Improved Block Compressed Sensing Image Reconstruction by Data Sorting

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Abstract

In Block Compressed Sensing (BCS), the image is divided into small blocks and sampled with the same operator. At the decoder side, each block is treated as an independent sub-CS reconstruction task. This often results in generating some blocking artifacts and losing global structure of the image. In this paper, we propose data sorting into the BCS framework to overcome the BCS problems and improve the reconstruction result. We refer this new block-based CS technique as sort-based BCS (SBCS). In this method, the original image is sorted in such a way that a smooth image is produced. Then, block-based sampling and reconstruction are applied on the smoothed image. We use iterative projected Land Weber (PL) and iterative soft thresholding (IST) algorithms for image reconstruction. Simulation results show that the proposed SBCS image reconstruction provides significant improvement over the existing block-based CS image reconstruction methods, in terms of both subjective and objective evaluations.

Keywords: compressed sensing; data sorting; iterative soft thresholding; image reconstruction; sparsity

1. Introduction

Over the past few years, a new framework called as compressed sensing (CS) [1] has been developed for simultaneous sampling and compression of signals at sub-Nyquist rates [2]. It is shown that under certain conditions, the signal can be reconstructed exactly from a small set of measurements. As applied to 2D images, CS faces several challenges including a computationally expensive reconstruction process and huge memory required to store the random sampling operator [3]. Several fast algorithms have been developed for CS reconstruction [3, 4]. The memory challenge was first addressed in [2] using a block-based sampling operation.

In Block-based compressed sensing (CS), the original image is divided into small blocks and each block is sampled independently using the same measurement operator. The main advantages of this method include: (a) Measurement operator can be easily stored and implemented through a random under sampled filter bank; (b) BCS is more suitable for real-time applications as the encoder does not need to send the sampled data until the whole image is measured; (c) Since each block is processed independently, the initial solution can be easily obtained and the reconstruction process can be substantially speeded up. Despite these advantages, dividing the image into blocks and treating each image block as an independent sub-CS reconstruction task would generate some blocking artifacts and reduce the visual quality. To reduce the blocking artifact and smooth the image, a smoothing filter (e.g. Wiener filter [2]) is applied in the spatial domain. However, this results in losing the details and fine structures of the reconstructed image.

CS theory shows that the sparsity degree of a signal plays a significant role in reconstruction. The higher sparsity degree of a signal, the higher reconstruction quality it will have [5].An image reconstruction can be formulated in form of an optimization problem under regularization-based framework. Classical regularization term reflects the local prior information of the image (e.g. local smoothness) and represents the local sparsity model. Local smoothness describes the closeness of neighboring pixels in 2D space domain of images, which means the intensities of the neighboring pixels are quite similar. In other words, local sparsity models are built on the assumption that images are locally smooth except at the edges [6]. Thus, they will demonstrate high effectiveness in reconstructing smooth areas. There exist various choices for local sparsity model. In these models, to seek a basis or domain where the image can exhibit a high degree of sparsity is a main challenge. Most of the conventional CS reconstruction methods use predefined bases (e.g. DCT, DWT and gradient domain). Each of these bases are efficient at describing a given class of images. For example, DWT and gradient domain are suitable for piecewise-smooth images. Thus, these bases provide sparser representations for smoother images.

Considering the above two points, in this paper, we propose a method to improve the BCS image reconstruction scheme via data sorting. In this method, the original image is sorted in such a way that a smooth image is produced. Then, block-based sampling and reconstruction are applied on the smoothed image. Since smoother images give a better match to the local sparsity models, better reconstruction will be obtained. The sort-based BCS (SBCS) can be used coupled with existing reconstruction algorithms. We use the iterative projected Land Weber (PL) [7] and iterative soft thresholding (IST) [8] algorithms for image reconstruction. The proposed SBCS image reconstruction scheme effectively reduces the blocking artifacts introduced in BCS even without using of the smoothing filter while preserving the details of the image. Experimental simulations verify that the proposed CS reconstruction based on SBCS outperforms equivalent reconstruction based on the BCS.

The rest of the paper is organized as follows: In section II, the compressed sensing problem as well as BCS technique reintroduced briefly. In section III, an improved BCS image reconstruction method is presented. The simulation results are described in section IV. Finally, the conclusions are provided in section V.

