Journal of Theoretical and Applied Physics
a SpringerOpen Journal

RESEARCH

Study on kink space-time in scale invariant theory with wet dark fluid

Bivudutta Mishra[*] and Pradyumn Kumar Sahoo

Abstract

In this paper, the kink space-time with wet dark fluid in scale invariant theory of gravitation is investigated. The gauge function β is considered as $\beta = \beta(r)$ and $\beta = \beta(t)$. The matter field is assumed to be perfect fluid. It has been found that perfect fluid does not survive in this theory in both the cases. Hence, the space-time in both the cases reduces to Minkowskian geometry and the space-time is flat.

Keywords: Kink space-time; Perfect fluid; Gauge function; Wet dark fluid; Scale invariant

AMS: 83F05

Introduction

There has been considerable interest in scale invariant theory owing to the scaling behaviour exhibited in high-energy particle scattering experiments (Callan et al. [1]). However, such theories are considered to be valid only in the limit of high energies or vanishing rest masses. This is because in elementary particle theories, rest masses are considered constants, and the scale invariance is generally valid only when the constant rest mass condition is relaxed.

Canuto et al. [2] formulated a scale covariant theory of gravitation by associating the mathematical operation of scale transformation with the physics, using different dynamical systems to measure space-times distances. For gravitational units, the gauge condition is chosen so that the standard Einstein equations are recovered.

In the alternative theory proposed by Brans and Dicke [3], there exists a variable gravitational parameter G. Another theory which admits a variable G is the scale covariant theory of Canuto et al. [2]. Dirac [4,5] rebuilt Weyl's unified theory by introducing notion of two metrics and an additional gauge function β. A scale invariant variation principle was proposed from which gravitational and electromagnetic field equations can be derived. It is concluded that an arbitrary gauge function is necessary in all scale invariant theories.

It is found that the scale invariant theory of gravitation agrees with general relativity up to the accuracy of observations made of up to now. Dirac [4,5], Hoyle and Narlikar [6] and Canuto et al. [2] have studied several aspects of the scale invariant theories of gravitation. But Wesson's [7] formulation is so far best to describe all the interactions between matter and gravitation in scale-free manner.

In the scale invariant theory of gravitation, Einstein's equation have been written in scale-independent way by performing the conformal or scale transformation as

$$\bar{g}_{ij} = \beta^2(x^k)g_{ij}, \tag{1}$$

where the gauge function $\beta(0 < \beta < 1)$ is in its most general formulation of all space-time coordinates. Thus, using the conformal transformation of the type given by Equation 1, Wesson [7] transforms the usual Einstein field equations into

$$G_{ij} + f_{ij} - \Lambda_0 g_{ij} = -\kappa T_{ij} \tag{2}$$

$$\beta f_{ij} = 2\beta\beta_{;ij} - 4\beta_{,i}\beta_{,j} - \left(g^{ab}\beta_{,a}\beta_{,b} - 2g^{ab}\beta_{;ab}\right) \tag{3}$$

$$G_{ij} \equiv R_{ij} - \frac{1}{2}Rg_{ij} \tag{4}$$

In these equations, G_{ij} is the conventional Einstein tensor involving g_{ij}. Semicolon and comma, respectively, denote covariant differentiation with respect to g_{ij} and

* Correspondence: bivudutta@yahoo.com
Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad 500078, Andhra Pradesh, India

Springer

partial derivatives with respect to coordinates. R_{ij} is the Ricci tensor and R is the Ricci scalar. The cosmological term Λg_{ij} of Einstein theory is now transformed to $\Lambda_0 \beta^2 g_{ij}$ in scale invariant theory with a dimensionless cosmological constants Λ_0. G and κ are respectively the Newtonian and Wesson's gravitational parameters. T_{ij} is the energy momentum tensor of the matter field and $\kappa = \frac{8\pi G}{c^4}$. A particular feature of this theory is that no independent equation for β exists.

