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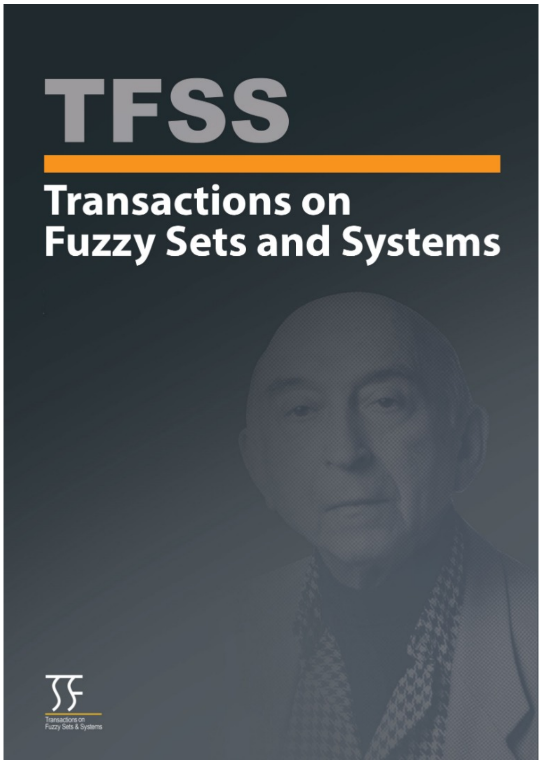

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Multidimensional Performance Assessment in Financial Contact Centers: An Interval-Valued Fuzzy Logic Approach to Debt Recovery Optimization

Muhammad Kamran , **Muhammad Tahir** , **Muhammad Farman*** , **Narzullayeva Umida Raxmatullayevna** , **Aseel Smerat**

(Special Section on "Dynamical Systems in Fuzzy Environments".)

Abstract. Businesses have seen increased difficulties in receiving payments due to the sluggish economic growth. This global tendency significantly impacts both affluent and impoverished nations. Enhancing debt collection practices is important to keeping economic stability. Debt collection firms and affiliated contact centers are enterprises that concentrate on securing payments for outstanding bills. Their proficiency simplifies the payment process. It is widely recognized that assessing the performance of a debt collection agency is challenging and often fraught with ambiguity. A performance measurement is a collection of statistically derived attributes that indicate the efficacy of an activity or its progress. Each project may be administered at an individual, team, departmental, or organizational level. The primary objective is to establish a systematic and unbiased methodology for debt collection by improving the performance of the call center. Initially, our research examined the issue of monitoring call center performance through an exploratory approach. The parent companies utilizing contact centers for debt collection and the call centers have convened numerous times. The proposed methodology offers an equitable assessment of performance criteria. This study initially presents indicator performance functions to ensure the outcomes are equitable. We formulate nine unique functions together with their corresponding parameters to objectively evaluate the effectiveness of the indicators, based on imprecise interval-valued Pythagorean fuzzy preference connections. This article is beneficial, as it presents a straightforward and adaptable method for monitoring the performance of a call center.

AMS Subject Classification 2020: 68T27; 68T37; 94D05
Keywords and Phrases: Call center, Performance measurement, Pythagorean fuzzy sets, Uncertainty modeling, Service quality.

1 Introduction

A business must expeditiously settle its debts to maintain financial stability and for long-term survival. Effective communication with consumers and prompt reimbursement necessitate specific skills [1]. The insolvency rate in Western Europe was projected to be 2.7% in 2019 [2]. The sluggish economic growth, decline in global trade, and issues within the industrial sector all contributed to this rate [3]. The prognosis for 2020 was far more adverse, with emerging nations facing heightened vulnerability. In recent years, call centers

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have undergone significant transformations. Numerous enterprises currently employ them to expedite debt collection [4]. Companies are progressively using various methods to communicate with clients, with contact centers being among the most significant. A call center is a technology-driven facility that manages incoming and outgoing calls. The call center employs operators who use sophisticated real-time tools to enhance operational efficiency. While some organizations retain in-house call center staff, outsourcing to third-party providers has emerged as the predominant practice. Effective oversight and administration are consistently required, regardless of the configuration. A systematic and objective performance management system is essential for enhancing payment processing [5].

One of the most effective methods for managers to assess the vitality of their operations is to utilize performance measurement (PM). The objective of this method is to monitor the performance of an activity consistently. Through the integration of performance, encompassing both objectives and relational frameworks, enterprises can efficiently and swiftly achieve their goals. Performance is variably defined across different contexts and organizations, as it is contingent upon the specific circumstances [6]. Researchers emphasize the need for correlations between activities and their outcomes to evaluate performance against a set standard. Performance evaluation comprises three primary components: relationship, goal, and characteristics [7]. A connection is the link between an individual, group, or organization and the external environment. The attributes encompass measurable factors such as cost, quality, and adaptability. The target denotes the expected level of performance from the organization under evaluation [8]. Project management is employed at several tiers within an organization. Individuals utilize it to identify methods for improvement and self-advancement, whereas superiors employ it to analyze historical outcomes and strategies for the future.

Numerous analytical techniques are utilized in PM studies for the evolution in corporate sustainability [9]. This category of studies includes three types: those that look at how important performance indicators are, those that look at how different indicators affect each other, and those that look at how performance elements affect each other. Business analysts ascertain the essential aspects requisite for effective project management and promote multi-criteria decision-making (MCDM) as the superior methodology for sustainable industry evaluation [10]. Key requirements include integrity, ensuring that the data is meaningful and reflects overall performance; granularity, allowing for the evaluation of specific organizational components; traceability, enabling the monitoring of performance over time; flexibility, permitting adjustments to the model to align with the company's needs; and the capacity to provide forward-looking insights [11].

Assessing a call center's success is difficult due to the interplay between subjective and objective factors in an unpredictable environment. Traditional methods of project management frequently overlook the subjective characteristics of indicator weights during computations. Numerous individuals discover that fuzzy set theory [12] enhances their decision-making in uncertain circumstances. Several challenges exist in employing standard fuzzy sets to depict linguistic notions [13]. Fuzzy multi-sets [14], hesitant fuzzy sets [15], intuitionistic fuzzy sets (IFSs) [16], pythagorean fuzzy sets (PFSs) [17] and type-2 fuzzy sets [18] are extensions designed to address uncertainties in complicated networks. Dombi aggregation operators (AOs) for intuitionistic linguistic decision-making difficulties are addressed by numerous authors to manage uncertainty more effectively [19]. Over the past two decades, scholars have extensively emphasized IFS and Interval valued IFSs (IVIFSs) [20] theories to tackle MADM issues by utilizing information of graphs and concept lattice measures. Xu and Chen [21] proposed weighted averaging and geometric aggregation operators for diverse intuitionistic fuzzy numbers (IFNs) in that context. Nonetheless, by incorporating the degree of reluctance of the IFNs into the analysis, Kumar et al. [22] presented the improved and interactive weighted AOs.

Peng [23] introduced several novel operations for interval-valued PFSs (IVPFSs). To address the MADM problem, Garg [24] established the concept of neutrality operations and the corresponding aggregation operators. Each set possesses distinct properties and restrictions attributable to bonding within the unit interval. This research examines the necessity for a comprehensive analytical tool to accurately reflect human opinion, given the ambiguity associated with evaluating call center performance. The proposed solution is based on IVPFSs,

which utilize imperfect preference relations (PR). The integration of membership and non-membership values differentiates IVPFSs from traditional techniques, [25]. This enhances the accuracy of the results, providing greater flexibility in data management and improving their alignment with expert opinions.
The primary objective of the study is to address the ambiguity associated with using linguistic parameters to evaluate the performance of a call center. Call centers are establishments where individuals communicate with consumers by phone. Individuals frequently evaluate their employment primarily based on qualitative criteria rather than quantitative metrics. The proposed approach integrates IVPFSs to address expert uncertainty, utilizes incomplete PR to enhance judgment accuracy, and applies pairwise comparisons to facilitate improved decision-making. Utilizing precision numbers or traditional fuzzy sets often results in the loss of expert assessments.
This research's main contributions can be summarized as follows:

- Developing a methodical and impartial approach to evaluate the efficiency of debt collection call centers, reducing the inherent subjective assessments.
- Introducing IVPFS parameters to precisely capture and manage expert uncertainty in performance evaluations while preserving the accuracy of judgements.
- Nine performance functions were developed along with criteria to objectively evaluate call center metrics, ensuring that assessments are equitable and consistent.
- Fixing the issue with qualitative performance criteria for call centers by developing ways to quantify subjective language evaluations.
- Integrating imprecise PR with pairwise comparisons to enhance judgement accuracy while ensuring data security.
- Offering a straightforward and adaptable solution specifically designed for debt collection contact centers, aimed at enhancing financial stability through improved performance assessment.

The study is summarised below:
In section 2, we defined the basic concepts and definitions that are essential for understanding the model. In section 3, we present the proposed technique of the model with all steps where we add an algorithm that shows the process of our technique. In section 4, we defined nine indicator performance functions, described their applications, and explained how they perform. In section 5, we support our study with some mathematical foundations that consist of theorems and propositions. Section 6 focuses on numerical evaluations and includes applications that utilise the proposed technique. In section 7, performance criteria are defined, and based on them, the ranking of alternatives is defined. The conclusion of the study is in section 8.