2. Background

2.1.Compressed sensing problem

Consider a finite-length signal $\mathbf{u} \in \mathbb{R}^N$. A number of M ($M \ll N$) linear, non-adaptive measurement of $\mathbf{u}$ are acquired through the following linear transformation:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{u} + \mathbf{e} \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^M$ is the measurement vector, $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is a measurement matrix and $\mathbf{e}$ denotes possible measurement noise vector with $\|\mathbf{e}\|_2 \le \epsilon$. The usual choice for the measurement matrix $\mathbf{\Phi}$ is a random matrix [3]. We wish to reconstruct the signal $\mathbf{u}$ from $\mathbf{y}$ by solving (1). Since $M \ll N$, the reconstruction of $\mathbf{u}$ from $\mathbf{y}$ is ill-posed in general. However, if $\mathbf{u}$ is spares (or compressible), then exact reconstruction is possible. Many signals themselves are not sparse, but when they are expressed in a convenient basis will have sparse representations. In this case, the signal $\mathbf{u}$ can be represented as

$$\mathbf{u} = \Psi^{-1}\mathbf{x} \quad (2)$$

where $\mathbf{\Psi}$is a sparsifying basis or dictionary and $\mathbf{x} \in \mathbb{R}^N$ is the coefficient vector that most entries of which are zero or close to zero.

To reconstruct the signal$\mathbf{u}$from$\mathbf{y}$,one could search for the sparsest coefficient vector (i.e. the vector $\mathbf{x}$ with the smallest l_0 norm) consistent with the measurement $\mathbf{y}$ by solving the optimization problem

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \quad subject\ to \quad \|\mathbf{y} - \mathbf{\Phi}\Psi^{-1}\mathbf{x}\|_2 \leq \epsilon \quad (3)$$

and then reconstruct $\mathbf{u}$ using (2). Unfortunately, this optimization problem is NP-hard that can only be solved using a combinatorial approach [9]. Thus, alternative procedures to find out a suboptimal solution have been proposed in recent years. One of these, is to relax the l_0 norm, replacing it by a continuous or even smooth approximation [10].Examples of such approximations includel_p norms for some $0 < p \leq 1$[11] or even smooth functions such as Logarithm [12], Exponential [13].A popular choice for the approximation function is l_1 norm which leads to the convex optimization problem

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1 \quad subject\ to \quad \|\mathbf{y} - \mathbf{\Phi}\Psi^{-1}\mathbf{x}\|_2 \leq \epsilon \quad (4)$$

This optimization problem known as basis pursuit denoising (BPDN) can be recast into the unconstrained optimization problem

$$\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{\Phi}\Psi^{-1}\mathbf{x}\|_2^2 + \lambda\|\mathbf{x}\|_1 \quad (5)$$

which the second term is a regularization term that reflects prior information of the signal andλ is the regularization parameter. With an appropriate choice of the λ, the problem in (5) will yield the same solution as that in (4) [14]. This problem can be solved by many efficient algorithms such as iterative soft thresholding (IST) algorithm. This algorithm is simple to be implemented; however, it converges quite slowly. Some accelerated versions of IST have been proposed, including two-step IST (TwIST) [15] and fast IST (FISTA) [16]. Other algorithms include iterative projected Land Weber (PL), iteratively reweighted least squares (IRLS) [17], augmented direction method of multipliers (ADMM) [18], split Bergman iterations (SBI) [19] and Douglas-Rachford splitting (DRS) [20].

2.2.Block-based compressed sensing (BCS)

Suppose an image with N pixels and we want to acquire M CS measurements. In BCS, the image is divided into $B \times B$ non-overlapping blocks and then each block is sampled independently with the same measurement matrix $\mathbf{\Phi}_B \in \mathbb{R}^{m_B \times B^2}$ where $m_B = \left\lfloor \frac{MB^2}{N} \right\rfloor$.Let $\boldsymbol{u}_i \in \mathbb{R}^{B^2}$ is the vectorized signal of the i-th block through raster scanning. The corresponding measurement vector $\boldsymbol{y}_i \in \mathbb{R}^M$will be as

