Transactions on Fuzzy Sets and Systems (TFSS)
URL: https://sanad.iau.ir/journal/tfss/
Online ISSN: 2821-0131
Vol.5, No.2, (2026), 362-383
DOI: https://doi.org/10.71602/tfss.2026.1205592

Article Type: Original Research Article

Analysis on the Neutrosophic Fuzzy Rough Multi-objective Quadratic Transportation Problem Using Various Membership Functions

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Abstract. This paper introduces a new variant of fuzzy set called a neutrosophic fuzzy rough set, which is developed by combining both rough set and neutrosophic fuzzy set theory for optimal benefit. An effective optimization of the multi-objective quadratic transportation problem is examined, with a chance of distinct solution vectors for each objective function. This paper employs both neutrosophic fuzzy rough numbers and MOQTP to model Neutrosophic Fuzzy Rough Multi-Objective Quadratic Transportation Problem (NFRMOQTP). In addition, we present a method for solving NFRMOQTP with a numerical example, which involves transforming the model into a single-objective quadratic transportation problem by utilizing various membership functions. Further, in order to verify the suggested approach, we contrast the outcomes with existing technique and the results are discussed.

AMS Subject Classification 2020: 49N10; 94D05; 90C08; 90C29
Keywords and Phrases: Quadratic transportation problem, Rough set theory, Neutrosophic set, Membership functions, Goal programming.

Abbreviations employed in this study	
MOTP	Multi Objective Transportation problem
MOQTP	Multi Objective Quadratic Transportation Problem
MOLTP	Multi Objective Linear Transportation Problem
GP	Goal Programming
WSM	Weighted Sum Method
Apr	Approximation
Lim	Limit
Max	Maximum
Min	Minimum
TNN	Triangular Neutrosophic Number
NFRN	Neutrosophic Fuzzy Rough Numbers
NFRMOQTP	Neutrosophic Fuzzy Rough Multi Objective Quadratic Transportation Problem
LPP	Linear Programming Problem

1 Introduction

In 1982 & 2002, Pawlak [1, 2] introduced a methodology for rough set theory. The core concept of rough set theory is the formation of equivalence classes within the given data for analysis and to elaborate on the capabilities of rough set theory and to propose rough fuzzy relations. Upon analyzing rough and fuzzy sets, Dubois and Prade [3] observed that these two theories provide distinct methodologies for managing imprecise

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Received: 2 May 2025; **Revised**: 17 May 2025; **Accepted**: 18 September 2025; **Available Online**: 23 November 2025; **Published Online**: 7 November 2026.

How to cite: Paraman Anukokila, Rajendran Nisanthini, Bheeman Radhakrishnan. Analysis on the neutrosophic fuzzy rough multi-objective quadratic transportation problem using various membership functions. *Transactions on Fuzzy Sets and Systems.* 2026; 5(2): 362-383. DOI: https://doi.org/10.71602/tfss.2026.1205592

information. Based on their research on rough sets and fuzzy sets, they introduced two novel hybrid solutions: fuzzy rough sets and rough fuzzy sets. Fuzzy set theory was developed by Zadeh [4] in 1965 as a way to mathematically define concepts that are ambiguous or imprecise. Fuzzy sets grant a potent mathematical representation that helps in the handling of ambiguous and imprecise data, allowing for a more precise and sensitive comprehension of intricate issue settings. Atanassov's fuzzy set theory [5] is expanded upon by intuitionistic fuzzy set theory. The uncertainty that cannot be classified as either membership or non-membership is known as indeterminacy, and it is not captured by the intuitionistic fuzzy sets. By employing neutrosophic sets, which add a third parameter to represent indeterminacy, Smarandache [6] was able to overcome this constraint. Neutrosophic sets offer a more thorough depiction of uncertainty and ambiguity in optimization problems by combining memberships for degree of truth, indeterminate, and falsehood. The neutrosophic set model is an essential tool for addressing genuine scientific and technical challenges.

Hitchcock [7] first formulated the transportation issue, a notable example of linear programming seen in several real decision-making scenarios. Reducing the cost of transportation or increasing profit is the aim of the transportation problem. Haley [8] first proposed the term solid transportation problem to describe the challenge of traditional transportation expansion. The recent literature addresses several extensions of transportation problems, such as the solid transportation problem, the fractional transportation problem, the multi-objective transportation problem, and multi objective fractional transportation problem [9–11]. In this way, when a decision-maker in a real-world transportation system must optimize quadratic cost/profit functions rather than linear ones, the result is quadratic objective functions, which give rise to MOQTP. Originally, Basirzadeh [12] suggested using fuzzy numbers in transportation problems. Subsequent studies use the transportation problem with different fuzzy parameters. We employ an expanded structure of fuzzy rough numbers, called neutrosophic fuzzy rough numbers in MOQTP issues, to characterize different viewpoints in a systematic computation.

1.1 Motivation of this study

- The motivation for neutrosophic fuzzy rough numbers is to provide a more robust mathematical tool for addressing uncertainty and ambiguous information in transportation and decision-making challenges by integrating the principles of neutrosophic fuzzy numbers and rough numbers.
- In the literature, Mahajan et al. [13] presents a MOQPP under intuitionistic fuzzy, since intuitionistic fuzzy cannot handle indeterminacy. We have employed a neutrosophic fuzzy environment to handle the indeterminacy and there is no study on MOQTP involving NFRN.
- The integration of an NFRN with the MOQTP results in a challenge characterized by ambiguity, information reliability, and competing aims. Addressing this issue is challenging and intriguing in its relevance.

1.2 Contribution of this study

In summary, the important contributions of the proposed strategy are:

- The notion of neutrosophic fuzzy rough number and its arithmetic operations are introduced and demonstrated through graphical representation.
- The model of MOQTP is developed within the framework of NFRN environment, which offers distinct advantages as compared to other uncertain environments.
- The MOQTP model is designed under the NFRN framework, which provides specific benefits over other uncertain settings. We suggest a strategy to address NFRMOQTP by progressively transforming the

original issue into the exact MOTP problem using the ranking function, and then turning the crisp MOTP problem into a single-objective transportation problem by various membership functions documented in the literature. The suggested technique flexibly employs membership functions as objective functions. The selection of a certain membership function is contingent upon the decision maker and the characteristics of the situation.