2 Basic Concepts and Definitions

This section systematically outlines the fundamental mathematical ideas utilized in our framework, proceeding from basic fuzzy sets to our novel Interval-Valued Pythagorean Fuzzy Preference Relationships (IVPFPRs).

Definition 2.1. *A fuzzy set (FS) [12], $\mathcal{A}$ in universe X characterizes through a membership function:*

$$\mathcal{A} = \{(x_\mu, \mathcal{T}_\mu) \mid x_\mu \in X\}, \quad \mathcal{T}_\mu : X \to [0,1]$$

where $\mathcal{T}_\mu(x_\mu)$ quantifies the membership degree of element x_μ in $\mathcal{A}$.
Example: *Consider $X = \{Call\ duration\}$. A FS "Long calls" might assign:*

$$\mathcal{T}_{long}(5min) = 0.2, \quad \mathcal{T}_{long}(10min) = 0.7$$

Definition 2.2. *An IFS [16] $\mathcal{I}$ in a universe of discourse X extends traditional FSs by incorporating both membership and non-membership degrees defined as:*

$$\mathcal{I} = \{(x_\mu, (\mathcal{T}_\mu, \mathcal{I}_\mu)) \mid x_\mu \in X\},$$

where:

$$\mathcal{T}_\mu, \mathcal{I}_\mu \in [0,1], \quad \mathcal{T}_\mu + \mathcal{I}_\mu \leq 1 \quad \forall x_\mu \in X$$

with the additional hesitation degree of x_μ:

$$\pi_\mu = 1 - \mathcal{T}_\mu - \mathcal{I}_\mu$$

Example: *For call center performance evaluation:*

$$(x_{quality}, (\mathcal{T}_{quality}, \mathcal{I}_{quality})) = (\textit{Customer Satisfaction}, 0.7, 0.2)$$

with hesitation $\pi_{quality} = 0.1$, indicating 70% positive evaluation, 20% negative evaluation and 10% missing information.

Definition 2.3. *A PFS [17], $\mathcal{P}$ is defined as:*

$$\mathcal{P} = \{(x_\mu, (\mathcal{T}_\mu, \mathcal{I}_\mu)) \mid x_\mu \in X\}$$

with constraints:

$$\mathcal{T}_\mu^2 + \mathcal{I}_\mu^2 \leq 1, \quad \mathcal{T}_\mu, \mathcal{I}_\mu \in [0,1]$$

Example: *For call quality assessment:*

$$(\mathcal{T}_{quality}, \mathcal{I}_{quality}) = (0.8, 0.5) \quad \textit{since } 0.8^2 + 0.5^2 = 0.89 \leq 1$$

Definition 2.4. *An IVFS [26], $\tilde{\mathcal{A}}$ models membership uncertainty using intervals:*

$$\tilde{\mathcal{A}} = \left\{ \left(x_\mu, [\tilde{\mathcal{T}}_\mu^L, \tilde{\mathcal{T}}_\mu^U] \right) \mid x_\mu \in X \right\}$$

where $0 \leq \tilde{\mathcal{T}}_\mu^L \leq \tilde{\mathcal{T}}_\mu^U \leq 1$.

Example: *"Satisfactory call duration" might be:*

$$\tilde{\mathcal{T}}_{satisfactory} = [0.6, 0.9]$$

indicating the membership degree lies somewhere between 0.6 and 0.9.

Definition 2.5. An IVPFS [23], $\tilde{\mathcal{P}}$ combines PFS and IVFS concepts:

$$\tilde{\mathcal{P}} = \left\{ \left(x_\mu, ([\tilde{\mathcal{T}}_\mu^L, \tilde{\mathcal{T}}_\mu^U], [\tilde{\mathcal{I}}_\mu^L, \tilde{\mathcal{I}}_\mu^U]) \right) \mid x_\mu \in X \right\}$$

with the generalized pythagorean condition:

$$(\tilde{\mathcal{T}}_\mu^U)^2 + (\tilde{\mathcal{I}}_\mu^U)^2 \leq 1$$

Definition 2.6. *An interval-valued pythagorean fuzzy preference relations (IVPFPR) $\tilde{\mathcal{Q}} = (\mathcal{G}_{\mu\nu})_{n\times n}$ is a matrix where each element $\mathcal{G}_{\mu\nu} = ([\tilde{\mathcal{T}}_{\mu\nu}^L, \tilde{\mathcal{T}}_{\mu\nu}^U], [\tilde{\mathcal{I}}_{\mu\nu}^L, \tilde{\mathcal{I}}_{\mu\nu}^U])$ satisfies:*

1. Boundary Conditions:

$$[\tilde{\mathcal{T}}^L_{\mu\nu}, \tilde{\mathcal{T}}^U_{\mu\nu}], [\tilde{\mathcal{I}}^L_{\mu\nu}, \tilde{\mathcal{I}}^U_{\mu\nu}] \subseteq [0,1]$$

2. Reciprocity:

$$\tilde{\mathcal{T}}_{\nu\mu} = \tilde{\mathcal{I}}_{\mu\nu}, \quad \tilde{\mathcal{I}}_{\nu\mu} = \tilde{\mathcal{T}}_{\mu\nu}$$

3. Consistency:

$$(\tilde{\mathcal{T}}^U_{\mu\nu})^2 + (\tilde{\mathcal{I}}^U_{\mu\nu})^2 \leq 1$$

4. Diagonal Neutrality:

$$\tilde{\mathcal{T}}_{\mu\mu} = \tilde{\mathcal{I}}_{\mu\mu} = ([0.5, 0.5], [0.5, 0.5])$$

Example: *For two alternatives A_1, A_2:*

$$\tilde{\mathcal{Q}} = \begin{pmatrix} ([0.5, 0.5], [0.5, 0.5]) & ([0.6, 0.8], [0.3, 0.4]) \\ ([0.3, 0.4], [0.6, 0.8]) & ([0.5, 0.5], [0.5, 0.5]) \end{pmatrix}$$

where $0.8^2 + 0.4^2 = 0.8 \leq 1$ satisfies the pythagorean condition.

Definition 2.7. *For missing elements $\mathcal{G}_{\mu\nu}$ in an incomplete IVPFPR:*

- *The arithmetic mean function is defined as below:*

$$\mathcal{G}_{\mu\nu} = \frac{\mathcal{G}^2_{\mu k} \oplus \mathcal{G}^2_{k\nu}}{2} \tag{1}$$

Example: If $\mathcal{G}_{12} = ([0.6, 0.7], [0.2, 0.3])$ and $\mathcal{G}_{23} = ([0.5, 0.6], [0.3, 0.4])$, then:

$$\mathcal{G}_{13} = \frac{([0.36, 0.49], [0.04, 0.09]) \oplus ([0.25, 0.36], [0.09, 0.16])}{2} = ([0.305, 0.425], [0.065, 0.125])$$

- *The geometric mean function is defined as below:*

$$\mathcal{G}_{\mu\nu} = \sqrt{\mathcal{G}^2_{\mu k} \otimes \mathcal{G}^2_{k\nu}} \tag{2}$$

The acceptability criterion states that an IVPFPR is complete if every row and column has at least one known off-diagonal element.

3 Proposed Technique

This section outlines the proposed approach for IVPFPR to measure the performance of the call centre as follows in Algorithm 1 in detail for better understanding:

Algorithm 1 Proposed Methodology

Input: Set of alternatives $A = \{A_1, A_2, \ldots, A_n\}$, linguistic scale (Table 1), expert evaluations.

Output: Ranked alternatives with IVPF scores.

procedure IVPFPR_DECISION

Step 1: Construct IVPFPR

Expert evaluates alternatives using linguistic scale (Table 1) to build IVPFPR $\tilde{\mathcal{Q}} = (\mathcal{G}_{\mu\nu})_{n\times n}$.

Each $\mathcal{G}_{\mu\nu} = (\tilde{\mathcal{T}}_{\mu\nu}, \tilde{\mathcal{I}}_{\mu\nu})$ where:

$$\tilde{\mathcal{T}}_{\mu\nu} = [\tilde{\mathcal{T}}^L_{\mu\nu}, \tilde{\mathcal{T}}^U_{\mu\nu}], \quad \tilde{\mathcal{I}}_{\mu\nu} = [\tilde{\mathcal{I}}^L_{\mu\nu}, \tilde{\mathcal{I}}^U_{\mu\nu}].$$

Ensure $(\tilde{\mathcal{I}}^U_{\mu\nu})^2 + (\tilde{\mathcal{T}}^U_{\mu\nu})^2 \leq 1$.