$$\boldsymbol{y}_i = \mathbf{\Phi}_B \boldsymbol{u}_i \quad (6)$$

where $\mathbf{\Phi}_B$ is an orthonormalized i.i.d Gaussian matrix. BCS is memory efficient as we just need to store an $m_B \times B^2$measurement operator$\mathbf{\Phi}_B$, rather than an $M \times N$ one [2]. At the decoder side, each block of the image is reconstructed from its corresponding measurement vector. This, i.e. treating each block as an independent sub-CS reconstruction task, would generate some blocking artifacts and reduce the visual quality. To reduce the blocking artifacts and smooth the image, a Wiener filter is applied in the spatial domain. Nonetheless, the reconstructed image still has a low quality.

3. Improved BCS image reconstruction

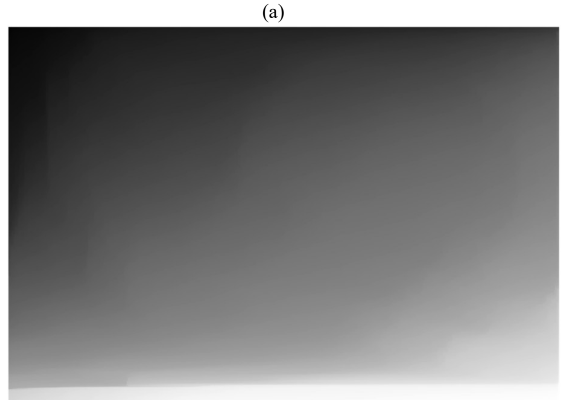
3.1. Data sorting

As mentioned before, the sparsity degree of an image for a given sparsifying transform plays a significant role in CS reconstruction. The higher sparsity degree of an image, the higher reconstruction quality it will have. Some sparsifying transforms such as DCT, DWT and gradient provide sparser representations for smoother images. Alternatively, better reconstructions for these images can be obtained. When the images of interest are nonsmoothed, we can sort them to produce smooth images. Consider, *Goldhill* image and the sorted image in horizontal and vertical directions of it as shown in Fig. 1(a) and Fig. 1(b), respectively. The histograms of the DCT and DWT coefficients of the original and sorted images are shown in Fig. 2.It is obvious to see that the histogram of the transform coefficients of the sorted image is very sharp and most coefficients are near zero. So, data sorting can lead to sparser representations of the data and hence higher quality reconstruction.

3.2. Sort-based BCS (SBCS) image reconstruction

In order to overcome the aforementioned BCS problems and improve the reconstruction result, we use the data sorting into the BCS framework. In this method, which referred as SBCS, the original image is sorted independently in horizontal and vertical directions before applying the block-based random sampling. Sorting the image helps to sparser representation of the image and hence higher quality reconstruction. The sorted image is divided into small blocks and each block is sampled independently using the same measurement operator.

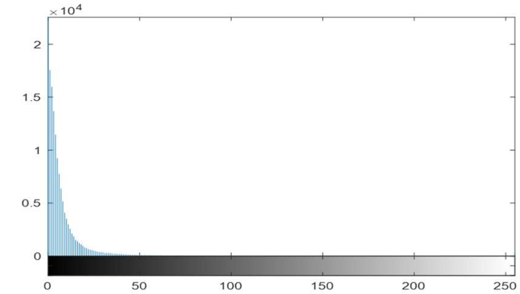
(a)



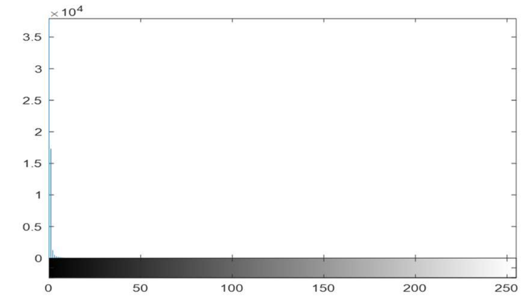
(b)

Fig. 1. (a) Goldhill image, (b) Sorted image of ‘a’

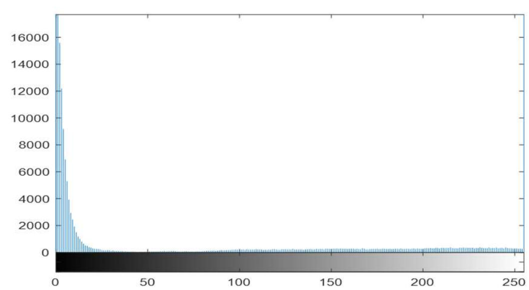
At the decoder side, each block of the sorted image is reconstructed from its corresponding measurement vector. The reconstructed sorted image then undergoes a sorting back process to generate the final reconstructed image. In this method, there is no need to apply Wiener filter, since the task of removing the blocking artifacts and retaining the visual quality is accomplished by the data sorting.