Beesham [8-10], Mohanty and Mishra [11,12], Mishra and Mohanty [13], Mishra [14-16], Reddy and Venkateswaralu [17] have investigated several aspects of scale invariant theory. However, kink space-time with wet dark fluid has not been considered, so far, in the scale invariant theory of gravitation. Hence in this paper, we consider the kink space-time with wet dark fluid (WDF) in the scale invariant theory of gravitation. In Section 'Wet dark fluid', the WDF is discussed in detail. In Section 'Metric and field equations', with kink metric the field equations and its solutions are obtained, when the gauge function $\beta = \beta(r)$ and $\beta = \beta(t)$. Some discussions and the concluding remarks are given in Section 'Concluding remarks'. Finally lists of references are mentioned at the end.

Wet dark fluid

Recently, there has been considerable interest in cosmological model with 'Dark energy' (DE) in general relativity because of the fact that our universe is currently undergoing an accelerated expansion which has been confirmed by host of observations, such as type I supernovae (SNeIa) (Riess et al. [18]; Perlmutter et al. [19]; Bahcall et al. [20]), Sloan Digital Sky Survey (SDSS) (Tegmark et al. [21]), and Wilkinson Microwave Anisotropy Probe (WMAP) (Bennet et al. [22]; Hinshaw et al. [23]; Nolta et al. [24]). Based on these observations, cosmologists have accepted the idea of dark energy. Cosmologists have proposed many candidates for dark energy to fit the current observations, such as cosmological constant, tachyon, quintessence, and phantom. Current studies to extract the properties of a dark energy component of the universe from observational data focus on the determination of its equation of state $w(t) = \frac{p}{\rho}$ which is not necessarily constant. The methods for restoration of the quantity $w(t)$ from expressional data have been developed (Sahni and Starobinsky [25]), and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmological time (Sahni et al. [26]). Recently, the parameter $w(t)$ has been calculated with some reasoning which reduced to some simple parameterization of the dependences by some authors (Huterer and Turner [27]; Weller and Albrecht [28]; Linden and Virey [29]; Krauss et al. [30]; Usmani et al. [31]; Chen et al. [32]).

These observations provide us a clear outline of the universe: it is flat and full of undamped form of energy density pervading the universe. The undamped energy called DE with negative pressure attributes to about 74% of the total energy density. The remaining 26% of the energy density consists of matter including about 22% dark matter density and about 4% baryon matter density. So understanding the nature of DE is one of the most challenging problems in modern astrophysics and cosmology. Recent cosmological observations contradict the matter-dominated universe with decelerating expansion, indicating that our universe experiences accelerated expansion. We are motivated to use the WDF as a model for DE which seems, from an empirical equation of state proposed by Tait [33] and Hayward [34], to treat water and aqueous solution.

The equation for WDF is

$$p_{\mathrm{WDF}} = \gamma \left(\rho_{\mathrm{WDF}} - \rho^* \right), \qquad (5)$$

where the parameters γ and ρ^* are taken to be positive and $0 \le \gamma \le 1$.

To find the WDF energy density, we use the energy conservation equation as

$$\dot{\rho}_{\mathrm{WDF}} + 3H \left(p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}} \right) = 0. \qquad (6)$$

Applying the equation of state $3H = \frac{\dot{v}}{v}$, Equation 6 reduces to

$$\rho_{\mathrm{WDF}} = \left(\frac{\gamma}{1+\gamma} \right) \rho^* + \frac{c}{v(1+\gamma)}, \qquad (7)$$

where c is the velocity of light and v is the volume expansion.

WDF has two components: one behaves as a cosmological constant and other as standard fluid with equation of state $p = \gamma \rho$. If we take $c > 0$ in Equation 7, this fluid will not violate the strong energy condition:

$$\begin{aligned} p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}} &= (1+\gamma)\rho_{\mathrm{WDF}} - \gamma\rho^* \\ &= (1+\gamma)\frac{c}{v(1+\gamma)} \ge 0 \end{aligned} \qquad (8)$$

Holman and Naidu [35] used the WDF as dark energy in the homogeneous, isotropic FRW case. Singh and Chaubey [36] studied Bianchi type I universe with WDF.