- Furthermore, the findings have been validated by comparison analysis using Gargs methodology [14].

2 Literature Review

The transportation problem has long been a valuable tool in business, manufacturing, marketing and advertising, and other fields of interest. Decision making and analysis rely heavily on it. Fuzzy set theory and transportation issues together provide an abundance of efficient and innovative decision analysis applications. Bellman and Zadeh [15] were the first to suggest novel concepts for optimization issues in the field of fuzzy sets. In a fuzzy setting, Zimmerman [16] presented the first and most important multi-objective linear programming model. Tanaka and Asai [17] described LPPs with fuzzy numbers. Using the interval type 2 fuzzy set, Pratihar et al. [18] investigated transportation costs, supply, and demand in relation to transportation problem. The MOTP was developed by Mahajan and Gupta [19] using the fully intuitionistic fuzzy technique. Bagheri et al. [20] addressed the uncertain MOTP with fuzzy costs by using the fuzzy arithmetic data envelopment analysis approach to handle the problem. Adhami and Ahmad [21] introduced a distinctive Pythagorean hesitant fuzzy programming method to address the Multi-Objective Transportation Problem (MOTP) amongst unpredictable supply, demand, and cost variables. Triangular intuitionistic fuzzy numbers were recently used as parameters in a research by Rani et al. [22] addressing the uncertain non-linear transportation problem. Akram et al. [23] used Fermatean fuzzy set theory to tackle the uncertainty inherent in a multi-objective transportation issue model. Sahoo [24] developed novel scoring systems based on Fermatean fuzzy logic and used them to address the transportation problem. Additionally, Ali et al. [25] used the solution programming approach to study a multi-objective capacitated fractional transportation problem.

Further extensions of the fuzzy scenario have been made to address quadratic programming issues using the fuzzy idea. Trade-offs between aims are necessary in these situations because achieving one goal at the expense of another prevents the emergence of the perfect solution. The idea of Pareto optimality is invoked in this trade-off, aiding in the provision of a compromise resolution. Numerous methods have been used to investigate QTP, and Jain et al. [26] developed an approach for addressing quadratic fractional integer programming problems. Mekhilef et al. [27] have delineated a method for addressing a multi-objective integer indefinite quadratic fractional problem. Separable quadratic problems, such as separable convex QTP with a given number of sources, are examined by Megiddo and Tamir [28]. Adlakha [29] provides a straight forward analytical method for resolving quadratic transportation difficulties. Recently, Singh et al. [30] created a brand-new multi-objective indefinite transportation model based on Fermatean fuzzy, proving its suitability for sustainable transportation.

Many researchers have proposed operators for fuzzy rough numbers in multi-criteria decision-making issues. Ibrahim [31] proposed using an interval-valued Fermatean fuzzy rough set based decision-making model to evaluate sustainability in mobility for autonomous vehicles within the context of smart city assessment. Akram [32,33] enhanced the CRITIC-REGIME approach for using Pythagorean fuzzy rough numbers in decision-making and introduced spherical fuzzy rough numbers as a foundation for multi-criteria group decision-making. Liu [34] has improved the BWM and MABAC methodologies for decision-making by using q-rung orthopair fuzzy rough numbers. Kamran et al. [35] introduced a neutrosophic Z-rough set approach for decision making problem using trigonometric operators.

The contemplated phases for this study are as follows: Section 3 reports the preliminary findings and their mathematical operations. Section 4 develops a mathematical formulation for NFRMOQTP along with a methodology for solving it. Section 5 presents practical examples, a comparison section, results, and discussion. Section 6 covers the conclusion and directions for future research.

3 Preliminaries

Definition 3.1. [36] *Let $\mathcal{W}$ be the universal disclosure set, then a single valued neutrosophic fuzzy set $\mathcal{N}$ on $\mathcal{W}$ is defined as $\mathcal{N} = \{(w, T_{\mathcal{N}}(w), I_{\mathcal{N}}(w), F_{\mathcal{N}}(w)) : w \in \mathcal{W}\}$, where $T_{\mathcal{N}}(w), I_{\mathcal{N}}(w), F_{\mathcal{N}}(w) \in [0,1]$ are truth, indeterminacy and falsity membership degrees, respectively, such that for every $w \in \mathcal{W}$, $0 \leq (T_{\mathcal{N}}(w) + I_{\mathcal{N}}(w) + F_{\mathcal{N}}(w)) \leq 3$.*

Example 3.2. A single valued neutrosophic fuzzy number $\mathcal{N} = (0.7, 0.3, 0.3)$.

Definition 3.3. [37] *A triangular single valued neutrosophic number, $\tilde{\mathcal{N}} = \{(a, b, c); \mu_{\tilde{\mathcal{N}}}, \nu_{\tilde{\mathcal{N}}}, \zeta_{\tilde{\mathcal{N}}}\}$ is a neutrosophic fuzzy set over the real domain $\mathbb{R}$, whose truth ($T_{\tilde{\mathcal{N}}}$), indeterminacy ($I_{\tilde{\mathcal{N}}}$) and falsity ($F_{\tilde{\mathcal{N}}}$) membership functions are given as follows:*

$$T_{\tilde{\mathcal{N}}}(w) = \begin{cases} \mu_{\tilde{\mathcal{N}}}\left(\frac{w-a}{b-a}\right) & a \leq w \leq b; \\ \mu_{\tilde{\mathcal{N}}} & w = b; \\ \mu_{\tilde{\mathcal{N}}}\left(\frac{c-w}{c-b}\right) & b < w \leq c; \\ 0 & w < a \text{ or } w > c. \end{cases}$$

$$I_{\tilde{\mathcal{N}}}(w) = \begin{cases} \frac{(b-w)+\nu_{\tilde{\mathcal{N}}}(w-a)}{(b-a)} & a \leq w \leq b; \\ \nu_{\tilde{\mathcal{N}}} & w = b; \\ \frac{(w-b)+\nu_{\tilde{\mathcal{N}}}(c-w)}{(c-b)} & b < w \leq c; \\ 1 & w < a \text{ or } w > c. \end{cases}$$

$$I_{\tilde{\mathcal{N}}}(w) = \begin{cases} \frac{(b-w)+\zeta_{\tilde{\mathcal{N}}}(w-a)}{(b-a)} & a \leq w \leq b; \\ \zeta_{\tilde{\mathcal{N}}} & w = b; \\ \frac{(w-b)+\zeta_{\tilde{\mathcal{N}}}(c-w)}{(c-b)} & b < w \leq c; \\ 1 & w < a \text{ or } w > c, \end{cases}$$

where, $\mu_{\tilde{\mathcal{N}}}, \nu_{\tilde{\mathcal{N}}}, \zeta_{\tilde{\mathcal{N}}}$ denote the greatest extent of truth-membership degree, least extent of indeterminacy and falsity membership degree respectively, which is represented in Figure 1.