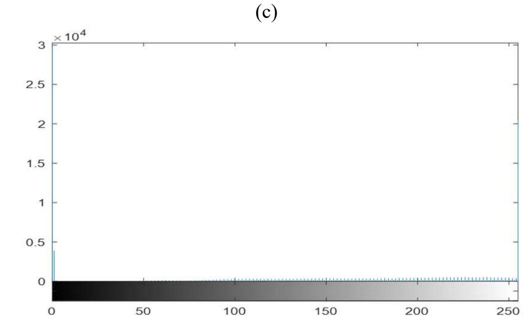
(a)



(b)



(c)



(d)

Fig. 2. Illustrations of improving image sparsity by data sorting. (a) The histogram of the DCT coefficients of the original image, (b) The histogram of the DCT coefficients of the sorted image, (c) The histogram of the DWT coefficients of the original image, (d) The histogram of the DWT coefficients of the sorted image.

Wiener filter can lead to accelerate the reconstruction. The existing reconstruction algorithms can be used coupled with the SBCS. In this paper, we use the PL and IST algorithms and refer the combination of these reconstruction algorithms and SBCS framework as SBCS-PL and SBCS-IST, respectively.

The PL algorithm forms the approximation to the image by successively projecting and thresholding [3]. This algorithm begins with an initial approximation$\mathbf{u}^0$and produces the approximation at iteration k+1 as

$$\breve{\mathbf{u}}^k = \mathbf{u}^k + \mathbf{\Phi}^T(\mathbf{y} - \mathbf{\Phi}\mathbf{u}^k) \quad (7)$$

$$\breve{\mathbf{x}}^k = \mathbf{\Psi}\breve{\mathbf{u}}^k \quad (8)$$

$$\bar{\mathbf{x}}^k = \begin{cases} 0 & if\ |\breve{\mathbf{x}}^k| < \tau^k \\ \breve{\mathbf{x}}^k & if\ |\breve{\mathbf{x}}^k| \geq \tau^k \end{cases} \quad (9)$$

$$\bar{\mathbf{u}}^k = \mathbf{\Psi}^{-\mathbf{1}}\bar{\mathbf{x}}^k \quad (10)$$

$$\mathbf{u}^{k+1} = \bar{\mathbf{u}}^k + \mathbf{\Phi}^T(\mathbf{y} - \mathbf{\Phi}\bar{\mathbf{u}}^k) \quad (11)$$

We initialize with $\mathbf{u}^0 = \mathbf{\Phi}^T\mathbf{y}$. As done in [3], we employ the universal thresholding method to set the $\tau$for hard thresholding in (9):

$$\tau^k = \lambda\sigma^k\sqrt{2\log K} \quad (12)$$

where λ is a constant control factor to manage convergence, K is number of the transform coefficients, and σ^k is estimated using a robust median estimator,

$$\sigma^k = \frac{median(|\breve{\mathbf{x}}^k|)}{0.6745} \quad (13)$$

The IST algorithm gives the solution at the iteration k+1 as (7)-(11), except that uses soft thresholding in the form of (14) rather than (9):

$$\bar{\mathbf{x}}^k = \begin{cases} \breve{\mathbf{x}}^k - \alpha & if \quad \breve{\mathbf{x}}^k \geq \alpha \\ 0 & if \quad |\breve{\mathbf{x}}^k| < \alpha \\ \breve{\mathbf{x}}^k + \alpha & if \quad \breve{\mathbf{x}}^k \leq -\alpha \end{cases} \quad (14)$$

In both SBCS-PL and SBCS-IST, we can use a spatial domain 3×3 Wiener filter in the beginning of each iteration to increase the image quality at very low measurement ratios (below 0.05) and accelerate the reconstruction process at higher measurement ratios. We refer the resulting methods as SBCS-SPL and SBCS-SIST.

