \\ &\quad + \lambda_0 v_1 n_{0\chi} + \lambda_0 n_0 v_{1\chi} + \lambda_0 v_0 n_{1\chi} = 0 \end{aligned} \tag{16}$$

$$\frac{1}{\lambda_0^2}\phi_{1\tau\tau} - n_{e2} + n_{c2} + n_{h2} + n_2 \mu Z = 0 \tag{17}$$

$$n_{e2} = n_{e0}\phi_2 T_{\mathrm{eff}}/ZT_e \tag{18}$$

$$n_{c2} = -n_{c0}\phi_2 T_{\mathrm{eff}}/ZT_c \tag{19}$$

$$n_{h2} = -n_{h0}\phi_2 T_{\mathrm{eff}}/ZT_h \tag{20}$$

$$\begin{aligned} &-\lambda_0 n_0 v_{2\tau} - \lambda_0 n_1 v_{1\tau} + n_0 v_0 v_{2\tau} + n_0 v_1 v_{1\tau} + n_1 v_0 v_{1\tau} \\ &\quad + \lambda_0 n_0 v_0 v_{1\chi} + \mu n_0 \phi_{2\tau} + \mu n_1 \phi_{1\tau} + \lambda_0 \mu n_0 \phi_{1\chi} \\ &\quad + \lambda_0 \mu n_1 \phi_{0\chi} + 3\sigma n_{2\tau} + 3\sigma \lambda_0 n_{1\chi} = 0 \end{aligned} \tag{21}$$

For KdV equation, the second-order quantities are eliminated using Eqs. (16)–(21) and substituting $v_1 = (\lambda_0 - v_0)n_1/n_0\, Z, \phi_1 = \left((\lambda_0 - v_0)^2 - 3\sigma\right)n_1/\mu n_0$, and $\lambda_0 = v_0 \pm \sqrt{\frac{n_0\mu^2 Z^2}{N_p T_{\mathrm{eff}}} + 3\sigma}$ which are derived from first-order equations. A modified form of KdV equation thus obtained is

$$n_{1\chi} + \alpha n_1 n_{1\tau} + \beta n_{1\tau\tau\tau} + \delta n_1 n_{0\chi} = 0 \tag{22}$$

together with $\alpha = \frac{3}{2\lambda_0^2 n_0}\left[(\lambda_0 - v_0) - \frac{\sigma}{(\lambda_0 - v_0)}\right]$,

$\beta = \frac{(\lambda_0 - v_0)}{2\lambda_0^4 \mu^2 Z n_0}\left[(\lambda_0 - v_0) - \frac{3\sigma}{(\lambda_0 - v_0)}\right]^2$ and

$\delta = \frac{1}{2\lambda_0 n_0}\left[(\lambda_0 - v_0) - \frac{3\sigma}{(\lambda_0 - v_0)}\right]$,

The presence of coefficient δ and hence the corresponding additional term is the result of density gradient present in the plasma. The solution of this KdV equation is analysed in two ways: one for weak inhomogeneity and the other for strong inhomogeneity of number density in plasma.

For weakly inhomogeneous plasma, we substitute $n_1 = g(\chi)n(\tau,\chi)$ and $g(\chi) = \exp\left[-\int \delta\left(\frac{\partial n_0}{\partial \chi}\right)\mathrm{d}\chi\right]$ so that Eq. (22) gets reduced to the following form

$$n_\chi + g\alpha n n_\tau + \beta n_{\tau\tau\tau} = 0 \tag{23}$$

This equation is solved using the transformation $\zeta = \tau - U\chi$, where U is a constant velocity [19]. The usual procedure of integration under the boundary conditions that $n \to 0$ and $n_\zeta \to 0$ at $\zeta \to \pm\infty$ yields the following solution

$$n(\zeta) = n_m \mathrm{sech}^2(\zeta/W) \tag{24}$$

where $n_m = 3U/g\alpha$ represents the amplitude of the soliton together with $g = \mathrm{e}^{-\delta'\left(\int \frac{1}{n_0}\mathrm{d}n_0\right)} = \mathrm{e}^{-\delta' \ln(n_0)} = \mathrm{e}^{-\ln(n_0)^{\delta'}} = \frac{1}{n_0^{\delta'}}$, where $\delta' = \frac{1}{2\lambda_0}\left[(\lambda_0 - v_0) - \frac{3\sigma}{(\lambda_0 - v_0)}\right]$. $W = 2\sqrt{\beta/U}$ represents the width of the soliton.