Example 3.4. A triangular single valued neutrosophic number $\tilde{\mathcal{N}} = \{(1, 2, 3); 0.7, 0.3, 0.3\}$.

Definition 3.5. [2] *Let $\mathcal{W}$ be a domain that is taken to be non-empty and where $S \subset \mathcal{W}$. Let Y be the symbol for an equivalency relation satisfying the indiscernible requirement on $\mathcal{W}$. A pair $(\mathcal{W}, Y)$ is called an approximation space or rough set space. The lower and upper approximations of S, denoted by $Y_L(S)$ and $Y_U(S)$ are defined as follows:*

$$Y_L(S) = \{w \in W | [w]_Y \subset S\},$$
$$Y_U(S) = \{w \in W | [w]_Y \cap S \neq \phi\}.$$

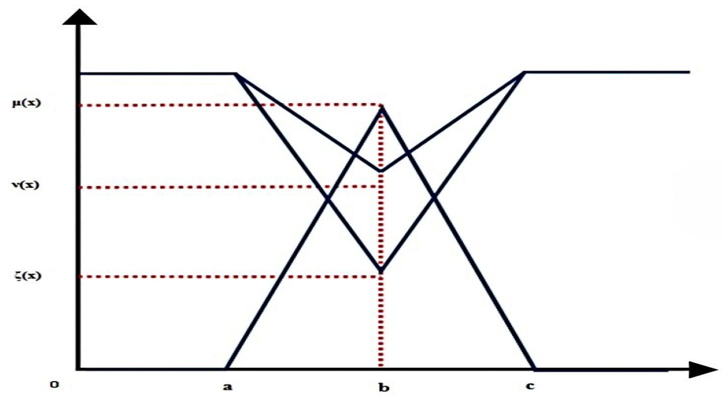

Figure 1: Triangular neutrosophic fuzzy number

Definition 3.6. [37] *Let us assume that $\mathcal{W}$ is the domain containing TNNs that arise from expert evaluations on a specific field. These values are segregated into n number of classes, arranged as $\tilde{\mathcal{N}}_1 < \tilde{\mathcal{N}}_2 < ... < \tilde{\mathcal{N}}_n$, where $\tilde{\mathcal{N}}_i = \{(a_i, b_i, c_i); \mu_{\tilde{\mathcal{N}}_i}, \nu_{\tilde{\mathcal{N}}_i}, \zeta_{\tilde{\mathcal{N}}_i}\}$ for each $1 < i < n$ indicates the TNN. Let $\tilde{\mathcal{R}} = \{\tilde{\mathcal{N}}_1, \tilde{\mathcal{N}}_2, ..., \tilde{\mathcal{N}}_n\}$ be the collection of all classes and $\tilde{\mathcal{R}}_a = \{\tilde{\mathcal{N}}_i^a :< i < n\}, \tilde{\mathcal{R}}_b = \{\tilde{\mathcal{N}}_i^b :< i < n\}, \tilde{\mathcal{R}}_c = \{\tilde{\mathcal{N}}_i^c :< i < n\}$ be the set of corresponding components, if we assume that B is a generic element from the domain, then the lower and upper approximation of each component is equal to Apr_L and Apr_U for each class $\tilde{\mathcal{N}}_i$. In terms of math, it is defined as*

$$Apr_L(\tilde{\mathcal{N}}_i^a) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_a \leq \tilde{\mathcal{N}}_i^a\}; Apr_U(\tilde{\mathcal{N}}_i^a) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_a \geq \tilde{\mathcal{N}}_i^a\}.$$
$$Apr_L(\tilde{\mathcal{N}}_i^b) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_b \leq \tilde{\mathcal{N}}_i^b\}; Apr_U(\tilde{\mathcal{N}}_i^b) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_b \geq \tilde{\mathcal{N}}_i^b\}.$$
$$Apr_L(\tilde{\mathcal{N}}_i^c) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_c \leq \tilde{\mathcal{N}}_i^c\}; Apr_U(\tilde{\mathcal{N}}_i^c) = \bigcup\{B \in \mathcal{W} | \tilde{\mathcal{R}}_c \geq \tilde{\mathcal{N}}_i^c\}.$$

The lower and upper bound of class $\tilde{\mathcal{N}}_i^a$ is the lower and upper bound of each component specified as follows:

$$Lim_L(\tilde{\mathcal{N}}_i^a) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^a)}{|Apr(\tilde{\mathcal{N}}_i^a)|}; \qquad Lim_U(\tilde{\mathcal{N}}_i^a) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^a)}{|Apr(\tilde{\mathcal{N}}_i^a)|};$$
$$Lim_L(\tilde{\mathcal{N}}_i^b) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^b)}{|Apr(\tilde{\mathcal{N}}_i^b)|}; \qquad Lim_U(\tilde{\mathcal{N}}_i^b) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^b)}{|Apr(\tilde{\mathcal{N}}_i^b)|};$$
$$Lim_L(\tilde{\mathcal{N}}_i^c) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^c)}{|Apr(\tilde{\mathcal{N}}_i^c)|}; \qquad Lim_U(\tilde{\mathcal{N}}_i^c) = \frac{\sum B \in Apr(\tilde{\mathcal{N}}_i^c)}{|Apr(\tilde{\mathcal{N}}_i^c)|},$$