4. Simulation results

In this section, the simulation results are presented to evaluate the proposed methods. We use two standard test images of size 512×512as shown in Fig. 3. All simulations are performed using MATLAB 2016a, on a 2GHZ laptop computer. In our implementations, we employ block size $B = 32$, as done in [2,3], maximum number of iterations $k_{max} = 200$, $\lambda = 6$, $\alpha = 8$, and the DCT as the sparsifying transform. Peak-Signal-to-Noise Ratio (PSNR) and the visual quality are employed to evaluate the quality of the reconstructed image. Since the quality of reconstruction can vary due to the randomness of the measurement matrix, all PSNR values are averaged over 5 independent trials. We compare our results with BCS-SPL-DDWT [3] and NLDR [21] methods. The BCS-SPL-DDWT is a block-based CS image recovery method that employs a smoothed version of PL (SPL) algorithm and the DDWT as the sparsifying transform. The smoothing is done by the Wiener filter. The NLDR is a block-based CS image recovery method which improves the CS reconstruction result by adding a nonlocal (NL) estimation step after the initial CS reconstruction from the IST algorithm.

(a) (b)

Fig. 3. Standard grayscale test images. (a) Lena, (b) Goldhill.

Fig. 4 and Fig. 5 show some visual results of CS reconstruction using 10% of measurements obtained by the SBCS-PL, SBCS-IST, SBCS-SPL and SBCS-SIST methods. It can be seen that the quality of the reconstructed images is quite high, and our proposed methods perform well in preserving the details of the images. Table I illustrates PSNR and reconstruction time with different measurement ratios, M/N. All values are obtained by averaging on the results of 5 independent trails. It is obvious that the proposed methods produce high PSNR values at all measurement ratios. The SBCS-PL and SBCS-SPL present a better reconstruction in the case of ratios 0.03 and 0.05, compared with the other methods. At higher measurement ratios, all our methods produce near similar PSNR values. The using Wiener filter increases the reconstructed image quality at very low measurement ratios (below 0.05) and accelerates the reconstruction process at higher measurement ratios. For the measurement ratios below 0.05, we use $\alpha = 20$ in SBCS-IST method.

Table 2 compares the PSNR values obtained by our methods and the BCS-SPL-DDWT and NLDR at different measurement ratios. The results indicate that the proposed methods considerably outperform the other methods in the most cases, with PSNR improvements of up to 20 and 15 dB, compared with BCS-SPL-DDWT and NLDR, respectively. The PSNR achieved by SBCS-IST at ratio 0.1 is still higher than the one by NLDR. The computation time needed for our methods is very low as compared to the mentioned methods. The BCS-SPL-DDWT and NLDR take about 1-5 and 10 minutes, respectively. These results verify the effectiveness of the SBCS in reducing the blocking artifacts introduced in BCS even without using of the smoothing filter while preserving the details of the image.

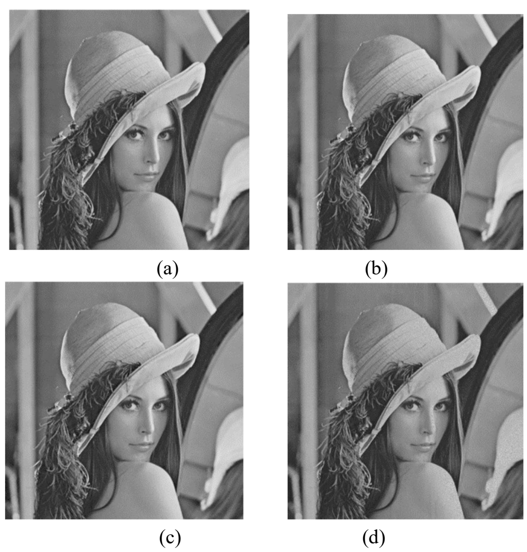


Fig.4. CS reconstructed image *Lena* with 10% measurement ratio. (a) SBCS-PL (PSNR=46.3 dB), (b) SBCS-IST (PSNR=34.43 dB), (c) SBCS-SPL (PSNR=47.6 dB), (d) SBCS-SIST (PSNR=50.8 dB).