Moreover, Chaubey [37] has investigated Bianchi type V universe with WDF and studied the solution for constant deceleration parameter, whereas in [38] studied the Bianchi type III and Kantowski-Sachs universe filled with dark energy from a WDF. In this paper, a new equation of state for the dark energy component of the universe has been used. Also Chaubey [39] investigated Bianchi type VI_0 universe filled with dark energy from a WDF and obtained the exact solutions to the corresponding field equations in quadrature form.

Metric and field equations

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij}^m = \left(p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}}c^2\right)U_i U_j + p_{\mathrm{WDF}}g_{ij} \tag{9}$$

together with

$$g_{ij}U^i U^j = -1, \tag{10}$$

where U^i is the four-velocity vector of the fluid, and p_{WDF} and ρ_{WDF} are the pressure and energy density of the WDF respectively.

A formulation of the general relativity theory is given in terms of three postulates about a mathematical model for space-time. This model is a manifold M with a metric g of Lorentz signature. The physical significance of the metric (space-time) is given by the first two postulates: those of local casualty and of local conservation of energy momentum. These postulates are common to both general and special theory of relativity and so are supported by the experimental evidence of the latter theory. The third postulate, the field equations for the metric g, is less well experimentally established. However, most of our results will depend only on the property of field equations that gravity is attractive for positive matter densities. This property is common to both general theory and alternative theories of relativity.

Recently, there has been some interest in exploring the conditions under which space-times with Finkelstein-Misner kinks are geodesically complete and, at the same time, satisfy reasonably strict energy condition. Our purpose in this paper is to construct this topic in relation to a broader class of kink space-time. The class of space-time that is under discussion is a special case of the general spherically space-time given by Letelier and Wang [40].

The kink space-time is considered as

$$ds^2 = -\cos2\alpha c^2 dt^2 - 2\sin2\alpha dr dt + \cos2\alpha dr^2 + r^2 d\Omega^2, \tag{11}$$

where $d\Omega^2 = d\theta^2 + \cos^2\theta d\phi^2$ and $\alpha = \alpha(r)$.

Case 1. When $\beta = \beta(r)$. With the above assumption of $\beta = \beta(r)$, the field Equation 2 for the metric Equation 11 with the co-moving coordinates $\left(0, 0, 0, \sqrt{\sec2\alpha}\right)$ can be written as

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin2\alpha\alpha_1 - 2\sin2\alpha\alpha_1\left(1 + 2\sin^2 2\alpha\right)\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - 2\cos2\alpha\frac{\beta_{11}}{\beta} + \frac{4}{r}\cos2\alpha\frac{\beta_1}{\beta}$$
$$+ \sec2\alpha\left(2\frac{\beta_{11}}{\beta} - 4\frac{\beta_1^2}{\beta^2}\right) - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{12}$$

$$\sin2\alpha\alpha_{11} + \frac{2}{r}\sin2\alpha\alpha_1 + 2\cos2\alpha\alpha_1^2 - 4\sin^3 2\alpha\alpha_1\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - \cos2\alpha\frac{\beta_{11}}{\beta} + \frac{2}{r}\cos2\alpha\frac{\beta_1}{\beta} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{13}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin2\alpha\alpha_1 + 2\sin2\alpha\alpha_1\left(1 - 2\sin^2 2\alpha\right)\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - 2\cos2\alpha\frac{\beta_{11}}{\beta} + \frac{4}{r}\cos2\alpha\frac{\beta_1}{\beta} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{14}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin2\alpha\alpha_1 + 2\sin2\alpha\alpha_1\left(1 - 2\sin^2 2\alpha\right)\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - 2\cos2\alpha\frac{\beta_{11}}{\beta} + \frac{4}{r}\cos2\alpha\frac{\beta_1}{\beta} - \Lambda_0$$
$$= \kappa c^2 \left(p_{\mathrm{WDF}} + c^2\rho_{\mathrm{WDF}}\right)\sec^2 2\alpha - \kappa c^2 p_{\mathrm{WDF}}. \tag{15}$$