Strongly inhomogeneous plasma

For plasma with strong density gradient, the solution of the modified KdV equation is obtained using sin-cosine method [20] with the variable transformation as follows

$$\psi = Q(\tau - U\chi), n_1(\tau,\chi) = n_1(\psi) \tag{25}$$

Here Q is the inverse of the width of the solitary wave [14] moving with velocity U in a frame. Equation (22) with the density gradient $n_{0\psi}$ will be:

$$-QU\frac{\mathrm{d}n_1}{\mathrm{d}\psi}+Q\alpha n1\frac{\mathrm{d}n_1}{\mathrm{d}\psi}+\beta Q^3\frac{\mathrm{d}^3n_1}{\mathrm{d}\psi^3}-QU\delta n_{0\psi}n_1=0 \quad (26)$$

The sine–Gordon equation can be used in solving nonlinear wave equation. The travelling wave solution of Eq. (26) is given by [14]

$$n_1(u)=\sum_{r=1}^{n}\left(S_r\sin u+P_r\cos u\right)\cos^{r-1}u+P_0 \quad (27)$$

$$\frac{\mathrm{d}u}{\mathrm{d}\psi}=\sin u$$

When the linear and nonlinear terms of Eq. (26) are compared, the solution is expressed as

$$n_1(u)=S_1\sin u+S_2\sin u\cos u+P_1\cos u+P_2\cos^2 u+P_0 \quad (28)$$

The above solution (28) is substituted in the differential Eq. (26), and the differential equation is transformed into trigonometric polynomial equation. Collecting the terms with similar power of $\sin^m u\ \cos^n u$(where m and $n=0,1,\ldots$), we have

$$Q\alpha P_0P_1+Q\alpha S_1S_2-2\beta Q^3P_1+QU\delta n_{0\psi}P_0-QUP_1=0 \quad (29a)$$

$$\left(Q\alpha P_0S_2+Q\alpha P_1S_1-5\beta Q^3S_2+QU\delta n_{0\psi}S_1-QUS_2\right)\sin u=0 \quad (29b)$$

$$\left(2Q\alpha P_0P_2+Q\alpha P_1^2-Q\alpha S_1^2+Q\alpha S_2^2-16\beta Q^3P_2+QU\delta n_{0\psi}P_1-2QUP_2\right)\cos u=0 \quad (29c)$$

$$\left(Q\alpha P_0S_1-2Q\alpha P_1S_2-2Q\alpha P_2S_1-5\beta Q^3S_1-QU\delta n_{0\psi}S_2-QUS_1\right)\sin u\cos u=0 \quad (29d)$$

$$\left(Q\alpha P_0P_1-3Q\alpha P_1P_2+4Q\alpha S_1S_2-8\beta Q^3P_1-QU\delta n_{0\psi}P_2-QUP_1\right)\cos^2 u=0 \quad (29e)$$

$$\left(2Q\alpha P_0S_2+2Q\alpha P_1S_1-3Q\alpha P_2S_2-28\beta Q^3S_2-2QUS_2\right)\sin u\cos^2 u=0 \quad (29f)$$

$$\left(2Q\alpha P_0P_2+Q\alpha P_1^2-2Q\alpha P_2^2-Q\alpha S_1^2+3Q\alpha S_2^2-40\beta Q^3P_2-2QUP_2\right)\cos^3 u=0 \quad (29g)$$

$$\left(3Q\alpha P_1S_2+3Q\alpha P_2S_1+6\beta Q^3S_1\right)\sin u\cos^2 u=0 \quad (29h)$$

$$\left(3Q\alpha P_1P_2-3Q\alpha S_1S_2+6\beta Q^3P_1\right)\cos^4 u=0 \quad (29i)$$

$$\left(4Q\alpha P_2S_2+24\beta Q^3S_2\right)\sin u\cos^4 u=0 \quad (29j)$$

$$\left(2Q\alpha P_2^2-2Q\alpha S_2^2+24\beta Q^3P_2\right)\cos^5 u=0 \quad (29k)$$

Using these equations, the results obtained are $P_0=\left(U+8\beta Q^2\right)/\alpha$, $P_1=-U\delta n_{0\psi}/\alpha$, $P_2=-2\beta Q^2/\alpha$, $S_1=S_2=0$.

From these results, we can rewrite Eq. (28) as

$$n_1(u)=P_1\cos u+P_2\cos^2 u+P_0=-P_2\sin^2 u+P_1\cos u+P_0+P_2$$

Hence, final solution is obtained as [14]:

$$n_1(\psi)=n_1(\tau,\chi)=-\left(\frac{U}{\alpha}\right)\mathrm{sech}^2\left[\frac{(\tau-U\chi)}{Q^{-1}}\right]-\left\{\left(\frac{U\delta n_{0\psi}}{\alpha}\right)\tanh\left[\frac{(\tau-U\chi)}{Q^{-1}}\right]+\left(\frac{2U}{\alpha}\right)\right\}$$

Here width of the solitary wave is Q^{-1} together with $Q^2=-U/2\beta$. The negative value of β will only give positive value for soliton width; hence, it indicates that only slow mode corresponds to the evolution of solitons in strongly inhomogeneous plasma. Since the coefficient of dispersive term β stays positive for the fast mode, the soliton width becomes imaginary. This is not physically acceptable, and hence, the fast mode does not evolve in the form of soliton. This is due to that fact that the balance between the effects of nonlinearity and dispersion is not attained in the plasma for the case of fast mode due to larger frequency of oscillations. For the case of slow mode, the solution can be written as $n_1=n_{\mathrm{main}}\left(\mathrm{sech}^2\mathrm{term}\right)+n_T(\text{second Term})$, which implies that with the main soliton there will be a tail-like structure because of the density gradient in the plasma.