Step 2: Estimate Missing Elements

for each unknown $\mathcal{G}_{\mu\nu}$ **do**

Compute using arithmetic mean (Eq. 1):

$$\mathcal{G}_{\mu\nu} = \frac{\mathcal{G}^2_{\mu k} \oplus \mathcal{G}^2_{k\nu}}{2}.$$

end for

Step 3: Aggregate Preferences

Apply interval-valued Pythagorean fuzzy weighted averaging (IVPFWA) AOs [27] (Eq. 3) to each row of $\tilde{\mathcal{Q}}$:

$$IPFWA(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \left\langle \left[\sqrt{1 - \prod_{\mu=1}^{n} (1 - \mathcal{X}^2_\mu)^{w_\mu}}, \sqrt{1 - \prod_{\mu=1}^{n} (1 - \mathcal{Y}^2_\mu)^{w_\mu}} \right], \left[\prod_{\mu=1}^{n} (\mathcal{Z}_\mu)^{w_\mu}, \prod_{\mu=1}^{n} (\mathcal{D}_\mu)^{w_\mu} \right] \right\rangle \tag{3}$$

Step 4: Determine Criteria Weights

Use IVPFWA (Eq. 4) to compute weights:

$$IVPFWA_\omega(\tilde{\mathcal{A}}_i) = \left(\left[\sqrt{1 - \prod (1 - \bar{\mathcal{X}}^2_\mu)^{w_\mu}}, \sqrt{1 - \prod (1 - \bar{\mathcal{Y}}^2_\mu)^{w_\mu}} \right], \left[\prod \bar{\mathcal{Z}}^2_\mu)^{w_\mu}, \prod \bar{\mathcal{D}}^2_\mu)^{w_\mu} \right] \right) \tag{4}$$

where $\mu = 1, \ldots, n$ and $\bar{\mathcal{X}}_\mu = \bar{\mathcal{X}}_{\sigma}(\nu)$. The weight vector $\omega_{\mathcal{G}} = (\omega_{\mathcal{G}_1}, \omega_{\mathcal{G}_2}, \ldots, \omega_{\mathcal{G}_n})$ is obtained using the ordered weighted aggregate (OWA) operator [28] as indicated in Eq 5:

$$\omega\mathcal{G}_\nu = \frac{e^-[(\mu - \mathcal{T}_{\mathcal{G}})^2/2\sigma^2_{\mathcal{G}}]}{\Sigma^n_{\mu=1} e^-[(\mu - \mathcal{T}_{\mathcal{G}})^2/2\sigma^2_{\mathcal{G}}]} \qquad , \nu = 1, 2, ..., n \tag{5}$$

Step 5: Defuzzification Defuzzified $\tilde{\mathcal{A}}_\nu = ([\mathcal{X}\nu, \mathcal{Y}\nu], [\mathcal{Z}\nu, \mathcal{D}\nu])$ for each $\nu = 1, 2, ..., n$ using the function P($\tilde{\mathcal{A}}$) from Eq. 6 and normalized the weights that were produced so that the sum would be 1.0.

$$P(\tilde{\mathcal{A}}) = \frac{\mathcal{X}^2 + \mathcal{Y}^2\sqrt{1 - \mathcal{X}^2 - \mathcal{Z}^2} + \mathcal{Y}^2 + \mathcal{X}^2\sqrt{1 - \mathcal{Y}^2 - \mathcal{D}^2}}{2} \tag{6}$$

Step 6: Calculate Scores

for each alternative A_μ **do**

Compute score $S(\tilde{\alpha})$ (Eq. 7):

$$S(\tilde{\alpha}) = \frac{\mathcal{X}^2 + \mathcal{Y}^2 - \mathcal{Z}^2 - \mathcal{D}^2}{2}. \tag{7}$$

end for
Return: Rank alternatives by descending $S(\tilde{\alpha})$.
end procedure

Table 1: Linguistic ratings and their corresponding IVPFNs

Linguistic Terms	IVPFNs
Extremely Low ($\mathcal{AL}$)	([0.5,0.5],[0.5,0.5])
Highly Low ($\mathcal{VL}$)	([0.11,0.26],[0.53,0.68])
Low ($\mathcal{L}$)	([0.23,0.38],[0.41,0.56])
Moderately Low ($\mathcal{ML}$)	([0.29,0.44],[0.35,0.50])
Approximately Equivalent ($\mathcal{AE}$)	([0.35,0.50],[0.29,0.44])
Moderately High ($\mathcal{MH}$)	([0.41,0.56],[0.0.23,0.38])
High ($\mathcal{H}$)	([0.47,0.62],[0.17,0.32])
Particularly High ($\mathcal{VH}$)	([0.53,0.68],[0.11,0.26])
Extremely High ($\mathcal{AH}$)	([0.59,0.74],[0.05,0.20])
Observation(s): For equivalence, we indicate ($\mathcal{EE}$)	([0.35,0.35],[0.35,0.35])

4 Indicator Performance Functions

Assessing agent performance necessitates advanced criteria that may differentiate varying levels of aptitude while upholding realistic expectations. Conventional binary evaluations frequently neglect to reflect the subtle distinctions in performance. Our approach resolves this issue by formulating a series of indicator performance metrics based on preference functions, hence enhancing the Preference ranking Organization framework. Figure 1 delineates the comprehensive taxonomy of nine performance function types, each tailored for distinct evaluation contexts. These routines correlate observed indicator values (x-axis) with dimensionless performance ratings (y-axis), facilitating standardised comparisons across many metrics.

4.1 Function Specifications and Mathematical Properties

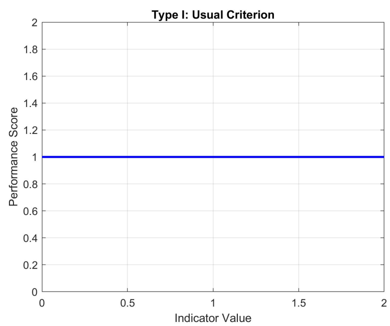
1. **Binary Criterion (Step Function)**

$$P_1(\mathcal{K}) = \mathbb{I}_{(0,\infty)}(\mathcal{K}) = \begin{cases} 1 & \text{if } \mathcal{K} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

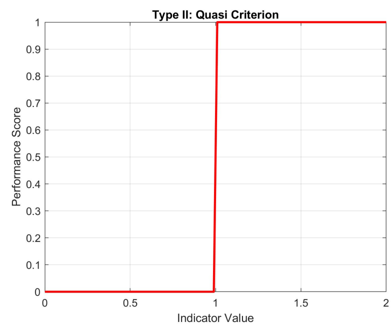
- *Application*: Basic pass/fail evaluation
- *Properties*: Borel measurable but discontinuous at 0
- *Validation*: $\mathbb{P}(\mathcal{K} > 0)$ gives probability of success

2. **Quasi-Criterion (Threshold)**

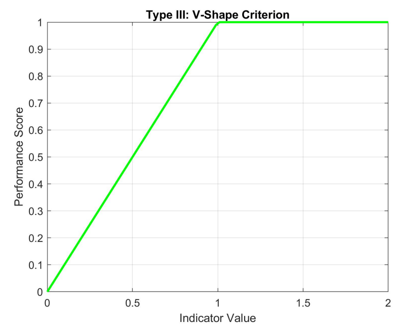
$$P_2(\mathcal{K}; \mathcal{X}) = \mathbb{I}_{(\mathcal{X},\infty)}(\mathcal{K}), \quad \mathcal{X} \in \mathbb{R}^+ \tag{9}$$



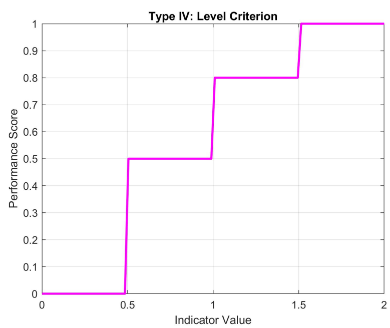
(a) Type I: Binary Threshold



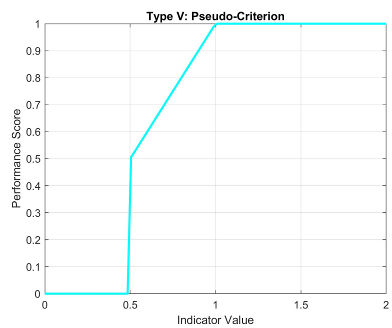
(b) Type II: Fixed Threshold



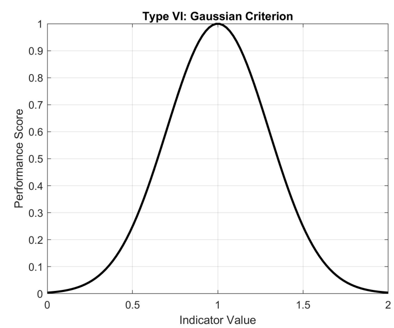
(c) Type III: Linear Ramp



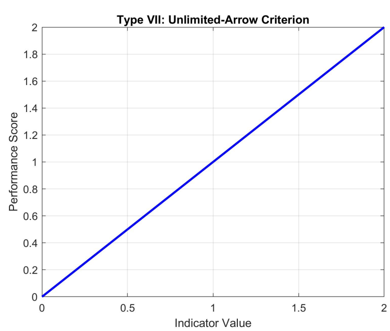
(d) Type IV: Discrete Levels



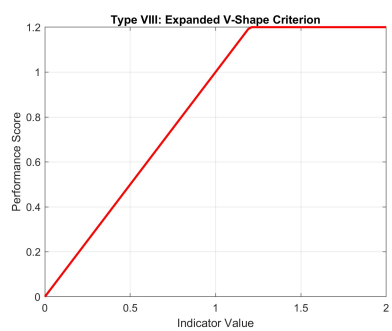
(e) Type V: Bounded Linear



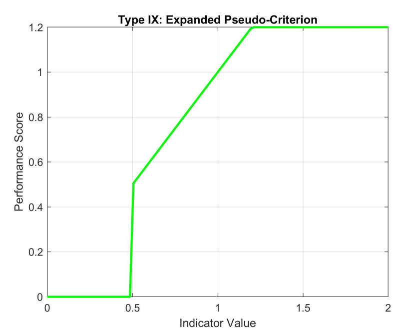
(f) Type VI: Gaussian



(g) Type VII: Unbounded Linear



(h) Type VIII: Partial Ramp



(i) Type IX: Extended Bounded

Figure 1: Taxonomy of indicator performance functions showing the mapping from raw measurements (x-axis) to normalized scores (y-axis)