where $|Apr(\tilde{\mathcal{N}}_i^l)|, l = a, b, c$, represents the cardinality of $Apr(\tilde{\mathcal{N}}_i^l)$. In Figure 2, a Neutrosophic fuzzy rough number is shown and stated as

$$\text{NFRN}(\tilde{\mathcal{N}}_i) = \left[\lceil Lim_L(\tilde{\mathcal{N}}_i^a), Lim_U(\tilde{\mathcal{N}}_i^a)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_i^b), Lim_U(\tilde{\mathcal{N}}_i^b)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_i^c), Lim_U(\tilde{\mathcal{N}}_i^c)\rfloor; \mu_{\tilde{\mathcal{N}}_i}, \nu_{\tilde{\mathcal{N}}_i}, \zeta_{\tilde{\mathcal{N}}_i}\right],$$

such that $0 \leq (\mu_{\tilde{\mathcal{N}}_i} + \nu_{\tilde{\mathcal{N}}_i} + \zeta_{\tilde{\mathcal{N}}_i}) \leq 3$.

Example 3.7. Let $\mathcal{W} = \{[(1, 2, 3); 0.7, 0.3, 0.3], [(4, 5, 6); 0.9, 0.2, 0.1], [(7, 8, 9); 0.8, 0.2, 0.2]\}$ be the universal set, consisting of triangular neutrosophic fuzzy numbers. The neutrosophic fuzzy rough number corresponding

to the class $\tilde{\mathcal{R}} = \{[(1,2,3); 0.7, 0.3, 0.3]\}$ is given as follows:
Approximation of $[(1,2,3); 0.7, 0.3, 0.3]$

$$Apr_L(1) = \{1\}; \quad Apr_L(2) = \{2\}; \quad Apr_L(3) = \{3\}.$$
$$Apr_U(1) = \{1,4,7\} \quad Apr_U(2) = \{2,5,8\} \quad Apr_U(3) = \{3,6,9\}.$$

Limits of $[(1,2,3); 0.7, 0.3, 0.3]$

$$Lim_L(1) = 1; \quad Lim_L(2) = 2; \quad Lim_L(3) = 3.$$
$$Lim_U(1) = 4; \quad Lim_U(2) = 5; \quad Lim_U(3) = 6.$$

The NFRN of $\tilde{\mathcal{R}} = \{[(1,2,3); 0.7, 0.3, 0.3]\}$ is $([1,4],[2,5],[3,6]; 0.7, 0.3, 0.3)$.

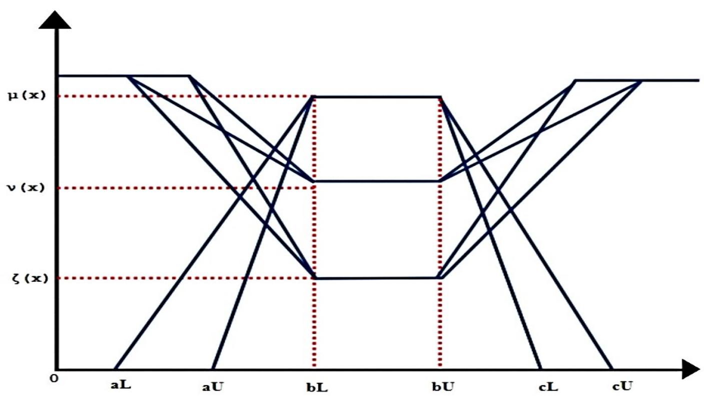

Figure 2: Triangular neutrosophic fuzzy rough number

Definition 3.8. [38] *A defuzzification technique for NFRN* $(\tilde{\mathcal{N}}) = \left[\lceil Lim_L(\tilde{\mathcal{N}}^a), Lim_U(\tilde{\mathcal{N}}^a)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}^b), Lim_U(\tilde{\mathcal{N}}^b)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}^c), Lim_U(\tilde{\mathcal{N}}^c)\rfloor; \mu_{\tilde{\mathcal{N}}}, \nu_{\tilde{\mathcal{N}}}, \zeta_{\tilde{\mathcal{N}}}\right]$ *is defined by*

$$\mathfrak{R}(\tilde{\mathcal{N}}) = \frac{1}{16}\left[Lim_L(\tilde{\mathcal{N}}^a) + Lim_U(\tilde{\mathcal{N}}^a) + Lim_L(\tilde{\mathcal{N}}^b) + Lim_U(\tilde{\mathcal{N}}^b) + Lim_L(\tilde{\mathcal{N}}^c) + Lim_U(\tilde{\mathcal{N}}^c)\right](2 + \mu_{\tilde{\mathcal{N}}} - \nu_{\tilde{\mathcal{N}}} - \zeta_{\tilde{\mathcal{N}}}). \quad (1)$$

Example 3.9. A NFRN $\tilde{\mathcal{N}} = ([70,75],[80,90],[95,110]; 0.8, 0.3, 0.2)$ is defuzzified as

$$\mathfrak{R}(\tilde{\mathcal{N}}) = \frac{1}{16}[(70 + 75 + 80 + 90 + 95 + 110)(2 + 0.8 - 0.3 - 0.2)] = 74.75$$

Definition 3.10. [37] *Let*
$\tilde{\mathcal{N}}_1 = \left[\lceil Lim_L(\tilde{\mathcal{N}}_1^a), Lim_U(\tilde{\mathcal{N}}_1^a)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_1^b), Lim_U(\tilde{\mathcal{N}}_1^b)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_1^c), Lim_U(\tilde{\mathcal{N}}_1^c)\rfloor; \mu_{\tilde{\mathcal{N}}_1}, \nu_{\tilde{\mathcal{N}}_1}, \zeta_{\tilde{\mathcal{N}}_1}\right]$,
$\tilde{\mathcal{N}}_2 = \left[\lceil Lim_L(\tilde{\mathcal{N}}_2^a), Lim_U(\tilde{\mathcal{N}}_2^a)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_2^b), Lim_U(\tilde{\mathcal{N}}_2^b)\rfloor, \lceil Lim_L(\tilde{\mathcal{N}}_2^c), Lim_U(\tilde{\mathcal{N}}_2^c)\rfloor; \mu_{\tilde{\mathcal{N}}_2}, \nu_{\tilde{\mathcal{N}}_2}, \zeta_{\tilde{\mathcal{N}}_2}\right]$
are the NFRNs. Then, the mathematical operations on these two NFRNs are defined as