Equations 14 and 15 yields

$$p_{\mathrm{WDF}} + \rho_{\mathrm{WDF}}c^2 = 0 \tag{16}$$

Now Equations 12, 13, 14, and 15 are reduced to three equations as

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin2\alpha\alpha_1 - 2\sin2\alpha\alpha_1\left(1 + 2\sin^2 2\alpha\right)\frac{\beta_1}{\beta} + \cos2\alpha\frac{\beta_1^2}{\beta^2}$$
$$- 2\cos2\alpha\frac{\beta_{11}}{\beta} + 2\cos2\alpha\frac{\beta_1}{\beta}\sec2\alpha\left(2\frac{\beta_{11}}{\beta} - 4\frac{\beta_1^2}{\beta^2}\right) - \Lambda_0$$
$$= -\kappa c^2 p_{\mathrm{WDF}} \tag{17}$$

$$\sin2\alpha\alpha_{11} + \frac{2}{r}\sin2\alpha\alpha_1 + 2\cos2\alpha\alpha_1^2 - 4\sin^3 2\alpha\alpha_1\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - 2\cos2\alpha\frac{\beta_{11}}{\beta} + \frac{2}{r}2\cos2\alpha\frac{\beta_1}{\beta} - \Lambda_0$$
$$= -\kappa c^2 p_{\mathrm{WDF}} \tag{18}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin2\alpha\alpha_1 + 2\sin2\alpha\alpha_1\left(1 - 2\sin^2 2\alpha\right)\frac{\beta_1}{\beta}$$
$$+ \cos2\alpha\frac{\beta_1^2}{\beta^2} - 2\cos2\alpha\frac{\beta_{11}}{\beta} + \frac{4}{r}\cos2\alpha\frac{\beta_1}{\beta} - \Lambda_0$$
$$= -\kappa c^2 p_{\mathrm{WDF}} \tag{19}$$

Due to the highly nonlinear nature of the field Equations 17, 18, and 19, we consider

$$\beta = \frac{1}{ar+b}, \tag{20}$$

where $\alpha(\neq 0)$ and b are real constants.

Now, the set of field Equations 17, 18, and 19 reduces to

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + \frac{2a}{ar+b}\sin 2\alpha\alpha_1 + \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1$$
$$+ \frac{3a^2}{(ar+b)^2}\cos 2\alpha + \frac{4a}{r(ar+b)}\cos 2\alpha - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}}$$
$$\tag{21}$$

$$2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta 4^2 -}{\beta^2} - A_c = -KC^2 p_{\mathrm{WDF}}$$
$$\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 - \frac{2a}{ar+b}\cos 2\alpha\alpha_1$$
$$- \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1 - \frac{3a^2}{(ar+b)^2}\cos 2\alpha - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}}$$
$$\tag{22}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - \frac{2a}{ar+b}\sin 2\alpha\alpha_1 + \frac{4a}{ar+b}\sin^3 2\alpha\alpha_1$$
$$- \frac{3a^2}{(ar+b)^2}\cos 2\alpha - \frac{4\alpha}{r(ar+b)}\cos 2\alpha - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}}$$
$$\tag{23}$$

Equations 21 and 23 yields α = constant; hence, the metric potential is a constant.

Case 2. When $\beta = \beta(r)$. Again, by use of commoving coordinates $\left(0,0,0,\sqrt{\sec 2\alpha}\right)$, the field Equation 2 for the metric Equation 11 can be written as

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - 2\sin 2\alpha\tan 2\alpha\alpha_1\frac{\beta_4}{\beta} + \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta}$$
$$+ 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta 4^2}{\beta^2} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{24}$$

$$\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 + 4\cos 2\alpha\alpha_1\frac{\beta_4}{\beta}$$
$$+ \frac{2}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta 4^2}{\beta^2} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}}$$
$$\tag{25}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - 2\sin 2\alpha\alpha_1\frac{\beta_4}{\beta} + \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta}$$
$$+ 2\cos 2\alpha\frac{\beta_{44}}{\beta} - \cos 2\alpha\frac{\beta 4^2}{\beta^2} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{26}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + 2(\sec 2\alpha + \cos 2\alpha)\alpha_1\frac{\beta_4}{\beta}$$
$$+ \frac{4}{r}\sin 2\alpha\frac{\beta_4}{\beta} + 2(\cos 2\alpha - \sec 2\alpha)\frac{\beta_{44}}{\beta} \tag{27}$$
$$+ (4\sec 2\alpha - \cos 2\alpha)\frac{\beta_4^2}{\beta^2} - \Lambda_0 = -\kappa c^4 \rho_{\mathrm{WDF}}\sec^2 2\alpha,$$

where the suffixes 1 and 4 denote ordinary differentiation with respect to r and t, respectively.