- *Application*: Testing against fixed standards
- *Properties*: Simple function with single discontinuity

3. **V-Shape Criterion (Linear Ramp)**

$$P_3(\mathcal{K};\mathcal{X}) = \min\left(1, \frac{\mathcal{K}}{\mathcal{X}}\right)_+, \quad \mathcal{X} > 0 \tag{10}$$

- *Application*: Proportional performance assessment
- *Properties*: Lipschitz continuous ($L = 1/\mathcal{X}$)
- *Derivative*: $P_3'(\mathcal{K}) = 1/\mathcal{X}$ on $(0, \mathcal{X})$

4. **Level Criterion (Discrete Steps)**

$$P_4(\mathcal{K}) = \sum_{i=1}^{4} m_i \mathbb{I}_{[x_i, x_{i+1})}(\mathcal{K}), \quad x_1 < \cdots < x_5 \tag{11}$$

- *Application*: Tiered performance classification
- *Properties*: Piecewise constant with finite range
- *Normalization*: Typically $0 = m_1 < \cdots < m_4 = 1$

5. **Pseudo-Criterion (Linear Transition)**

$$P_5(\mathcal{K};\mathcal{X},\mathcal{Y}) = \begin{cases} 0 & \mathcal{K} \le \mathcal{X} \\ \frac{\mathcal{K}-\mathcal{X}}{\mathcal{Y}-\mathcal{X}} & \mathcal{X} < \mathcal{K} < \mathcal{Y} \\ 1 & \mathcal{K} \ge \mathcal{Y} \end{cases} \tag{12}$$

- *Application*: Graduated performance evaluation
- *Properties*: Absolutely continuous, differentiable a.e.
- *Parameters*: $\mathcal{X}$=lower bound, $\mathcal{Y}$=upper bound

6. **Gaussian Criterion**

$$P_6(\mathcal{K};\mu,\sigma) = \exp\left(-\frac{(\mathcal{K}-\mu)^2}{2\sigma^2}\right) \tag{13}$$

- *Application*: Natural performance distributions
- *Properties*: C^∞ smooth, maximum at $\mathcal{K} = \mu$
- *Normalization*: Peak value always 1 at optimal point

7. **Unlimited Linear Criterion**

$$P_7(\mathcal{K};\alpha) = \alpha\mathcal{K}, \quad \alpha > 0 \tag{14}$$

- *Application*: Unbounded performance scaling
- *Caveat*: Requires normalization for cross-comparison
- *Properties*: Linear operator preserving ordering

8. **Expanded V-Shape**

$$P_8(\mathcal{K};\mathcal{X},\mathcal{Y},t) = \begin{cases} \mathcal{K}/\mathcal{X} & 0 \le \mathcal{K} \le \mathcal{X} \\ 1 + \frac{t-1}{\mathcal{Y}-\mathcal{X}}(\mathcal{K}-\mathcal{X}) & \mathcal{X} < \mathcal{K} \le \mathcal{Y} \\ t & \mathcal{K} > \mathcal{Y} \end{cases} \tag{15}$$

- *Application*: Performance with diminishing returns
- *Parameters*: $\mathcal{X}$=linear phase bound, t=asymptote
- *Continuity*: Requires $t \geq 1$ for monotonicity

9. **Extended Pseudo-Criterion**

$$P_9(\mathcal{K};\mathcal{X},\mathcal{Y},\mathcal{Z},t) = \begin{cases} 0 & \mathcal{K} \le \mathcal{X} \\ \frac{\mathcal{K}-\mathcal{X}}{\mathcal{Y}-\mathcal{X}} & \mathcal{X} < \mathcal{K} \le \mathcal{Y} \\ 1 + \frac{t-1}{\mathcal{Z}-\mathcal{Y}}(\mathcal{K}-\mathcal{Y}) & \mathcal{Y} < \mathcal{K} \le \mathcal{Z} \\ t & \mathcal{K} > \mathcal{Z} \end{cases} \tag{16}$$

- *Application*: Multi-phase performance evaluation
- *Design*: Combines Types V and VIII characteristics
- *Constraints*: $\mathcal{X} < \mathcal{Y} < \mathcal{Z}$ and $t \geq 1$

Every performance function meets the criteria for measurable functions, rendering them appropriate for probabilistic analysis. The hierarchy advances from basic binary discriminators (Types I-II) to continuous transformations (Types III-VI) and culminates in sophisticated multi-regime functions (Types VII-IX). Essential mathematical features indicate that all functions are Borel measurable, facilitating integration over performance distributions. Types I-V and VII-IX are inherently non-decreasing. The maximum performance score is generally standardised at 1, with the exception of Type VII. The choice of suitable performance functions is contingent upon the evaluation environment, specifically Type I or II for binary judgements, Types III, V, or VI for continuous metrics, and Types VIII or IX for complex performance landscapes.

5 Mathematical Foundations

This section establishes the foundation for the theoretical framework of our IVPFPRs methodology. We present significant theorems and essential claim that affirm the mathematical soundness of our methodology.

Theorem 5.1 (Consistency Preservation in IVPFPR Completion)**.** *Let $\tilde{\mathcal{Q}} = (\mathcal{G}_{\mu\nu})_{n\times n}$ be an incomplete IVPFPR where missing elements are estimated using either the arithmetic mean (Eq. 1) or geometric mean (Eq. 2) method. The completed IVPFPR maintains the pythagorean condition:*

$$(\tilde{\mathcal{I}}^U_{\mu\nu})^2 + (\tilde{\mathcal{T}}^U_{\mu\nu})^2 \le 1, \quad \forall \mu,\nu \in \{1,...,n\}$$

Proof. Consider an unknown element $\mathcal{G}_{\mu\nu}$ estimated from known adjoining elements $\mathcal{G}_{\mu k}$ and $\mathcal{G}_{k\nu}$.
Case 1: Arithmetic Mean Completion (Eq. 1)

$$\mathcal{G}_{\mu\nu} = \frac{\mathcal{G}^2_{\mu k} \oplus \mathcal{G}^2_{k\nu}}{2}$$

For any two IVPFNs $\mathcal{G}_{\mu k} = ([\mathcal{X}_1, \mathcal{Y}_1], [\mathcal{Z}_1, \mathcal{D}_1])$ and $\mathcal{G}_{k\nu} = ([\mathcal{X}_2, \mathcal{Y}_2], [\mathcal{Z}_2, \mathcal{D}_2])$ satisfying $\mathcal{Y}_i^2 + \mathcal{D}_i^2 \leq 1$ (for $i = 1, 2$), their weighted average preserves the condition because:

$$\begin{aligned}\left(\frac{\mathcal{Y}_1^2 + \mathcal{Y}_2^2}{2}\right) + \left(\frac{\mathcal{D}_1^2 + \mathcal{D}_2^2}{2}\right) &= \frac{(\mathcal{Y}_1^2 + \mathcal{D}_1^2) + (\mathcal{Y}_2^2 + \mathcal{D}_2^2)}{2} \\ &\leq \frac{1+1}{2} = 1\end{aligned}$$

Case 2: Geometric Mean Completion (Eq. 2)

$$\mathcal{G}_{\mu\nu} = \sqrt{\mathcal{G}_{\mu k}^2 \otimes \mathcal{G}_{k\nu}^2}$$

The product of numbers in $[0, 1]$ remains in $[0, 1]$, and specifically:

$$(\mathcal{Y}_1\mathcal{Y}_2)^2 + (\mathcal{D}_1\mathcal{D}_2)^2 \leq \mathcal{Y}_1^2\mathcal{Y}_2^2 + \mathcal{D}_1^2\mathcal{D}_2^2 \leq (\mathcal{Y}_1^2 + \mathcal{D}_1^2)(\mathcal{Y}_2^2 + \mathcal{D}_2^2) \leq 1$$

Thus, both completion methods preserve the fundamental pythagorean fuzzy constraint. This theorem ensures that our completion methods adhere to the fundamental IVPFPR axioms. The arithmetic mean remains constant via convex combination, but the geometric mean maintains constancy through multiplicative preservation. □

Theorem 5.2 (Strict Monotonicity of IVPF Score Function)**.** *The score function* $S(\tilde{\alpha}) = \frac{\mathcal{X}^2+\mathcal{Y}^2-\mathcal{Z}^2-\mathcal{D}^2}{2}$ *for an IVPFN* $\tilde{\alpha} = ([\mathcal{X}, \mathcal{Y}], [\mathcal{Z}, \mathcal{D}])$ *has the following monotonicity properties:*