Results and discussion

The analysis of all the three types of the modes and their evolution as solitons in dusty plasma is done using the following parameters: dust number density $n_0=10^{12}\ \mathrm{m}^{-3}$, cold ion number density $n_{c0}=10^{13}\ \mathrm{m}^{-3}$, electron number density using quasineutrality equation $n_{e0}=n_{c0}+n_{h0}+\mu Zn_0$, charge on dust $Z=1000$, nature of dust charge $\mu=+1$ (positive charge) mass of the dust $m=10^{-23}$ kg, the constant velocity for soliton $U=1$, temperature in eV for dust as $K_BT_D=0.01$ eV, for electron as $K_BT_e=5$ eV, for cold ion as $K_BT_c=0.1$ eV, for hot ion as $K_BT_h=1$ eV,

Figure 1 represents the soliton structure in homogeneous plasma, Fig. 2 in weakly inhomogeneous plasma and Fig. 3 in strongly inhomogeneous plasma. It is clear from the graphs that, for the same number density of dust, the charge on the dust plays a major role in modifying the soliton's profile in plasmas. All the three solitons are the

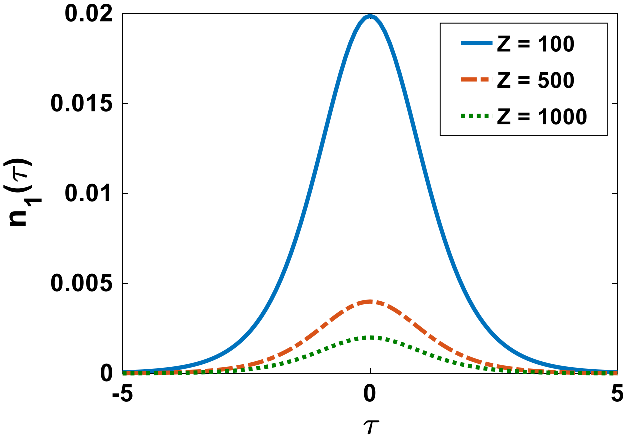

Fig. 1 Modification of soliton structure in homogeneous plasma with the variation of charge on the dust Z, when $K_BT_d=0.01$ eV, $K_BT_e=5$ eV, $K_BT_h=1$ eV, $K_BT_c=0.1$ eV, $n_0=10^{12}$ m^{-3}, $n_{c0}=10^{13}$ m^{-3}, $n_{h0}=1000\times n_{c0}$, $n_{e0}=n_{c0}+n_{h0}+\mu n_0Z$, $v_0=10^{-1}C_s$ and $U=1$

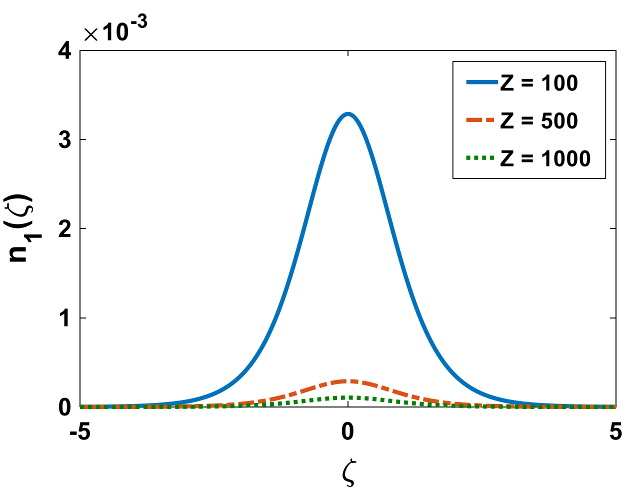

Fig. 2 Modification of soliton structure in weakly inhomogeneous plasma with the variation of charge on the dust Z for the same parameters as in Fig. 1

compressive solitons. In the present plasma of two ion species, the increasing charge on the dust particles leads to the decrease in peak amplitude of all the solitons and the solitons evolve with smaller size; similar effects of dust parameters on the soliton amplitude and width were observed by Dorranian group [6, 7]. This is due to the reduced restoring force owing to the reduction in electron number density. For the homogeneous plasma, when the charge on the dust is doubled (increased from 500 to 1000), the peak amplitude falls by 49.9%, whereas it reduces to 63.65% for weakly inhomogeneous plasma and 50.05% for strongly inhomogeneous plasma. In the case of strongly inhomogeneous plasma, the soliton evolves with significant tailing structure for the higher charge on

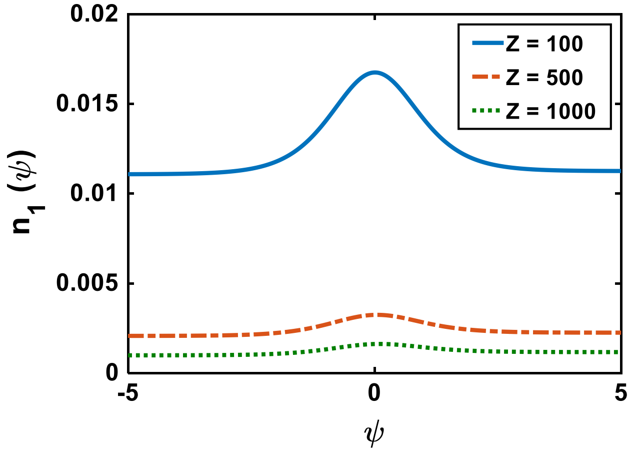

Fig. 3 Modification of soliton structure in strongly inhomogeneous plasma with the variation of charge on the dust Z for the same parameters as in Fig. 1