For a simple formulation of scale invariant theory, the gauge function is taken as

$$\beta = \frac{1}{ct} \tag{28}$$

Using the gauge function in Equation 28, the set of field Equations 24, 25, 26, and 27 reduces to

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\frac{\sin 2\alpha\tan 2\alpha\alpha_1}{t}$$
$$- \frac{2}{r}\frac{\sin 2\alpha}{t} + \frac{3\cos 2\alpha}{t^2} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{29}$$

$$\sin 2\alpha\alpha_{11} + \frac{2}{r}\sin 2\alpha\alpha_1 + 2\cos 2\alpha\alpha_1^2 - \frac{4\cos 2\alpha\alpha_1}{t} - \frac{2}{r}\frac{\sin 2\alpha}{t}$$
$$+ \frac{3\cos 2\alpha}{t^2} - \Lambda_0 = -\kappa c^2 p_{\mathrm{WDF}} \tag{30}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 - \frac{2\cos 2\alpha\alpha_1}{t} - \frac{4}{r}\frac{\sin 2\alpha}{t} + \frac{3\cos 2\alpha}{t^2} - \Lambda_0$$
$$= -\kappa c^2 p_{\mathrm{WDF}} \tag{31}$$

$$\frac{2}{r^2}\sin^2\alpha + \frac{2}{r}\sin 2\alpha\alpha_1 + 2(\sec 2\alpha + \cos 2\alpha)\alpha_1\frac{\beta_4}{\beta}$$
$$- \frac{4}{r}\frac{\sin 2\alpha}{t} + \frac{(5\cos 2\alpha - 8\sec 2\alpha)}{t^2} - \Lambda_0$$
$$= -\kappa c^4 \rho_{\mathrm{WDF}}\sec^2 2\alpha. \tag{32}$$

Equations 29 and 31 yield
$\alpha_1 = 0$, *i.e.* α = constant.
Thus, the metric potential is a constant.

Concluding remarks

It is a general belief that Einstein theory of general relativity is just a leading order of more general theory of gravity. This is due to the fact that although general relativity has passed many experimental tests, however, it fails to explain some recent observations such as accelerated expansion. Among various extensions of general relativity, the scale invariant gravity has got much

attention as one of promising candidates for explaining the current accelerating phases in the evolution of the universe.

In the formulation of scale invariant theory of gravitation, Wesson assumed (1) the metric is diagonal and spherically symmetric; (2) the gauge function β depends only one coordinate; and (3) the energy momentum tensor is that of a perfect fluid. Here, the scale invariant theory was taken along with the WDF. The gauge function was taken as functions of r and t in two different cases. In both the cases, it is found that space-time reduces to Minkowskian geometry, and the matter field does not survive in any case in this theory. Hence, the space-time is flat.

Competing interests

Authors' contributions

All calculations, derivations of the various results and their verifications were carried out by BM and PKS. Both authors read and approved the final manuscript.

Authors' information

BM and PKS are working as associate and assistant professors in the Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad, India.