1. *Strictly increasing in membership bounds* $\mathcal{X}, \mathcal{Y}$
2. *Strictly decreasing in non-membership bounds* $\mathcal{Z}, \mathcal{D}$

Proof. Compute the partial derivatives:

$$\begin{aligned}\frac{\partial S}{\partial \mathcal{X}} &= \mathcal{X} \geq 0 \quad \text{with equality iff } \mathcal{X} = 0 \\ \frac{\partial S}{\partial \mathcal{Y}} &= \mathcal{Y} \geq 0 \quad \text{with equality iff } \mathcal{Y} = 0 \\ \frac{\partial S}{\partial \mathcal{Z}} &= -\mathcal{Z} \leq 0 \quad \text{with equality iff } \mathcal{Z} = 0 \\ \frac{\partial S}{\partial \mathcal{D}} &= -\mathcal{D} \leq 0 \quad \text{with equality iff } \mathcal{D} = 0\end{aligned}$$

The strict inequalities when $\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \mathcal{D} > 0$ prove the claimed monotonicity. This holds because:

- Increasing membership degrees $(\mathcal{X}, \mathcal{Y})$ always increases the score
- Increasing non-membership degrees $(\mathcal{Z}, \mathcal{D})$ always decreases the score

This characteristic ensures that our scoring methodology functions logically: alternatives with a greater number of members and fewer non-members consistently receive superior ratings. □

Theorem 5.3 (IVPFPR Acceptability Criterion)**.** *An incomplete IVPFPR* $\tilde{Q}$ *is acceptable for completion if and only if:*

$$\forall \mu \in \{1, ..., n\}, \exists \nu \neq \mu \text{ such that } \mathcal{G}_{\mu\nu} \text{ is known}$$

Proof. Necessity (⇒): If any row/column has only unknown off-diagonal elements, there exists no adjoining pairs to estimate missing values via Eqs. 1 or 2, making completion impossible.

Sufficiency (⇐): With at least one known element per row/column, we can:

1. Construct estimation chains: $\mathcal{G}_{\mu\nu}$ via $\mathcal{G}_{\mu k}$ and $\mathcal{G}_{k\nu}$ iteratively fill all missing values while preserving Theorem 5.1

This provides a method to ascertain whether a partially populated preference relation may be appropriately completed prior to initiating any calculations. □

Proposition 5.4 (Diagonal Element Characterization)**.** *For any consistent IVPFPR $\tilde{\mathcal{Q}}$, the diagonal elements satisfy:*

$$\tilde{\mathcal{T}}_{\mu\mu} = \tilde{\mathcal{I}}_{\mu\mu} = ([0.5, 0.5], [0.5, 0.5]) \quad \forall\mu$$

Proof. From IVPFPR definition:

1. **Reflexivity**: $\tilde{\mathcal{T}}_{\mu\mu} = \tilde{\mathcal{I}}_{\mu\mu}$ (self-comparison symmetry)
2. **Indifference**: Complete uncertainty requires equal membership/non-membership
3. **Pythagorean Condition**:

$$0.5^2 + 0.5^2 = 0.5 \leq 1 \quad \text{(satisfied)}$$

Thus, exact 0.5 intervals represent perfect neutrality in self-comparisons. □

Diagonal elements serve as consistency checks deviations indicate potential errors in preference elicitation.

Proposition 5.5 (Transitive Completion)**.** *Given an incomplete IVPFPR $\tilde{\mathcal{Q}}$, the estimated values $\mathcal{G}_{\mu\nu}$ constructed via:*

$$\mathcal{G}^-_{\mu\nu} = \min_k \left\{ \mathcal{G}^-_{\mu k} \otimes \mathcal{G}^-_{k\nu} \right\} \tag{17}$$

$$\mathcal{G}^+_{\mu\nu} = \max_k \left\{ \mathcal{G}^+_{\mu k} \otimes \mathcal{G}^+_{k\nu} \right\} \tag{18}$$

satisfy the weak stochastic transitivity property:

$$(\mathcal{G}_{\mu k} \succeq 0.5) \wedge (\mathcal{G}_{k\nu} \succeq 0.5) \Rightarrow \mathcal{G}_{\mu\nu} \succeq 0.5$$

where $\succeq$ denotes the interval dominance relation.

Proof. For any k, the $\otimes$ operation (Pythagorean product) preserves ordering:

$$a \geq 0.5, b \geq 0.5 \Rightarrow a \otimes b = \sqrt{a^2 + b^2 - a^2 b^2} \geq 0.5$$

The min/max operations in Eqs. (17)–(18) maintain this property for interval endpoints. □

6 Numerical Evaluation with Applications

Integrating the aforementioned approaches with the current legacy system is crucial for executing call center-preferential assessments. The actual implementation is executed in three stages: strategy, gathering information, and computing. The planning phase delineates the performance structure of the measuring system. The criteria for assessing and determining their respective weights and target measures, together with other pertinent parameters, are eventually defined. The data collection step consists of two stages: first, data is retrieved from resources and analysed to yield observed indicator values. The leaders of the team are responsible for assessing agents through qualitative evaluations, then gathering quantitative data. The computing phase signifies the final stage in which the entire achievement score is determined. Performance ratings for indicators are initially calculated using the functions and requirements defined during the strategy phase. The scores are computed based on the weight allocated to each indicator. The specialists analyzed a call center for debt collection and evaluated its practical implementation. In the preliminary stage of organization, experts define six performance metrics, as seen in Table 2.

Table 2: Professionals evaluations linguistic matrix

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$
Professional 1	$\mathcal{EE}$	$\mathcal{MH}$	$\mathcal{VH}$	$\mathcal{H}$	$\mathcal{AH}$	$\mathcal{VH}$
Professional 2	$\mathcal{EE}$	$\mathcal{MH}$	$\mathcal{H}$	$\mathcal{H}$	$\mathcal{AH}$	$\mathcal{AH}$
Professional 3	$\mathcal{EE}$	$\mathcal{E}$	$\mathcal{VH}$	$\mathcal{MH}$	$\mathcal{VH}$	$\mathcal{VH}$

Monetized Value of Collection $\mathcal{P}_1$: The total sum gathered by the agent throughout the appraisal period, indicating the financial effectiveness in recovery.

Collection Efficiency Ratio $\mathcal{P}_2$: The ratio of funds raised to the entirety designated for the representative's portfolio, indicating their ability to meet objectives.

Promise-to-Pay Value $\mathcal{P}_3$: Total monetary value of agreements made by agents in which debtors promise to pay in the future, reflecting the negotiation skills of the agent.

Customer Satisfaction Index $\mathcal{P}_4$: The number of customer complaints or dissatisfaction reports on the agent's behavior describes the service quality and professionalism of the agents.

Call Performance Rating $\mathcal{P}_5$: This is the rating given by the quality team based on calls that were evaluated from calls selected randomly. This reflects the agent's capability for communication and rendering customer service.

Agent Engagement Hours $\mathcal{P}_6$: Engagement hours of the agent, which means total hours put in by the agent within the review period, which shows the agent's commitment and participation.

To achieve this goal, we use IVPFPR, which are inadequate. Each expert is initially directed to compare the six indications in pairs according to the method described. At the outset, the specified method calls for all experts to compare the six signs in pairs. Instead of filling out the whole comparison matrix, the expert evaluated $\mathcal{P}_1$ using five criterions. Table 2 is obtained from the total linguistic assessments. The evaluations are converted into the IVPFS, and the values that are unclear in an analysis matrix for pairs are completed using Eq. 1. Tables 3 to 5 present a comprehensive set of comparisons made pairwise among the leading researchers.

Table 3: Professional 1's matrix for pairwise comparisons

Professional	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$
$\mathcal{P}_1$	(0.35, 0.35), (0.35, 0.35)	(0.41, 0.56), (0.23, 0.38)	(0.58, 0.68), (0.11, 0.26)	(0.47, 0.62), (0.17, 0.32)	(0.59, 0.74), (0.05, 0.20)	(0.53, 0.68), (0.11, 0.26)
$\mathcal{P}_2$	(0.23, 0.38), (0.41, 0.56)	(0.35, 0.35), (0.35, 0.35)	(0.167, 0.303), (0.091, 0.191)	(0.137, 0.264), (0.099, 0.21)	(0.201, 0.346), (0.085, 0.177)	(0.167, 0.303), (0.091, 0.192)
$\mathcal{P}_3$	(0.11, 0.26), (0.53, 0.68)	(0.091, 0.191), (0.167, 0.303)	(0.35, 0.35), (0.35, 0.35)	(0.1165, 0.226), (0.155, 0.282)	(0.180, 0.308), (0.142, 0.251)	(0.147, 0.265), (0.147, 0.265)
$\mathcal{P}_4$	(0.47, 0.62), (0.17, 0.32)	(0.137, 0.264), (0.099, 0.208)	(0.117, 0.226), (0.155, 0.282)	(0.35, 0.35), (0.35, 0.35)	(0.189, 0.325), (0.112, 0.212)	(0.155, 0.282), (0.117, 0.423)
$\mathcal{P}_5$	(0.59, 0.74), (0.05, 0.20)	(0.201, 0.346), (0.085, 0.177)	(0.180, 0.308), (0.142, 0.251)	(0.189, 0.325), (0.112, 0.212)	(0.35, 0.35), (0.35, 0.35)	(0.142, 0.251), (0.181, 0.505)
$\mathcal{P}_6$	(0.53, 0.68), (0.11, 0.26)	(0.167, 0.303), (0.091, 0.190)	(0.147, 0.265), (0.147, 0.265)	(0.155, 0.282), (0.117, 0.423)	(0.180, 0.505), (0.142, 0.251)	(0.35, 0.35), (0.35, 0.35)

Source(s): We offer straightforward computations for the values.