1. *Addition:*

$$\begin{aligned}\tilde{\mathcal{N}}_1 + \tilde{\mathcal{N}}_2 = &\lceil Lim_L(\tilde{\mathcal{N}}_1^a) + Lim_L(\tilde{\mathcal{N}}_2^a), Lim_U(\tilde{\mathcal{N}}_1^a) + Lim_U(\tilde{\mathcal{N}}_2^a)\rfloor,\\ &\lceil Lim_L(\tilde{\mathcal{N}}_1^b) + Lim_L(\tilde{\mathcal{N}}_2^b), Lim_U(\tilde{\mathcal{N}}_1^b) + Lim_U(\tilde{\mathcal{N}}_2^b)\rfloor,\\ &\lceil Lim_L(\tilde{\mathcal{N}}_1^c) + Lim_L(\tilde{\mathcal{N}}_2^c), Lim_U(\tilde{\mathcal{N}}_1^c) + Lim_U(\tilde{\mathcal{N}}_2^c)\rfloor;\\ &min\{\mu_{\tilde{\mathcal{N}}_1}, \mu_{\tilde{\mathcal{N}}_2}\}, max\{\nu_{\tilde{\mathcal{N}}_1}, \nu_{\tilde{\mathcal{N}}_1}\}, max\{\zeta_{\tilde{\mathcal{N}}_1}, \zeta_{\tilde{\mathcal{N}}_2}\}.\end{aligned}$$

2. *Subtraction:*

$$\begin{aligned}\tilde{\mathcal{N}_1} - \tilde{\mathcal{N}_2} =& \lceil Lim_L(\tilde{\mathcal{N}}_1^a) - Lim_U(\tilde{\mathcal{N}}_2^c), Lim_U(\tilde{\mathcal{N}}_1^a) - Lim_L(\tilde{\mathcal{N}}_2^c)\rfloor,\\ & \lceil Lim_L(\tilde{\mathcal{N}}_1^b) - Lim_U(\tilde{\mathcal{N}}_2^b), Lim_U(\tilde{\mathcal{N}}_1^b) - Lim_L(\tilde{\mathcal{N}}_2^b)\rfloor,\\ & \lceil Lim_L(\tilde{\mathcal{N}}_1^c) - Lim_U(\tilde{\mathcal{N}}_2^a), Lim_U(\tilde{\mathcal{N}}_1^c) - Lim_L(\tilde{\mathcal{N}}_2^a)\rfloor;\\ & min\{\mu_{\tilde{\mathcal{N}}_1}, \mu_{\tilde{\mathcal{N}}_2}\}, max\{\nu_{\tilde{\mathcal{N}}_1}, \nu_{\tilde{\mathcal{N}}_1}\}, max\{\zeta_{\tilde{\mathcal{N}}_1}, \zeta_{\tilde{\mathcal{N}}_2}\}.\end{aligned}$$

3. *Scalar Multiplication:*

$$\begin{aligned}\lambda\tilde{\mathcal{N}_1} =& [\lceil \lambda Lim_L(\tilde{\mathcal{N}}_1^a), \lambda Lim_U(\tilde{\mathcal{N}}_1^a)\rfloor, \lceil \lambda Lim_L(\tilde{\mathcal{N}}_1^b), \lambda Lim_U(\tilde{\mathcal{N}}_1^b)\rfloor,\\ & \lceil \lambda Lim_L(\tilde{\mathcal{N}}_1^c), \lambda Lim_U(\tilde{\mathcal{N}}_1^c)\rfloor; \mu_{\tilde{\mathcal{N}}_1}, \nu_{\tilde{\mathcal{N}}_1}, \zeta_{\tilde{\mathcal{N}}_1}].\end{aligned}$$

Example 3.11. Consider two NFRNs

$\tilde{\mathcal{N}_1} = ([70, 75], [80, 90], [95, 110]; 0.8, 0.3, 0.2)$ and $\tilde{\mathcal{N}_2} = ([60, 70], [75, 85], [86, 96]; 0.9, 0.1, 0.1)$ with $\lambda = 2$. Then

1. $\tilde{\mathcal{N}_1} + \tilde{\mathcal{N}_2} = ([130, 145], [155, 175], [181, 206]; 0.8, 0.3, 0.2)$.
2. $\tilde{\mathcal{N}_1} - \tilde{\mathcal{N}_2} = ([-26, -11], [-5, 15], [25, 50]; 0.8, 0.3, 0.2)$.
3. $\lambda.\tilde{\mathcal{N}_1} = ([140, 150], [160, 180], [190, 220]; 0.8, 0.3, 0.2)$.

4 Neutrosophic fuzzy rough multi-objective quadratic transportation problem

The implementation of k objectives with precise parameters in MOQTP may not always accurately reflect real-world scenarios. As a result, the MOQTP formulation, in which all of the parameters are NFRNs, is more suited to tracking membership/non-membership degrees in real-world scenarios. Under a completely NFRN context, the problem (MOQTP) can be represented as follows:

$$\begin{cases} \text{Max/Min} \;\; \tilde{Z}^k(\tilde{y}) = \displaystyle\sum_{j=1}^{m} p_j^k y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}^k y_i y_j, \quad k = 1, 2, \ldots, K \\ \text{subject to} \\ \quad \displaystyle\sum_{j=1}^{m} a_{ij} y_j \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b_i, \quad \text{for} \;\; i = 1, 2, \ldots, n. \\ \qquad\qquad y_j \ge 0, \qquad \text{for} \;\; j = 1, 2, \ldots, m. \end{cases}$$