Table 1 Effect of Z on α and β of homogeneous and inhomogeneous plasmas including the coefficient δ in inhomogeneous plasma

Z	Homogeneous plasma		Inhomogeneous plasma		
	α	β	α	β	δ
100	150.9170	0.4911	− 179.07	− 0.6915	53.5009
500	750.1831	0.4996	− 924.6476	− 0.7590	277.3491
1000	1500.1	0.4999	− 1851.2	− 0.7613	555.3392

the dust particles. This is understood based on the drastic change in the coefficient δ when Z increases from 100 to 1000 (Table 1). This table also justifies the reduction in soliton amplitude with higher value of Z, as the nonlinearity coefficient α goes much higher.

Figures 4 and 5 show the effect of density of cold ions on solitons profiles. Our calculation shows that there is hardly any impact of cold ion density on the soliton structure in homogeneous plasma. But for weakly inhomogeneous plasma, the structure sees a significant change for higher value of cold ion density; here the increase in peak amplitude of soliton is 11.65% for the enhanced ion density by 100 times (10^{15} to 10^{17} m$^{-3)}$. For the same variation in cold ion density, the rise in amplitude in strongly inhomogeneous plasma is 2.84%. Based on this, we can conclude that significant difference is observed in the soliton structure when the cold ion density is much higher than the dust density. The variation of soliton amplitude with n_{c0} is in accordance with the variation of nonlinearity coefficient α with n_{c0}. Table 2 shows a small increment in α with n_{c0} in homogeneous plasma, whereas this coefficient is reduced much significantly with n_{c0} in inhomogeneous plasma. Hence, there is a less change in soliton amplitude in homogeneous plasma and a fast increment in amplitude in weakly inhomogeneous

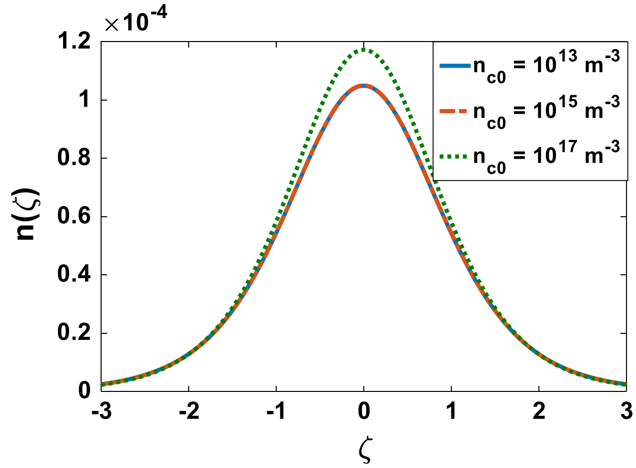

Fig. 4 Modification in soliton structure in weakly inhomogeneous plasma with the variation of cold ion density for the same parameters $K_BT_d=0.01$ eV,$K_BT_e=5$ eV, $K_BT_h=1$ eV, $K_BT_c=0.1$ eV, $n_0=10^{12}$ m^{-3}, n_{h0} = 10^{16} m^{-3}, $n_{e0}=n_{c0}+n_{h0}+\mu n_0 Z$, $v_0=10^{-1}C_s$ and $U=1$

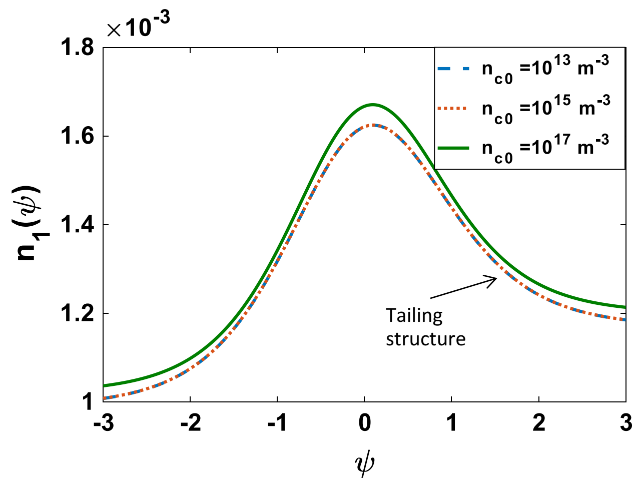

Fig. 5 Modification in soliton structure in strongly inhomogeneous plasma with the variation of cold ion density for the same parameters as in Fig. 4

plasma. The change in the tailing structure in strongly inhomogeneous plasma can be investigated based on the variation of coefficient δ with n_{c0}. Since there is a little change in δ in comparison with α, the soliton amplitude changes at a faster rate and the tail does not change much.