Table 3 shows that the values that are highlight are obtained by using Eq. 1. To illustrate, the following method is used to determine $\tilde{G_{23}}$:

$$\frac{\mathcal{G}_{21}^2 \bigoplus \mathcal{G}_{13}^2}{2} = \frac{(0.23^2, 0.38^2), (0.41^2, 0.56^2) \bigoplus (0.53^2, 0.68^2), (0.11^2, 0.26^2)}{2} = (0.1669, 0.3034), (0.0901, 0.1906)$$

$$\begin{pmatrix}
\frac{\mathcal{G}_{21}^2 \bigoplus \mathcal{G}_{13}^2}{2} = (0.1669, 0.3034), (0.0901, 0.1906) & \frac{\mathcal{G}_{21}^2 \bigoplus \mathcal{G}_{14}^2}{2} = (0.1369, 0.2644), (0.0985, 0.208) \\
\frac{\mathcal{G}_{21}^2 \bigoplus \mathcal{G}_{15}^2}{2} = (0.2005, 0.346), (0.0853, 0.1768) & \frac{\mathcal{G}_{21}^2 \bigoplus \mathcal{G}_{16}^2}{2} = (0.1669, 0.3034), (0.0901, 0.1906) \\
\frac{\mathcal{G}_{31}^2 \bigoplus \mathcal{G}_{12}^2}{2} = (0.0901, 0.1906), (0.1669, 0.3034) & \frac{\mathcal{G}_{31}^2 \bigoplus \mathcal{G}_{14}^2}{2} = (0.1165, 0.226), (0.1549, 0.2824) \\
\frac{\mathcal{G}_{31}^2 \bigoplus \mathcal{G}_{15}^2}{2} = (0.1801, 0.308), (0.1417, 0.2512) & \frac{\mathcal{G}_{31}^2 \bigoplus \mathcal{G}_{16}^2}{2} = (0.1465, 0.265), (0.1465, 0.265) \\
\frac{\mathcal{G}_{41}^2 \bigoplus \mathcal{G}_{12}^2}{2} = (0.0985, 0.208), (0.1369, 0.2644) & \frac{\mathcal{G}_{41}^2 \bigoplus \mathcal{G}_{13}^2}{2} = (0.1549, 0.2824), (0.1165, 0.4234) \\
\frac{\mathcal{G}_{41}^2 \bigoplus \mathcal{G}_{15}^2}{2} = (0.1885, 0.325), (0.1117, 0.2122) & \frac{\mathcal{G}_{41}^2 \bigoplus \mathcal{G}_{16}^2}{2} = (0.1549, 0.2824), (0.1165, 0.4234) \\
\frac{\mathcal{G}_{51}^2 \bigoplus \mathcal{G}_{12}^2}{2} = (0.0853, 0.1768), (0.2005, 0.346) & \frac{\mathcal{G}_{51}^2 \bigoplus \mathcal{G}_{13}^2}{2} = (0.1417, 0.2512), (0.1801, 0.505) \\
\frac{\mathcal{G}_{51}^2 \bigoplus \mathcal{G}_{14}^2}{2} = (0.1117, 0.2122), (0.1885, 0.325) & \frac{\mathcal{G}_{51}^2 \bigoplus \mathcal{G}_{16}^2}{2} = (0.1417, 0.2512), (0.1801, 0.505) \\
\frac{\mathcal{G}_{61}^2 \bigoplus \mathcal{G}_{12}^2}{2} = (0.0901, 0.1906), (0.1669, 0.3034) & \frac{\mathcal{G}_{61}^2 \bigoplus \mathcal{G}_{13}^2}{2} = (0.1465, 0.265), (0.1465, 0.4624) \\
\frac{\mathcal{G}_{61}^2 \bigoplus \mathcal{G}_{14}^2}{2} = (0.1165, 0.4234), (0.1549, 0.2824) & \frac{\mathcal{G}_{61}^2 \bigoplus \mathcal{G}_{15}^2}{2} = (0.1801, 0.505), (0.1417, 0.2512)
\end{pmatrix}$$

Table 4: The matrix for pairwise comparisons by professional 2

Professional	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$
$\mathcal{P}_1$	(0.35,0.35) (0.35,0.35)	(0.41,0.56) (0.23,0.38)	(0.47,0.62) (0.17,0.32)	(0.47,0.32) (0.17,0.32)	(0.59,0.74) (0.05,0.20)	(0.59,0.74) (0.05,0.20)
$\mathcal{P}_2$	(0.23,0.38) (0.41,0.56)	(0.35,0.35) (0.35,0.35)	(0.14,0.26) (0.10,0.21)	(0.14,0.26) (0.10,0.21)	(0.20,0.35) (0.09,0.18)	(0.20,0.35) (0.09,0.18)
$\mathcal{P}_3$	(0.17,0.32) (0.47,0.62)	(0.10,0.21) (0.14,0.26)	(0.35,0.35) (0.35,0.35)	(0.12,0.24) (0.12,0.24)	(0.19,0.33) (0.10,0.21)	(0.19,0.33) (0.11,0.21)
$\mathcal{P}_4$	(0.17,0.32) (0.47,0.62)	(0.10,0.21) (0.14,0.26)	(0.12,0.24) (0.12,0.24)	(0.35,0.35) (0.35,0.35)	(0.19,0.33) (0.11,0.21)	(0.19,0.32) (0.11,0.29)
$\mathcal{P}_5$	(0.05,0.20) (0.59,0.74)	(0.09,0.18) (0.20,0.35)	(0.11,0.21) (0.19,0.33)	(0.11,0.21) (0.19,0.33)	(0.35,0.35) (0.35,0.35)	(0.18,0.29) (0.18,0.29)
$\mathcal{P}_6$	(0.05,0.20) (0.59,0.74)	(0.09,0.18) (0.20,0.35)	(0.11,0.21) (0.19,0.33)	(0.11,0.21) (0.19,0.33)	(0.18,0.29) (0.18,0.29)	(0.35,0.35) (0.35,0.35)

The values that are highlighted in Table 4 are obtained by using Eq. 1. To illustrate, the following method is used to determine $\mathcal{G}_{23}$:

$$\begin{pmatrix} \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1369, 0.2644), (0.0985, 0.208) & \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{14}}{2} = (0.1369, 0.2644), (0.0985, 0.208) \\ \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{15}}{2} = (0.2005, 0.346), (0.0853, 0.1768) & \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{16}}{2} = (0.2005, 0.346), (0.0853, 0.1768) \\ \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0985, 0.208), (0.1369, 0.2644) & \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{14}}{2} = (0.1249, 0.2434), (0.1249, 0.2434) \\ \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1885, 0.325), (0.1117, 0.2122) & \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1885, 0.325), (0.1117, 0.2122) \\ \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0985, 0.208), (0.1369, 0.2644) & \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1249, 0.2434), (0.1249, 0.2434) \\ \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1885, 0.325), (0.1117, 0.2122) & \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1885, 0.325), (0.1117, 0.2122) \\ \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1117, 0.2122), (0.1885, 0.325) & \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{14}}{2} = (0.1117, 0.2122), (0.1885, 0.325) \\ \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1753, 0.2938), (0.1753, 0.2938) & \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{12}}{2} = (0.1753, 0.2938), (0.1753, 0.2938) \\ \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1117, 0.2122), (0.1885, 0.325) & \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{14}}{2} = (0.1117, 0.2122), (0.1885, 0.325) \end{pmatrix}$$

Table 5: Matrix for pairwise comparisons by professional 3

Professional	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$
$\mathcal{P}_1$	(0.35,0.35), (0.35,0.35)	(0.35,0.50), (0.29,0.44)	(0.53,0.68), (0.11,0.26)	(0.41,0.056), (0.23,0.38)	(0.53,0.68), (0.11,0.26)	(0.53,0.68), (0.11,0.26)
$\mathcal{P}_2$	(0.29,0.44), (0.35,0.50)	(0.35,0.35), (0.35,0.35)	(0.1825,0.328), (0.0673,0.1588)	(0.1261,0.2536), (0.0877,0.1972)	(0.1825,0.328), (0.0673,0.1588)	(0.1825,0.328), (0.0673,0.1588)
$\mathcal{P}_3$	(0.11,0.26), (0.53,0.68)	(0.0673,0.1588), (0.1825,0.328)	(0.35,0.35), (0.35,0.35)	(0.0901,0.1906), (0.1669,0.3034)	(0.1465,0.265), (0.1465,0.265)	(0.1465,0.265), (0.1465,0.265)
Source(s): We offer straightforward computations for the values.						