Here $p_j^k, q_{ij}^k, a_{ij}, b_i$ are NFRNs, for all i and j and $[q_{ij}^k]$ symmetric positive semi-definite for all k, which yields that $\tilde{Z}^k$ is a convex objective function.
We examine a MOQTP with objective parameters, coefficient matrix, and right-hand sides represented as NFRN numbers as follows:

$$\begin{cases} \text{Max/Min} \;\; \tilde{Z}^k(\tilde{y}) = \{\tilde{Z}_1^{\mathcal{R}}, \tilde{Z}_2^{\mathcal{R}}, \ldots, \tilde{Z}_r^{\mathcal{R}}\}. \\ \text{subject to} \\ \quad \displaystyle\sum_{j=1}^{m} (a_{ij})^{\mathcal{R}} y_j \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} (b_i)^{\mathcal{R}}, \quad \text{for} \quad i = 1, 2, \ldots, n. \\ \qquad\qquad y_j \ge 0, \qquad \text{for} \quad j = 1, 2, \ldots, m, \end{cases} \tag{2}$$

where $\tilde{Z}_l^{\mathcal{R}} = \sum_{j=1}^{m} p_j^k y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}^k y_i y_j$, for all $l = 1, 2, ..., r$, is a decision vector.

4.1 Solution Methodology

The steps of the proposed neutrosophic fuzzy rough multi objective quadratic transportation problem are structured in Figure 3 and also described below:

Step 1: Select the neutrosophic fuzzy rough multi objective quadratic transportation problem as mentioned in model.

$$\left\{\begin{array}{l} \text{Max/Min} \quad \tilde{Z}^k(\tilde{y}) = \sum_{j=1}^{m}[([\underline{p}_j^1, \overline{p}_j^1], [\underline{p}_j^2, \overline{p}_j^2], [\underline{p}_j^3, \overline{p}_j^3]); \mu_j^p, \nu_j^p, \zeta_j^p]^k y_j \\ \qquad\qquad + \sum_{i=1}^{n}\sum_{j=1}^{m}[([\underline{q}_{ij}^1, \overline{q}_{ij}^1], [\underline{q}_{ij}^2, \overline{q}_{ij}^2], [\underline{q}_{ij}^3, \overline{q}_{ij}^3]); \mu_{ij}^q, \nu_{ij}^q, \zeta_{ij}^q]^k y_i y_j, \\ \text{subject to} \\ \sum_{j=1}^{m}[([\underline{a}_{ij}^1, \overline{a}_{ij}^1], [\underline{a}_{ij}^2, \overline{a}_{ij}^2], [\underline{a}_{ij}^3, \overline{a}_{ij}^3]); \mu_{ij}^a, \nu_{ij}^a, \zeta_{ij}^a] y_j \\ \qquad \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} [([\underline{b}_{ij}^1, \overline{b}_{ij}^1], [\underline{b}_{ij}^2, \overline{b}_{ij}^2], [\underline{b}_{ij}^3, \overline{b}_{ij}^3]); \mu_{ij}^b, \nu_{ij}^b, \zeta_{ij}^b]_i, \\ \qquad y_j \geq 0, \\ \text{for} \quad i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m, \quad k = 1, 2, \dots, K. \end{array}\right. \tag{3}$$

Step 2: Determine the crisp value of each array for the chosen NFRMOQTP using defuzzication technique in equation 1.

Step 3: Approach the crisp MOQTP as a collection of k distinct single-objective sub problems, where the k sub problems produce the solution of $y_{(k)}$,

$$\left\{\begin{array}{l} \text{Max/Min} \quad \tilde{Z}^k(\tilde{y}) = \sum_{j=1}^{m} p_j^k y_j + \sum_{i=1}^{n}\sum_{j=1}^{m} q_{ij}^k y_i y_j, \quad k = 1, 2, \dots, K. \\ \text{subject to} \\ \qquad \sum_{j=1}^{m} a_{ij} y_j \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b_i, \quad \text{for} \quad i = 1, 2, \dots, n. \\ \qquad\qquad y_j \geq 0, \qquad \text{for} \quad j = 1, 2, \dots, m. \end{array}\right. \tag{4}$$

Step 4: Verify that all $y_{(k)}, k = 1, 2, ..., K$ is equal; if it is, move on to Step 8.

Step 5: If not, construct a payoff matrix of order $K \times K$ which provides the minimum and maximum values for each goal function. For the k^{th} goal of $\tilde{Z}^k(\tilde{y}_K)$, let $\mathcal{L}_k$ and $\mathcal{U}_k$ represent the lowest and maximum values, respectively:

$$\begin{aligned} \mathcal{U}_k =& \text{Max}\{\tilde{Z}^k(\tilde{y}_1), \tilde{Z}^k(\tilde{y}_2), ..., \tilde{Z}^k(\tilde{y}_K)\}; \\ \mathcal{L}_k =& \text{Min}\{\tilde{Z}^k(\tilde{y}_1), \tilde{Z}^k(\tilde{y}_2), ..., \tilde{Z}^k(\tilde{y}_K)\}; \quad k = 1, 2, ..., K. \end{aligned}$$

Step 6: Find y of $\tilde{Z}^k$ to achieve $\tilde{Z}^k \approx \mathcal{L}_k$ for all k. To tackle the fuzzy constraint, we shall use different membership function as mentioned in subsection 4.1 to determine the optimum compromise solution.

Step 7: Further, proceed to solve any of the following problems in equations 5, 6 and 7 to obtain the necessary solution.

Step 8: Enter the optimum compromise solution y into the model of equation 3. This provides the NFR-MOQTP's fuzzy optimum value at the end.

4.2 Membership Functions

The membership function characterizes the extent of fulfilment and compromise between the optimal and minimal values of the goal functions. The decision maker's interests and the problem's nature will determine whether the membership functions, representing the degree of attainability to desired values, are linear or non-linear. The decision-maker often seeks abrupt changes in the levels of attainability, which linear membership functions are incapable of delivering. Consequently, non-linear membership functions may prove to be more successful in some situations. In this context, we analyse different types of membership functions.