Acknowledgements

References

1. Callan, CG, Coleman, S, Jackiw, R: A new improved energy-momentum tensor. Annals Physics **59**(1), 42–73 (1970)
2. Canuto, VM, Hseih, SH, Adams, PJ: Scale covariant theory of gravitation and astrophysical applications. Phys Rev Lett **39**(88), 429–432 (1977)
3. Brans, CH, Dicke, RH: Mach's principle and a relativistic theory of gravitation. Phys Rev **124**, 925–935 (1961)
4. Dirac, PAM: Long range forces and broken symmetries. Proc R Soc Lon **A333**, 403–418 (1973)
5. Dirac, PAM: Cosmological model and the large number hypothesis. Proc R Soc Lon **A338**, 439–446 (1974)
6. Hoyle, F, Narlikar, JV: Action at a Distance in Physics and Cosmology. Freeman, San Francisco U.S.A (1974)
7. Wesson, PS: Scale-invariant gravity - a reformulation and an astrophysical test. Mon Not R Astro Soc **197**, 157–165 (1981)
8. Beesham, A: Bianchi type I cosmological model in scale covariant theory. Clasic Quant Grav **3**, 481–486 (1986)
9. Beesham, A: Friedmann-Robertson-Walker vacuum solution in the scale covariant theory. Astophys Space Sci **119**, 415–420 (1986)
10. Beesham, A: Friedmann cosmology with a cosmological constant in the scale covariant theory. Clasic Quant Grav **3**, 1027–1030 (1986)
11. Mohanty, G, Mishra, B: Bianchi type VI$_1$ cosmological model in scale invariant theory. Czech J Phys **52**(6), 765–773 (2002)
12. Mohanty, G, Mishra, B: Scale invariant theory for Bianchi type VIII and IX space-times with perfect fluid. Astrophys Space Sci **283**, 67–74 (2003)
13. Mishra, B, Mohanty, G: Bianchi type V cosmological model in scale invariant theory. Bulg J Phys **34**, 252–259 (2007)
14. Mishra, B: Non-static plane symmetric Zeldovich fluid model in scale invariant theory. Chinese Phys Lett **21**(12), 2359–2361 (2004)
15. Mishra, B: Static plane symmetric Zeldovich fluid model in scale invariant theory. Turk J Phys **32**, 357–361 (2008)
16. Mishra, B: Scale invariant theory of gravitation in non-diagonal Bianchi type II space-time. The ICFAI Univ J Phys **4**, 59–64 (2009)
17. Reddy, DRK, Venkateswarlu, R: Birkhoff-type theorem in scale covariant theory of gravitation. Astrophys Space Sci **136**, 191–194 (1987)
18. Riess, AG, Filippenko, AV, Challis, P, Clocchiatti, A, Diercks, A, Garnavich, PM, Gilliland, RL, Hogan, CJ, Jha, S, Kirshner, RP, Leibundgut, B, Phillips, MM, Reiss, D, Schmidt, BP, Schommer, RA, Smith, RC, Spyromilio, J, Stubbs, C, Suntzeff, NB, Tonry, J, 3: Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron J **116**, 1009 (1998)
19. Perlmutter, S, Aldering, G, Goldhaber, G, Knop, RA, Nugent, P, Castro, PG, Deustua, S, Fabbro, S, Goobar, A, Groom, DE, Hook, IM, Kim, AG, Kim, MY, Lee, JC, Nunes, NJ, Pain, R, Pennypacker, CR, Quimby, R, Lidman, C, Ellis, RS, Irwin, M, McMahon, RG, Ruiz-Lapuente, P, Walton, N, Schaefer, B, Boyle, BJ, Filippenko, AV, Matheson, T, Fruchter, AS, Panagia, N, Newberg, HJM, 2, et al.: Measurements of Ω and Λ from 42 high-redshift supernovae. Astrophys J **517**, 565 (1999)
20. Bahcall, NA, Ostriker, JP, Perlmutter, S, Steinhardt, PJ: The cosmic triangle: revealing the state of the universe. Science **284**, 1481–1488 (1999)