The values that are highlighted in Table 5 are obtained by using Eq. 1. To illustrate, the following method is used to determine $\tilde{G_{23}}$:

$$\begin{pmatrix} \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1825, 0.328), (0.0673, 0.1588) & \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{14}}{2} = (0.1261, 0.2536), (0.0877, 0.1992) \\ \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1825, 0.328), (0.0673, 0.1588) & \frac{\mathcal{G}^2_{21}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1825, 0.328), (0.0673, 0.1588) \\ \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0673, 0.1588), (0.1825, 0.328) & \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{14}}{2} = (0.0901, 0.1906), (0.1669, 0.3034) \\ \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1465, 0.265), (0.1465, 0.265) & \frac{\mathcal{G}^2_{31}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1465, 0.265), (0.1465, 0.265) \\ \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0877, 0.1972), (0.1261, 0.2536) & \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1669, 0.3034), (0.0901, 0.1906) \\ \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1669, 0.3034), (0.0901, 0.1906) & \frac{\mathcal{G}^2_{41}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1669, , 0.3034), (0.0901, 0.1906) \\ \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0673, 0.1588), (0.1825, 0.328) & \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1465, 0.265), (0.1465, 0.265) \\ \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{14}}{2} = (0.0901, 0.1906), (0.1669, 0.3034) & \frac{\mathcal{G}^2_{51}\bigoplus\mathcal{G}^2_{16}}{2} = (0.1465, 0.265), (0.1465, 0.265) \\ \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{12}}{2} = (0.0673, 0.1588), (0.1825, 0.328) & \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{13}}{2} = (0.1465, 0.265), (0.1465, 0.265) \\ \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{14}}{2} = (0.0901, 0.1906), (0.1669, 0.3034) & \frac{\mathcal{G}^2_{61}\bigoplus\mathcal{G}^2_{15}}{2} = (0.1465, 0.265), (0.1465, 0.265) \end{pmatrix}$$

Upon generating the complete pairwise evaluation matrix frameworks, the subsequent evaluation phase involves gathering every single column of their priority coincidences as previously indicated. The preference correlations are presented in an accumulated manner in Table 6. The ordered preference linkages are presented in Table 7.

For $\mathcal{P}_1$: The assessment values that are aggregated for $\mathcal{P}_1$ are calculated as outlined below, with similar calculations applied to $\mathcal{P}_2$ through $\mathcal{P}_6$ as detailed in Table 6.

$$\begin{aligned} &\sqrt{1-[(1-0.35^2)(1-0.41^2)(1-0.53^2)(1-0.47^2)(1-0.59^2)(1-0.53^2)]^{1/6}} \\ &= \sqrt{1-[(0.8775)(0.8319)(0.7191)(0.7791)(0.6519)(0.7191)]^{1/6}} \\ &= 0.49 \end{aligned}$$

$$\begin{aligned} &\sqrt{1-[(1-0.35^2)(1-0.56^2)(1-0.68^2)(1-0.62^2)(1-0.74^2)(1-0.68^2)]^{1/6}} \\ &= \sqrt{1-[(0.8775)(0.6864)(0.5376)(0.6156)(0.4524)(0.5376)]^{1/6}} \\ &= 0.63 \end{aligned}$$

$$[(0.35)(0.23)(0.11)(0.17)(0.05)(0.11)]^{1/6} \quad = 0.14$$

$$[(0.35)(0.38)(0.26)(0.32)(0.20)(0.26)]^{1/6} \quad = 0.29$$

Table 6: Consolidated preference relations

	Professional 1	Professional 2	Professional 3
$\mathcal{P}_1$	(0.49,0.63), (0.14,0.29)	(0.491,0.63), (0.13,0.29)	(0.46,0.60), (0.46,0.60)
$\mathcal{P}_2$	(0.22,0.33), (0.15,0.25)	(0.22,0.33), (0.15,0.253)	(0.23,0.34), (0.12,0.23)
$\mathcal{P}_3$	(0.19,0.27), (0.215,0.33)	(0.20,0.30), (0.18,0.294)	(0.18,0.26), (0.22,0.34)
$\mathcal{P}_4$	(0.20,0.30), (0.18,0.36)	(0.20,0.30), (0.18,0.294)	(0.213,0.431), (0.15,0.265)
$\mathcal{P}_5$	(0.15,0.25), (0.25,0.44)	(0.18,0.25), (0.25,0.375)	(0.18,0.26), (0.22,0.34)
$\mathcal{P}_6$	(0.19,0.35), (0.215,0.365)	(0.18,0.25), (0.25,0.375)	(0.213,0.431), (0.15,0.265)
Source(s): We offer straightforward computations for the values.			

Table 7: Ordering preference relations

	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\mathcal{P}_4$	$\mathcal{P}_5$	$\mathcal{P}_6$
1	(0.491,0.63), (0.13,0.29)	(0.23,0.34), (0.12,0.23)	(0.20,0.30), (0.181,0.294)	(0.20,0.30), (0.181,0.294)	(0.18,0.26), (0.22,0.34)	(0.19,0.35), (0.215,0.365)
2	(0.49,0.63), (0.14,0.29)	(0.22,0.33), (0.15,0.253)	(0.19,0.27), (0.215,0.33)	(0.20,0.30), (0.18,0.36)	(0.18,0.25), (0.25,0.375)	(0.213,0.431), (0.15,0.265)
3	(0.46,0.60), (0.18,0.32)	(0.22,0.33), (0.15,0.25)	(0.18,0.26), (0.22,0.34)	(0.213,0.431), (0.15,0.265)	(0.18,0.25), (0.25,0.44)	(0.18,0.25), (0.25,0.375)

The subsequent step involves sorting the aggregated values through the application of the score function. The score functions from $\mathcal{P}_1$ to $\mathcal{P}_6$ are computed in the following manner: For $\mathcal{P}_1$:

$$S(\mathcal{P}_{\infty\mu}) = \begin{cases} \frac{0.49^2+0.63^2-0.14^2-0.29^2}{2} = 0.164, & \text{if } \mu = 1 \quad (\text{rank} = 3) \\ \frac{0.491^2+0.63^2-0.13^2-0.29^2}{2} = 0.268, & \text{if } \mu = 2 \quad (\text{rank} = 1) \\ \frac{0.46^2+0.60^2-0.18^2-0.32^2}{2} = 0.218, & \text{if } \mu = 3 \quad (\text{rank} = 2) \end{cases}$$

For $\mathcal{P}_2$:

$$S(\mathcal{P}_{2_\mu}) = \begin{cases} \frac{0.22^2+0.33^2-0.15^2-0.25^2}{2} = 0.03615, & \text{if } \mu = 1 \quad (\text{rank} = 2) \\ \frac{0.22^2+0.33^2-0.15^2-0.253^2}{2} = 0.035, & \text{if } \mu = 2 \quad (\text{rank} = 3) \\ \frac{0.23^2+0.34^2-0.12^2-0.23^2}{2} = 0.0506, & \text{if } \mu = 3 \quad (\text{rank} = 1) \end{cases}$$

For $\mathcal{P}_3$:

$$S(\mathcal{P}_{3_\mu}) = \begin{cases} \frac{0.19^2+0.27^2-0.215^2-0.33^2}{2} = -0.0231, & \text{if } \mu = 1 \quad (\text{rank} = 2) \\ \frac{0.20^2+0.3^2-0.181^2-0.294^2}{2} = 0.0054, & \text{if } \mu = 2 \quad (\text{rank} = 1) \\ \frac{0.18^2+0.26^2-0.22^2-0.34^2}{2} = -0.032, & \text{if } \mu = 3 \quad (\text{rank} = 3) \end{cases}$$

For $\mathcal{P}_4$:

$$S(\mathcal{P}_{4_\mu}) = \begin{cases} \frac{0.20^2+0.30^2-0.18^2-0.36^2}{2} = -0.016, & \text{if } \mu = 1 \quad (\text{rank} = 3) \\ \frac{0.20^2+0.3^2-0.18^2-0.294^2}{2} = 0.005582, & \text{if } \mu = 2 \quad (\text{rank} = 2) \\ \frac{0.213^2+0.431^2-0.15^2-0.265^2}{2} = 0.0812, & \text{if } \mu = 3 \quad (\text{rank} = 1) \end{cases}$$

For $\mathcal{P}_5$:

$$S(\mathcal{P}_{5_\mu}) = \begin{cases} \frac{0.18^2+0.25^2-0.25^2-0.44^2}{2} = -0.0806, & \text{if } \mu = 1 \quad (\text{rank} = 3) \\ \frac{0.18^2+0.25^2-0.25^2-0.375^2}{2} = -0.054, & \text{if } \mu = 2 \quad (\text{rank} = 2) \\ \frac{0.18^2+0.26^2-0.22^2-0.34^2}{2} = -0.032, & \text{if } \mu = 3 \quad (\text{rank} = 1) \end{cases}$$

For $\mathcal{P}_6$:

$$S(\mathcal{P}_{6_\mu}) = \begin{cases} \frac{0.19^2+0.35^2-0.215^2-0.365^2}{2} = -0.010, & \text{if } \mu = 1 \quad (\text{rank} = 2) \\ \frac{0.18^2+0.25^2-0.25^2-0.375^2}{2} = -0.054, & \text{if } \mu = 2 \quad (\text{rank} = 3) \\ \frac{0.213^2+0.431^2-0.15^2-0.265^2}{2} = 0.069, & \text{if } \mu = 3 \quad (\text{rank} = 1) \end{cases}$$