- **Linear Membership Function** [39]
  A linear membership function of the maximization/minimization type is defined as

$$\mu_{\mathrm{U}_l}(\mathrm{Z}^k) = \begin{cases} 0, & \mathrm{Z}^k \leq \mathrm{L}_k, \\ \frac{\mathrm{Z}^k - \mathrm{L}_k}{\mathrm{U}_k - \mathrm{L}_k}, & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 1, & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

$$\mu_{\mathrm{L}_l}(\mathrm{Z}^k) = \begin{cases} 1, & \mathrm{Z}^k \leq \mathrm{L}_k, \\ \frac{\mathrm{U}_k - \mathrm{Z}^k}{\mathrm{U}_k - \mathrm{L}_k}, & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 0, & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

  Note that if $\mathrm{U}_k = \mathrm{L}_k$ for some Z^k then $\mu_{\mathrm{L}_l}(\mathrm{Z}^k) = 1$.

- **Parabolic Membership Function** [40]
  A parabolic membership function of the maximization/minimization type is defined as

$$\mu_{\mathrm{U}_p}(\mathrm{Z}^k) = \begin{cases} 0 & \mathrm{Z}^k \leq \mathrm{L}_k, \\ 1 - \left(\frac{\mathrm{U}_k - \mathrm{Z}^k}{\mathrm{U}_k - \mathrm{L}_k}\right)^2 & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 1 & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

$$\mu_{\mathrm{L}_p}(\mathrm{Z}^k) = \begin{cases} 1 & \mathrm{Z}^k \leq \mathrm{L}_k, \\ 1 - \left(\frac{\mathrm{Z}^k - \mathrm{L}_k}{\mathrm{U}_k - \mathrm{L}_k}\right)^2 & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 0 & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

  Note that if $\mathrm{U}_k = \mathrm{L}_k$ for some Z^k then $\mu_{\mathrm{L}_p}(\mathrm{Z}^k) = 1$.

- **Exponential Membership Function** [13]
  A exponential membership function of the maximization/minimization type for the shape parameter $\mathcal{S}_k$ is defined as

$$\mu_{U_e}(Z^k) = \begin{cases} 0, & Z^k \leq L_k, \\ \frac{exp\{-\mathcal{S}_k\left(\frac{U_k - Z^k}{U_k - L_k}\right)\} - exp\{-\mathcal{S}_k\}}{1 - exp\{-\mathcal{S}_k\}} & L_k \leq Z^k \leq U_k, \\ 1, & Z^k \geq U_k. \end{cases}$$

$$\mu_{L_e}(Z^k) = \begin{cases} 1, & Z^k \leq L_k, \\ \frac{exp\{-\mathcal{S}_k\left(\frac{Z^k - L_k}{U_k - L_k}\right)\} - exp\{-\mathcal{S}_k\}}{1 - exp\{-\mathcal{S}_k\}} & L_k \leq Z^k \leq U_k, \\ 0, & Z^k \geq U_k. \end{cases}$$

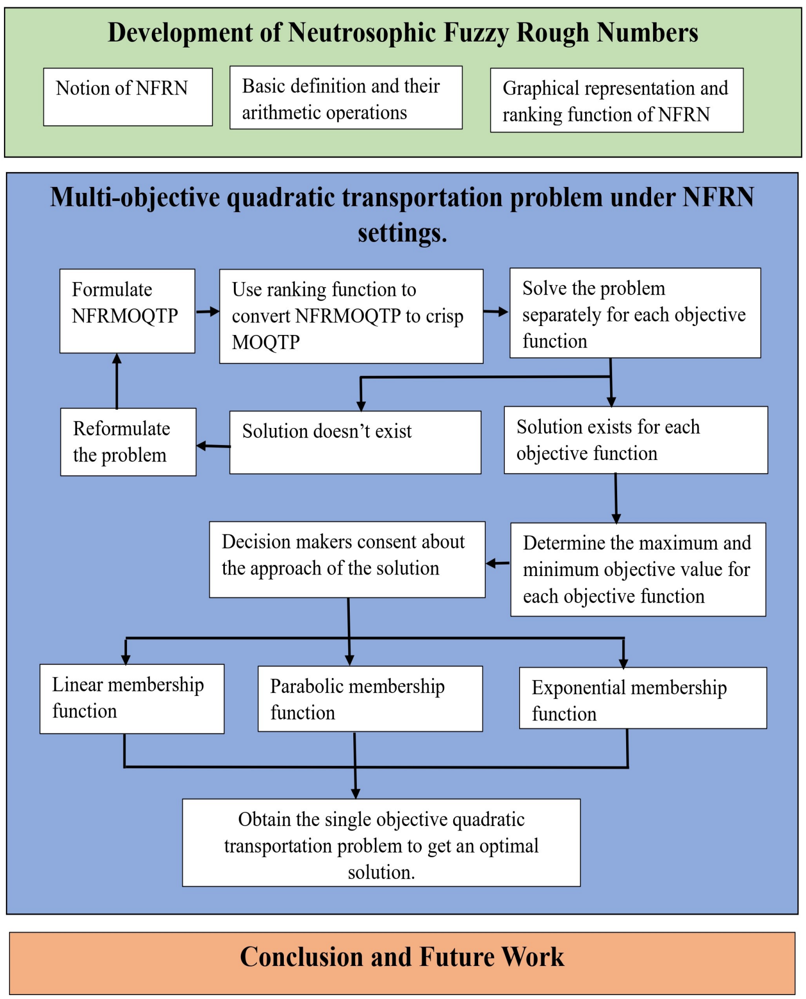

Figure 3: Overview of the article

The decision maker is most satisfied, as was previously indicated, if every objective function reaches their desired level. However, the majority of the goals are incompatible in practice. It is therefore nearly impossible to reach the aspiration level's maximum attainability. Thus, we make an effort to reach the aspiration level. The following goal programming is what the equation 4 converts to for this purpose.

$$\begin{cases} \text{Find } \; \{y_1, y_2, ..., y_m\} \\ \text{subject to} \\ \quad \mathrm{Z}^k \approx \mathrm{U}_k \\ \quad \mathrm{Z}^k \approx \mathrm{L}_k \\ \quad \text{constraints of equation 4.} \end{cases}$$

The purpose is to attain these objectives by using membership functions for the objective values to approach the aspiration level, thereby maximising the decision maker's satisfaction. A balance among these objectives is necessary to achieve the desired values. We use Zimmermanns approach for this purpose [16]. This strategy allows the problem in equation (4) to be addressed as,

$$\begin{cases} \text{Max } \; \theta \\ \text{subject to} \\ \quad \theta \leq \mu_{\mathrm{U}} \mathrm{Z}^k \leq 1, \\ \quad \theta \leq \mu_{\mathrm{L}} \mathrm{Z}^k \leq 1, \\ \quad 0 \leq \theta \leq 1. \\ \quad \text{constraints of equation 4.} \end{cases}$$

The problem in equation 4 assumes many shapes based on the decision makers' preferences via the application of distinct membership functions. We examine the application of three distinct types of membership functions previously addressed.