The role of cold ions to the soliton evolution is further clarified through Figs. 6, 7 and 8, where we have set the ratio of cold ion density to that of hot ion as 1:000. Under this situation of hot ion density much higher than that of dust and cold ions, the soliton profile is found to modify in a dramatic manner. In the homogeneous plasma, Fig. 6 shows the peak to reduce by 36.7% when the cold ion density is

Table 2 Effect of n_{c0} on α and β of homogeneous plasma and α, β and δ of inhomogeneous plasma

n_{c0} (m^{-3})	Homogeneous plasma		Inhomogeneous plasma		
	α	β	α	β	δ
10^{13}	1500.1	0.4999	−1851.2	−0.7613	555.3392
10^{15}	1509.2	0.4912	−1790.8	−0.6916	535.0383
10^{17}	2384.2	0.2324	−566.2967	−0.0131	113.2396

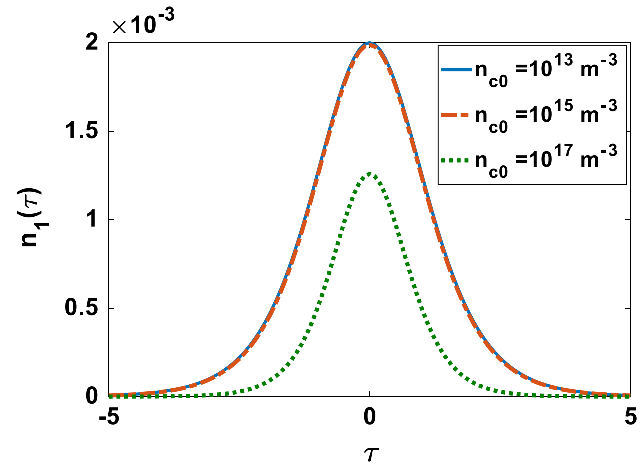

Fig. 6 Homogeneous plasma: Sketch of soliton profile with the variation of cold ion density, when the hot ion density is in the ratio of 1000:1 with the cold ions. The other parameters are $K_B T_d=0.01$ eV,$K_BT_e=5$ eV, $K_BT_h=1$ eV, $K_BT_c=0.1$ eV, $Z=1000$, $n_0=10^{12}$ m^{-3}, $n_{e0}=n_{c0}+n_{h0}+\mu n_0 Z$, $v_0=10^{-1}C_s$ and $U=1$

increased by 100 times (10^{15} to 10^{17} m^3), whereas this was observed only 0.5%. Now for the same change in density n_{c0}, the peak soliton amplitude is increased by 2500% in weakly inhomogeneous plasma. However, this enhancement in the amplitude is 216.23% in the case of strongly inhomogeneous plasma. In inhomogeneous plasma, the opposite behaviour of soliton amplitude and soliton width is also noticed, consistent to the previous case.

Figures 9, 10 and 11 clearly depict that increase in the value of dust number density increases the soliton amplitude in homogeneous plasma, whereas it decreases the soliton amplitude in inhomogeneous plasmas. The impact of charge and number density of the dust on the soliton structure in homogeneous plasma and weakly inhomogeneous plasma is depicted in Figs. 9 and 10. Figure 11 is devoted to the case of strongly inhomogeneous plasma, where the effect of dust density and density gradient is shown. For homogeneous and weakly inhomogeneous plasmas, Z has similar effect on the solitons. However, dust density n_0 imparts different effects. The increase in dust density in homogeneous plasma increases the peak amplitude of the soliton; if the

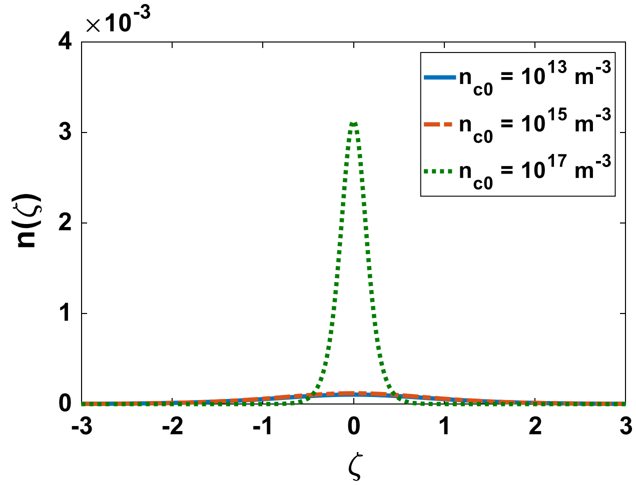

Fig. 7 Weakly inhomogeneous plasma: sketch of soliton profile with the variation of cold ion density, when the hot ion density is in the ratio of 1000:1 with the cold ions. The parameters are the same as in Fig. 6