The fuzzy weights for each indicator are calculated employing Equations 4 and 5. The uncertain weights are subsequently defuzzified in accordance with Equation 6. Table 8 displays both weights, the fuzzy information and crisp data associated with the indications. The IVPFS based weight of $\mathcal{P}_1$, specifically (0.483, 0.623) and (0.147, 0.298), is determined through the following calculation:

$$\sqrt{1-[(1-0.491^2)^{0.243}(1-0.49^2)^{0.514}(1-0.46^2)^{0.243}]} = 0.483.$$

$$\sqrt{1-[(1-0.24^2)^{0.243}(1-0.24^2)^{0.514}(1-0.21^2)^{0.243}]} = 0.623.$$

$$(0.13)^{0.243} \times (0.14)^{0.514} \times (0.18)^{0.243} = 0.147$$

$$(0.29)^{0.243} \times (0.29)^{0.514} \times (0.32)^{0.243} = 0.298.$$

Table 8: Weights of the criterion that are and normalization

Indicator	IVPFS weights	Defuzzified weights	Normalized weights
$\mathcal{P}_1$	(0.483,0.623),(0.147,0.298)	0.563	0.455
$\mathcal{P}_2$	(0.21,0.34),(0.142,0.25)	0.156	0.126
$\mathcal{P}_3$	(0.194,0.275),(0.207,0.321)	0.10	0.081
$\mathcal{P}_4$	(0.202,0.335),(0.173,0.318)	0.149	0.120
$\mathcal{P}_5$	(0.180,0.253),(0.242,0.381)	0.093	0.075
$\mathcal{P}_6$	(0.20,0.377),(0.180,0.312)	0.177	0.143

7 Performance Criteria

These criteria are applied in a weighted performance evaluation model, as defined by Oztaysi, Basar, et al. [29] and demonstrated in the provided Tables 9-10. The final weighted score helps in ranking call center agents or operations based on observed data.

Table 9: Performance metrics and their corresponding factor

	Weights	Operator Type	Objective	Additional factors
$\mathcal{P}_1$	0.455	Type VIII	500K	$\mathcal{X}=500\text{K}, \mathcal{Y}=600\text{K}, \tau= 1.2$
$\mathcal{P}_2$	0.126	Type V	25	$\mathcal{X}=10, \mathcal{Y}=25$
$\mathcal{P}_3$	0.081	Type VIII	800K	$\mathcal{X}=800\text{K}, \mathcal{Y}=1000\text{K}, \tau= 1.2$
$\mathcal{P}_4$	0.120	Type III (Inv.)	0	$\mathcal{X}=4$
$\mathcal{P}_5$	0.075	Type IV	9	$\mathcal{X}_1=3$, $\mathcal{M}_1=0.2$, $\mathcal{X}_2=5$,$\mathcal{M}_2=0.5$,$\mathcal{X}_3=7$, $\mathcal{M}_3=0.8$,$\mathcal{X}_4=9$,$\mathcal{M}_4=1$
$\mathcal{P}_6$	0.143	Type IX	150	$\mathcal{X}=50, \mathcal{Y}=150, \mathcal{Z}=180, \tau=1.2$

Table 10: Assessment of a call center agent's performance metrics

	Weights	Operator Type	Objective	Measured Value	Score	Weighted Result
$\mathcal{P}_1$	0.455	Type VIII	500K	550K	1.1	0.5
$\mathcal{P}_2$	0.126	Type V	25	20	0.66	0.083
$\mathcal{P}_3$	0.081	Type VIII	800K	700K	0.875	0.071
$\mathcal{P}_4$	0.120	Type III (Inv.)	0	3	0.25	0.03
$\mathcal{P}_5$	0.075	Type IV	9	8	0.8	0.06
$\mathcal{P}_6$	0.143	Type IX	150	160	1.1	0.157
		Overall				0.901

Using Eq. 6, the defuzzified weights are then computed.

$$P(\tilde{\mathcal{A}}) = \frac{\mathcal{X}^2 + \mathcal{Y}^2\sqrt{1-\mathcal{X}^2-\mathcal{Z}^2} + \mathcal{Y}^2 + \mathcal{X}^2\sqrt{1-\mathcal{Y}^2-\mathcal{D}^2}}{2}$$

$$P((\tilde{\mathcal{A}})_\mu) = \begin{cases} \mathcal{P}_1 = 0.563, \\ \mathcal{P}_2 = 0.156, \\ \mathcal{P}_3 = 0.1, \\ \mathcal{P}_4 = 0.149, \\ \mathcal{P}_5 = 0.093, \\ \mathcal{P}_6 = 0.177 \end{cases}$$

We then normalize the defuzzified values by divided each by the sum of all the weights, which makes sure that the aggregate of the weights is one. Table 8 shows the normalized weights in the column. The cumulative total of defuzzified weights is 1.238. We now seek to adjust the weights, as the overall weight should equal 1.

The calculation is derived from the $formula = \frac{Total \quad Weight}{Each \quad Weight}$:

$$\text{Weights for } \quad \mathcal{P}_\mu) = \begin{cases} \mathcal{P}_1 = 0.455, \\ \mathcal{P}_2 = 0.126, \\ \mathcal{P}_3 = 0.081, \\ \mathcal{P}_4 = 0.120, \\ \mathcal{P}_5 = 0.075, \\ \mathcal{P}_6 = 0.143 \end{cases}$$

7.1 Performance Measurement Implementation

After determining the indicator weights, domain experts proceed to define performance functions and their associated parameters. Table 9 details the specific parameters configured for our case study. The target values shown in Table 8 represent optimal benchmark values for each performance indicator. These parameters can be directly mapped to their corresponding performance curves. For instance, the $\mathcal{P}_4$ indicator tracks customer complaint volumes, where lower values indicate better performance. This inverse relationship is modeled using a Type III inverse function. The planning phase concludes once all performance functions and parameters are properly configured. During the data collection phase, current performance metrics are automatically gathered from operational systems. Table 10 presents a sample evaluation of a call center agent, showing both individual indicator scores and the composite performance rating. The "Observed Value" column contains actual measurements collected during the monitoring period. Performance scores are calculated by:

1. Inputting observed values into their respective performance functions.
2. Determining the corresponding y-value output in indicator curves.

For example, when evaluating $\mathcal{P}_1$ criteria:

- Target value: 500K
- Observed value: 550K
- Resulting performance score: 1.10

The overall agent performance (0.901) is computed through weighted aggregation:

$$\text{Overall Score} = \sum(\text{Indicator Score} \times \text{Weight}) \tag{19}$$

Validation was conducted by:

- Sharing sample calculations with management
- Collecting structured feedback

The managerial assessment confirmed several benefits:

- The function-based approach successfully incorporates managerial expertise into objective measurements

- Collaborative target-setting improves agent understanding of expectations
- The weighting system effectively communicates performance priorities
- The methodology differentiates agent performance levels with high precision

Management concluded that the system provides an effective framework for performance evaluation that meets operational requirements while maintaining fairness and transparency.

7.2 Limitations

While our IVPFS-based framework demonstrates significant advantages, several limitations should be acknowledged:

- Interval-valued Pythagorean fuzzy operations need more calculations than regular methods, especially when dealing with imperfect preference relations. The temporal complexity increases quadratically as the number of choices rises, which could affect real-time applications for extremely large call centers.
- Expert opinions are what make pairwise comparisons accurate. Our method uses fuzzy intervals to make things less subjective, but the quality of the results still depends on how well the evaluators can give consistent preference relations.
- The framework needs at least one known off-diagonal element for each row and column in incomplete IVPFPRs. When sparse datasets fail to meet this criterion, we must discover new methods to bridge the gaps. The interval-valued outputs are technically sound, but practitioners who are used to clear performance measurements may need more explanation, which could slow their acceptance in some operational settings.

8 Conclusion

This research introduces an innovative IVPFS based framework for assessing contact center performance, specifically addressing the challenge of uncertainty quantification in operational evaluations. Through empirical validation, we demonstrate that our approach effectively captures the inherent vagueness and imprecision in performance data while offering three key advantages over conventional methods: (1) enhanced measurement precision through advanced uncertainty modeling, (2) greater adaptability in handling data ambiguity, and (3) improved decision support for managerial assessments. The study makes three significant contributions to operational performance measurement. First, it establishes a mathematically rigorous yet practical solution for uncertainty in call center evaluations. Second, it provides empirical evidence of the framework's superiority through comparative analysis. Third, it lays the groundwork for future research in fuzzy-set-based management systems. Beyond performance evaluation, the PFS framework shows potential for broader applications in call center management, including workforce optimization, resource allocation, and customer relationship management. The integration of real-time data streams and advanced analytics could further enhance the system's predictive capabilities and responsiveness. Practically, this approach enables more accurate performance evaluations that can inform staffing decisions, training programs, and service quality improvements. The framework maintains methodological consistency while adapting to diverse operational contexts. Future research directions should explore:

- Integration with machine learning for adaptive performance benchmarking
- Application in other service industries facing similar measurement challenges

- Expansion of the framework's predictive capabilities through real-time data integration

The successful implementation in our case studies suggests strong potential for widespread adoption across customer service operations, offering a robust solution to the complex challenges of performance measurement in uncertain environments.

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