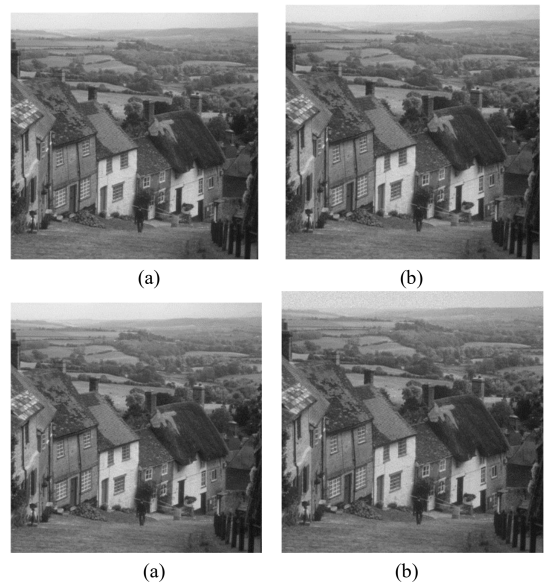


Fig. 5. CS reconstructed image *Goldhill* with 10% measurement ratio. (a) SBCS-PL (PSNR=49.2 dB), (b) SBCS-IST (PSNR=33.75 dB), (c) SBCS-SPL (PSNR=45.44 dB), (d) SBCS-SIST (PSNR=50.96 dB).

5. Conclusions

The motivation of this paper is to improve the BCS based image reconstruction scheme. We used data sorting into the BCS framework and named this new block-based CS technique as SBCS.We adopted the iterative projected Land Weber (PL) and iterative soft thresholding (IST) for image reconstruction. Experimental results demonstrated that the proposed CS reconstruction based on the SBCS outperforms equivalent reconstruction based on the BCS and effectively reduces the blocking artifacts introduced in BCS while preserving the details of the image. Our method is able to reconstruct an image exactly and quickly using fewer measurements.

Table 1. PSNR *and reconstruction time performance*

Image	M/N	SBCS-PL		SBCS-IST		SBCS-SPL		SBCS-SIST	
		PSNR	***Time***	***PSNR***	***Time***	***PSNR***	***Time***	***PSNR***	***Time***
Lena	0.03	33.73	12.89	18.95	12	40.16	14.68	29.51	14.36
	0.05	44.95	11.54	24.81	12.44	42.37	12.75	42.05	14.85
	0.1	46.29	12.65	34.43	13.01	47.60	11.58	50.80	13.58
	0.2	53.20	12.53	48.19	14.32	54.69	6.37	54.02	7.8
	0.3	56.34	13.77	53.91	14.04	57.02	4.71	55.68	6.22
	0.4	57.62	11.33	55.82	11.42	58.41	4.27	56.97	5.77
Goldhill	0.03	34.54	11.83	19.44	12	38.18	14.88	23.49	14.17
	0.05	40.70	12.12	23.22	12.37	42.64	12.63	40.24	14.35
	0.1	49.19	12.42	33.75	13.2	45.44	13.05	50.96	11.87
	0.2	55.17	13.9	47.85	13.05	51.65	6.29	54.05	7.30
	0.3	57.02	11.6	54.52	12.74	55.23	4.65	55.67	6.42
	0.4	58.18	10.06	56.14	10.32	57.43	3.58	57.14	5.61

Table 2. PSNR comparison of CS reconstruction methods.

Image	M/N	SBCS-PL	SBCS-IST	SBCS-SPL	SBCS-SIST	BCS-SPL-DDWT[3]	NLDR [21]
Lena	0.1	46.29	34.43	47.60	50.80	28.31	33.67
	0.2	53.20	48.19	54.69	54.02	31.37	36.33
	0.3	56.34	53.91	57.02	55.68	33.50	37.82
	0.4	57.62	55.82	58.41	56.97	35.20	39.02
	0.5	58.88	57.25	59.53	58.18	36.78	40.16
Goldhill	0.1	49.19	33.75	45.44	50.96	26.96	28.94
	0.2	55.17	47.85	51.65	54.05	28.93	32.10
	0.3	57.02	54.52	55.23	55.67	30.45	33.99
	0.4	58.18	56.14	57.43	57.14	31.79	35.61
	0.5	59.30	57.61	58.96	58.34	33.11	37.20

References

[1] D. L. Donoho, "Compressed sensing," IEEE Trans. on Information Theory, vol. 52, pp. 1289–1306, April 2006.