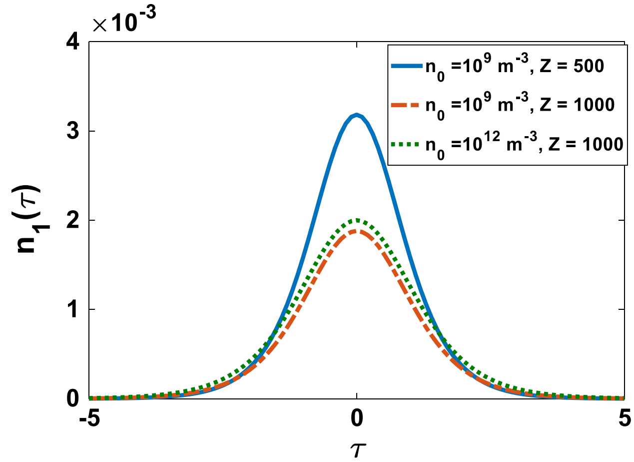

Fig. 9 Homogeneous plasma: plot depicts the effect of dust charge and number density together on soliton structure, when $K_B T_d = 0.01$ eV, $K_B T_e = 5$ eV, $K_B T_h = 1$ eV, $K_B T_c = 0.1$ eV, $n_{c0} = 10^{13}$ m^{-3}, $n_{h0} = 1000 \times n_{c0}$, $n_{e0} = n_{c0} + n_{h0} + \mu n_0 Z$, $v_0 = 10^{-1} C_s$ and $U = 1$

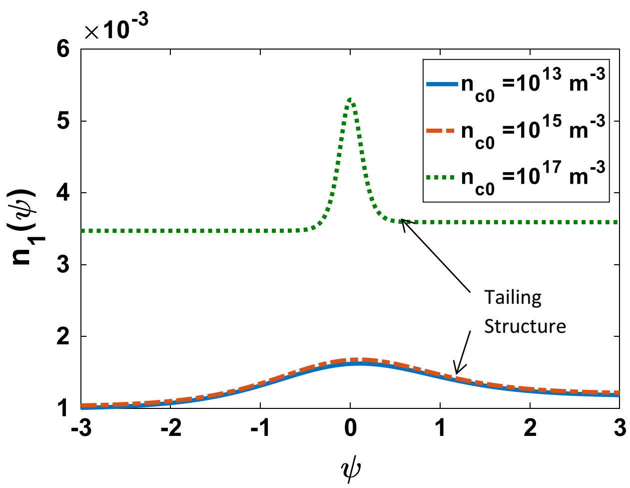

Fig. 8 Strongly inhomogeneous plasma: Sketch of soliton profile with the variation of cold ion density, when the hot ion density is in the ratio of 1000:1 with the cold ions. The parameters are the same as in Fig. 6

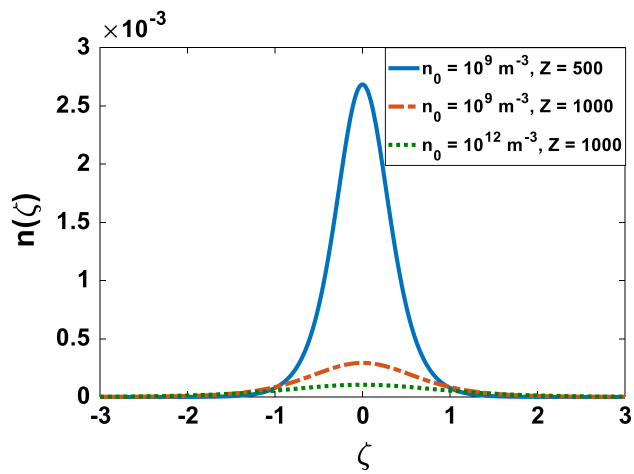

Fig. 10 Weakly inhomogeneous plasma: plot depicts the effect of dust charge and number densities together on soliton structure for the parameters the same as in Fig. 9

dust density is increased by 1000 times (from 10^9 to 10^{12}) the peak amplitude increases by 6.38%, whereas reverse effect is seen in weakly inhomogeneous plasma. Here the peak amplitude of the soliton decreases by 64.16%, when the dust density is increased by 1000 times (from 10^9 to 10^{12}). If strong inhomogeneous plasma is closely observed, then the dust density is found to show similar result as that in weakly inhomogeneous plasma. In this case, the peak amplitude of soliton decreases by 24.36% for the same enhancement in the density gradient. The density gradient is found to enhance the soliton amplitude in strongly inhomogeneous plasma. Moreover, the soliton evolves with much significant tailing structure, consistent to the observation made by Singh et al. [14] in a relativistic plasma. The tailing structure is investigated in Fig. 12, where 3 times increase in density gradient results in 16.67% enhancement in tailing structure, and 9 times increase in density gradient causes the tailing structure to raise by 58.33%.

Now we investigate the potential profiles corresponding to the above solitons, and the variation of soliton energy in different plasmas. Figure 13 shows the impact of dust charge on the soliton energy in the three kinds of plasmas. The energy in all the cases is found to decrease with the increase in the dust charge. The soliton with lowest energy evolves in weakly inhomogeneous plasma, whereas the largest energy is realised

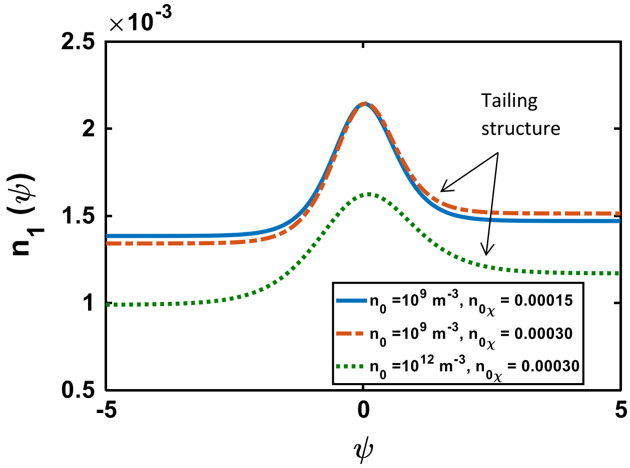

Fig. 11 Strongly inhomogeneous plasma: plot depicts the effect of density gradient and number density together on soliton structure, when $Z = 1000$ and the other parameters the same as in Fig. 9