21. Tegmark, M, Eisenstein, D, Strauss, M, Weinberg, D, Blanton, M, Frieman, J, Fukugita, M, Gunn, J, Hamilton, A, Knapp, G, Nichol, R, Ostriker, J, Padmanabhan, N, Percival, W, Schlegel, D, Schneider, D, Scoccimarro, R, Seljak, U, Seo, H, Swanson, M, Szalay, A, Vogeley, M, Yoo, J, Zehavi, I, Abazajian, K, Anderson, S, Annis, J, Bahcall, N, Bassett, B, Berlind, A, Brinkmann, J, 12, et al.: Cosmological constraints from the SDSS luminous red galaxies. Phys Rev D **74**, 1–34 (2006). 123507
22. Bennet, CL, Halpern, M, Hinshaw, G, Jarosik, N, Kogut, A, Limon, M, Meyer, SS, Page, L, Spergel, DN, Tucker, GS, Wollack, E, Wright, EL, Barnes, C, Greason, MR, Hill, RS, Komatsu, E, Nolta, MR, Odegard, N, Peirs, HV, Verde, L, Weiland, JL: First-year Wilkinson Microwave Anisotropy Probe Observations: preliminary maps and basic results. Astrophys J Suppl Ser **148**, 1–21 (2003)
23. Hinshaw, G, Weiland, JL, Hill, RS, Odegard, N, Larson, D, Bennett, CL, Dunkley, J, Gold, B, Greason, MR, Jarosik, N, Komatsu, E, Nolta, MR, Page, L, Spergel, DN, Wollack, E, Halpern, M, Kogut, A, Limon, M, Meyer, SS, Tucker, GS, Wright, EL: Five-year Wilkinson Microwave Anisotropy Probe Observations: data processing, sky maps and basic results. Astrophys J Suppl Ser **180**, 225–245 (2009)
24. Nolta, MR, Dunkley, J, Hill, RS, Hinshaw, G, Komatsu, E, Larson, D, Page, L, Spergel, DN, Bennett, CL, Gold, B, Jarosik, N, Odegard, N, Weiland, JL, Wollack, E, Halpern, M, Kogut, A, Limon, M, Meyer, SS, Tucker, GS, Wright, EL, 2: Five-year Wilkinson Microwave Anisotropy Probe Observations: angular power spectra. Astrophys. J. Suppl. **180**, 296–305 (2009)
25. Sahni, V, Starobinsky, A: Reconstructing dark energy. Int J Mod Phys **D15**(12), 2105–2132 (2006)
26. Sahni, V, Shafieloa, A, Starobinsky, A: Two new diagnostics of dark energy. Phys Rev D **78**(103502), 1–11 (2008)
27. Huterer, D, Turner, MS: Probing dark energy: methods and strategies. Phys Rev. D **64**(123527), 1–20 (2001)
28. Weller, J, Albrecht, A: Future supernovae observation as a probe of dark energy. Phys Rev D **65**(103512), 1–21 (2002)
29. Linden, S, Virey, JM: Test of the Chevallier-Polarski-Linder parameterization for rapid dark energy equation of state transitions. Phys Rev D **78**(023526), 1–8 (2008)
30. Krauss, LM, Jones-Smith, K, Hutter, D: Dark energy, a cosmological constant, type Ia supernovae. New J Phys **9**, 141 (2007)
31. Usmani, AA, Ghosh, PP, Mukhopadhayay, U, Ray, PC, Ray, S: The dark energy equation of state. Mon Not Roy Astron Soc Lett **386**, L92–L95 (2008)
32. Chen, CW, Gu, JA, Chen, P: Consistency test of dark energy model. Mod Phys Lett A **24**, 1649–1657 (2009)
33. Tait, PG: The Voyage of HMS Challenger. H.M.S.O, London (1998)
34. Hayward, ATJ: Compressibility equations for liquids: a comparative study. Brit J Appl Phys **18**(7), 965–977 (1967)
35. Holman, R, Naidu, S: Dark energy from wet dark fluid. Astrophysics arXiv. , 0408102 (2005). doi:arXiv:astro-ph/0408102
36. Singh, T, Chaubey, R: Bianchi type I universe with wet dark fluid. Pramana J Physics **71**(3), 447–458 (2008)
37. Chaubey, R: Bianchi type V universe with wet dark fluid. Astrophysics Space Sci **321**, 241–246 (2009)

38. Chaubey, R: Bianchi type III and KS universe with wet dark fluid.
     Int J Astronomy Astrophys **1**(2), 25–38 (2011)
39. Chaubey, R: Bianchi type VI_0 universe with wet dark fluid. Natural Sci
     3(10), 817–826 (2011)
40. Letelier, PS, Wang, AZ: Spherically symmetric thin shells in Brans-Dicke
     theory of gravity Phys. Rev D **48**, 631–646 (1993)