[2] L. Gan, "Block Compressed Sensing of Natural Images ," in Proc. of the 15th Intl. Conf. on Digital Signal Processing (DSP 2007), 2007.

[3] S. Mun and J. E. Fowler, "Block Compressed Sensing of Images Using Directional Transforms," 16th IEEE Intl. Conf. on Image Processing (ICIP 2009), 2009.

[4] J. Haupt and R. Nowak, "Signal reconstruction from noisy random projections," IEEE Trans. on Information Theory, vol. 52, no. 9, pp. 4036–4048, September 2006.

[5] J. Zhang, D. Zhao and et al., "Image Compressive Sensing Recovery via Collaborative Sparsity," IEEE Journal on Emerging and Selected Topics in Circuits and Systems, vol. 2, no. 3, Sep. 2012.

[6] J. Zhang, D. Zhao and et al., "Image Restoration Using Joint Statistical Modeling in a Space Transform Domain," IEEE Trans. on Circuits and systems for Video Technology, vol. 24, No. 6, 2014.

[7] H. J. Trussell and M. R. Civanlar, "The Landweber Iteration and Projection onto Convex Sets," IEEE Trans. on Acoustics, Speech and Signal Processing, vol. Assp-33, no. 6, 1985.

[8] I. Daubechies, M. Defrise, and C. De-Mol, "An Iterative Thresholding Algorithm for Linear

Inverse Problems with a Sparsity Constraint," Pure Appl. Math., vol. 57, no. 11, pp. 1413-1457, Nov. 2004.

[9] J. J. Fuchs, "On Sparse Representations in Arbitrary Redandants Bases," IEEE Trans. on Information Theory, vol. 50, no. 6, June 2004.

[10] M. Elad, "Sparse and Redundant Representations," springer, 2010.

[11] L. E. Frank and J. H. Friedman, "A statistical view of some chemometrics regression tools," Technometrics, vol. 35, no. 2, pp. 109–135, 1993.

[12] J. H. Friedman, "Fast sparse regression and classification," Int. J. Forecasting, vol. 28, no. 3, pp. 722–738, Jul./Sep. 2012.

[13] C. Gao, N. Wang, Q. Yu, and Z. Zhang, "A feasible nonconvex relaxation approach to feature selection," in Proc. 25th AAAI Conf. Artif. Intell., pp. 356–361, 2011.

[14] K. Hayashi, M. Nagahara and T. Tanaka, "A User's Guide to Compressed Sensing for Communication Systems," IEEE Trans. on commun., no. 3, March 2013.

[15] J. Bioucas-Dias and M. Figueiredo, "A new TwIST: Two step iterative shrinkage/ thresholding algorithms for image restoration," IEEE Trans. on Image Processing, vol.16, no. 12, pp. 2992-3004, Dec. 2007.

[16] A. Beck and M. Tebulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," SIAM J. Imaging Sci., vol. 2, no. 1, pp. 183-202, Jan. 2009.

[17] R. Chartrand and W. Yin, "Iteratively reweighted algorithms for compressive sensing, " in Proc. ICASSP, Las Vegas, USA, pp. 3869-3872, Mar.-Apr. 2008.

[18] S. Boyd and et.al., "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," Foundations and Trends in Machine Learning, vol. 3, no. 1, 2010.

[19] T. Goldstein and S. Osher, "The split Bregman method for $\ell 1$ regularized problems," SIAM J. Imag. Sci., vol. 2, no. 2, pp. 323-343, Apr. 2009.

[20] P. L. Combettes and J.C. Pesquet, "Proximal splitting methods in signal processing," in Fixed-Point Algorithms for Inverse Problems in Science and Engineering. Berlin, Germany: Springer-Verlag, pp. 185–212, 2011.

[21] S. Li and H. Qi, "A Douglas-Rachford Splitting Approach to Compressed Sensing Image Recovery Using Low Rank Regularization," IEEE Trans. on image processing, vol. 24, No. 11, 2015.