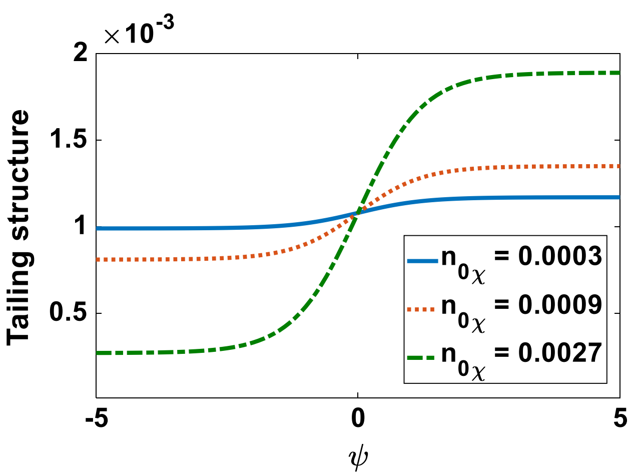

Fig. 12 Strongly inhomogeneous plasma: the tailing structure is observed with respect to the density gradient, when $K_BT_d = 0.01$ eV,$K_BT_e = 5$ eV, $K_BT_h = 1$ eV, $K_BT_c = 0.1$ eV, $n_{c0} = 10^{13}$ m^{-3},$n_0 = 10^{12}$ m^{-3}, $n_{h0} = 1000\ \times n_{c0}$, $n_{e0} = n_{c0} + n_{h0} + \mu n_0 Z$, $v_0 = 10^{-1}C_s$ and $U = 1$

in homogeneous plasma. Figure 14 shows the potential profiles in homogeneous, weakly inhomogeneous and strongly inhomogeneous plasmas. This is calculated using the formula $\phi_1 = \frac{Z}{\mu}\left(\lambda_0^2 - 3\sigma\right)\left(n_m \mathrm{sech}^2(\tau/W)\right)$ for homogeneous plasma and $\phi_1 = \left[\frac{1}{\mu n_0}\left(\lambda_0 - v_0\right)^2 - 3\sigma\right]\left(n_m \mathrm{sech}^2(\zeta/W)\right)$ for weakly inhomogeneous plasma and $\phi_1 = \left[\frac{1}{\mu n_0}\left(\lambda_0 - v_0\right)^2 - 3\sigma\right]$ $\left[-\left(\frac{U}{\alpha}\right)\mathrm{sech}^2\left[\frac{(\tau - U\chi)}{Q^{-1}}\right] - \left\{\left(\frac{U\delta n_{0\psi}}{\alpha}\right)\tanh\left[\frac{(\tau - U\chi)}{Q^{-1}}\right] + \left(\frac{2U}{\alpha}\right)\right\}\right]$ for strongly homogeneous plasma. The trend of potential profiles is found to be similar to the soliton energy in various plasmas.

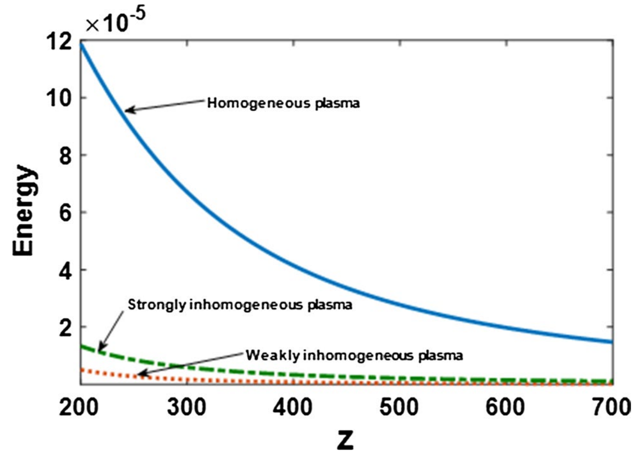

Fig. 13 Plot depicts the effect of dust charge on energy of soliton, when $K_BT_d = 0.01$ eV,$K_BT_e = 5$ eV, $K_BT_h = 1$ eV, $K_BT_c = 0.1$ eV, $n_{c0} = 10^{13}$ m^{-3},$n_0 = 10^{10}$ m^{-3}, $n_{h0} = 1000\ \times n_{c0}$, $n_{e0} = n_{c0} + n_{h0} + \mu n_0 Z$, $v_0 = 10^{-1}C_s$ and $U = 1$

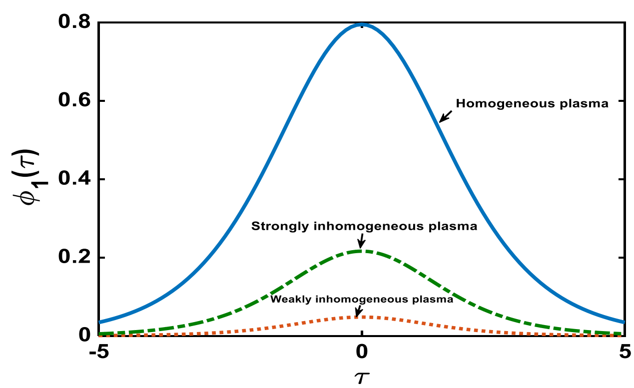

Fig. 14 Plot depicts the effect on potential profiles of homogeneous, weakly inhomogeneous and strongly inhomogeneous plasmas, when $K_BT_d = 0.01$ eV, $K_BT_e = 5$ eV, $K_BT_h = 1$ eV, $K_BT_c = 0.1$ eV, $n_{c0} = 10^{13}$ m^{-3},$n_0 = 10^{10}$ m^{-3}, $n_{h0} = 1000\ \times n_{c0}$, $n_{e0} = n_{c0} + n_{h0} + \mu n_0 Z$, $v_0 = 10^{-1}C_s$ and $U = 0.4$

Finally, we discuss the situation of electron depleted plasma, where all the electrons are attached to the dust particles. Figures 15 and 16 show the impact of n_0 on the soliton structure in electron depleted plasma for homogeneous case and strongly inhomogeneous case, respectively. Under this situation also, the dust charge reduces the size of the solitons in both kinds of the plasmas, similar to the earlier observations. However, the reduction is huge in the electron depleted plasmas. The most interesting result is that the reduction in soliton amplitude is exactly the same (~50%) in both the homogeneous and inhomogeneous electron depleted plasmas, when the dust charge is increased from 500 to 1000. There is no significant impact of other parameters on the soliton structures in electron depleted plasmas for all the three cases (figures not shown).

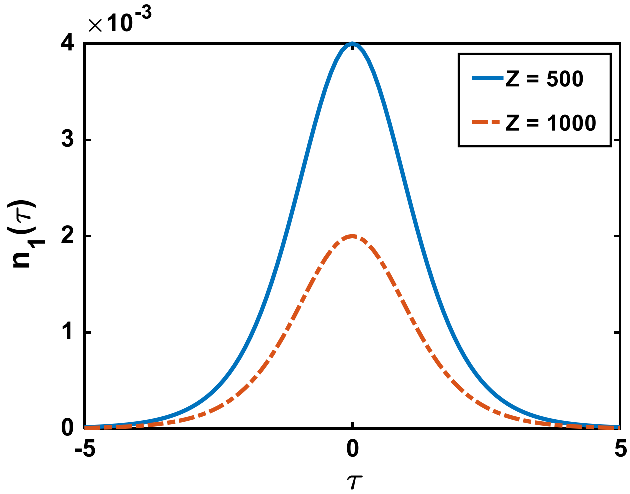

Fig. 15 Homogeneous plasma: Variation of soliton structure with Z for electron depleted plasma ($n_{e0}=0$), for the parameters the same as in Fig. 1

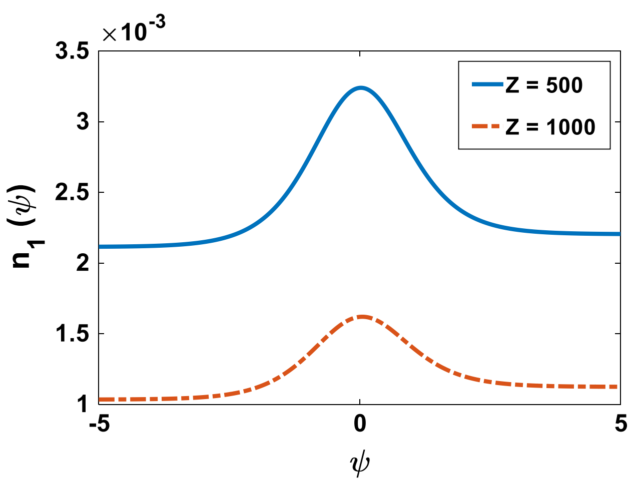

Fig. 16 Strongly inhomogeneous plasma: Variation of soliton structure with Z for electron depleted plasma ($n_{e0}=0$), for the parameters the same as Fig. 1

Conclusions

We have investigated the dust acoustic soliton evolution in homogeneous plasma, weakly inhomogeneous plasma and strongly inhomogeneous plasma having two-temperature ions. The dust charge is found to reduce the size of the solitons in all kinds of plasmas. The impact of cold ions on the soliton profiles becomes much significant when the density of hot ions remains larger than that of the cold ions. The dust density affects the soliton structure in an opposite manner in the homogeneous and inhomogeneous plasmas; solitons evolve with bigger size in homogeneous plasmas and of smaller size in inhomogeneous plasmas in the presence of more dust particles. The density gradient creates a tailing like structure with the main soliton. In the electron depleted plasmas, both homogeneous and inhomogeneous, the impact of dust charge on the soliton profile is huge and both the solitons attain exactly the same reduction in their amplitudes for the same enhancements in the dust charge.

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