If the decision maker aims to attain the goal level by transitioning linearly from the optimal answer to the compromise solution, the linear membership function is applicable. Utilizing linear membership functions for both maximization and minimization goal functions, equation 4 is formulated as follows,

$$\begin{cases} \text{Max } \; \theta \\ \text{subject to} \\ \quad \mathrm{Z}^k - \mathrm{L}_k \geq \theta(\mathrm{U}_k - \mathrm{L}_k), \\ \quad \mathrm{U}_k - \mathrm{Z}^k \geq \theta(\mathrm{U}_k - \mathrm{L}_k), \\ \quad 0 \leq \theta \leq 1 \; \text{ and } \; \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k. \\ \quad \text{constraints of equation 4.} \end{cases} \tag{5}$$

To attain the ambition level, the decision maker may use the parabolic membership function to transition from the optimal option to the compromise solution. Utilizing the parabolic membership function for both maximization and minimization goal functions, the equation 4 is reformulated as follows:

$$\begin{cases} \text{Max } \; \theta \\ \text{subject to} \\ \quad (\mathrm{U}_k - \mathrm{L}_k)^2 \geq (\mathrm{U}_k - \mathrm{Z}^k)^2 + \theta(\mathrm{U}_k - \mathrm{L}_k)^2 \\ \quad (\mathrm{U}_k - \mathrm{L}_k)^2 \geq (\mathrm{Z}^k - \mathrm{L}_k)^2 + \theta(\mathrm{U}_k - \mathrm{L}_k)^2 \\ \quad 0 \leq \theta \leq 1 \; \text{ and } \; \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k. \\ \quad \text{constraints of equation 4.} \end{cases} \tag{6}$$

The decision maker may use the exponential membership function to transition from the optimal option to the compromise solution in order to attain the goal level target. Utilizing the exponential membership function for both maximization and minimization goal functions, equation (4) is reformulated as follows:

$$\begin{cases} \text{Max } \theta \\ \text{subject to} \\ \quad (exp\{-\mathcal{S}_k \left(\frac{\mathrm{Z}^k - \mathrm{L}_k}{\mathrm{U}_k - \mathrm{L}_k}\right)\} - exp\{-\mathcal{S}_k\}) \geq (1 - exp\{-\mathcal{S}_k\}) \\ \quad (exp\{-\mathcal{S}_k \left(\frac{\mathrm{U}_k - \mathrm{Z}^k}{\mathrm{U}_k - \mathrm{L}_k}\right)\} - exp\{-\mathcal{S}_k\}) \geq (1 - exp\{-\mathcal{S}_k\}) \\ \quad 0 \leq \theta \leq 1 \text{ and } \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k. \\ \quad \text{constraints of equation (4).} \end{cases} \tag{7}$$

Now solve either of the equations (5), (6), (7) to get the required solution.

5 Numerical Example

Now we provide a working model to validate our suggested method. The suggested approach with several membership functions is used to solve the NFR-MOQTP structured issue.

Example 5.1. Let us consider a manufacturing company ABC having a set of items in two branches has to be transported to three whole sale sellers. There are two origins, *Q1* and *Q2*, and three destinations, $D1$, $D2$, and $D3$. Given the inevitability of uncertainty, the supply and demand for produced items are shown in Tables 1 and 2, respectively. The company's profit is regulated according to natural market observations and the transportation cost, arising from unforeseen circumstances, are shown as shown in Table 3. The firm aims to reduce costs and enhance total profit.

Table 1: Data for the supply of items per unit

Q1	([300,310][320,330][340,350];0.7,0.3,0.3)
Q2	([360,380][390,410][420,450];0.7,0.3,0.3)

Table 2: Data for the demand of items per unit

D1	D2	D3
([200, 210][215, 225][230, 250]; 0.7, 0.3, 0.3)	([220, 230][235, 245][250, 260]; 0.7, 0.3, 0.3)	([240, 250][260, 270][280, 290]; 0.7, 0.3, 0.3)

Table 3: Profit and Cost of the Company Per Unit

Q1	T.Profit	T.Cost
D1	$-0.00107Y$ + ([80, 86][90, 100][110, 125]; 0.8, 0.2, 0.1)	([60, 70][75, 85][86, 96]; 0.9, 0.1, 0.1)
D2	$-0.00102Y$ + ([125, 135][138, 148][150, 160]; 0.9, 0.3, 0.1)	([70, 75][80, 90][95, 110]; 0.8, 0.3, 0.2)
D3	$-0.00110Y$ + ([112, 120][125, 135][140, 150]; 0.8, 0.1, 0.2)	([55, 65][70, 80][85, 95]; 0.9, 0.2, 01)
Q2	T.Profit	T.Cost
D1	$-0.00109Y$ + ([100, 110][115, 125][130, 140]; 0.9, 0.2, 0.2)	([80, 90][85, 95][100, 110]; 0.8, 0.1, 0.2)
D2	$-0.00115Y$ + ([87, 95][99, 120][125, 140]; 0.8, 0.2, 0.2)	([50, 60][65, 75][80, 90]; 0.7, 0.2, 0.1)
D3	$-0.00113Y$ + ([72, 80][85, 95][100, 120]; 0.8, 0.1, 0.2)	([60, 65][69, 80][85, 95]; 0.8, 0.3, 0.2)

Solution: The following NFRMOQTP can be employed to structure the problem in order to obtain maximum profit and minimal cost. Let y_{11}, y_{12}, y_{13} and y_{21}, y_{22}, y_{23} be the number of items from the origin *Q1*,*Q2* to the destination $D1$, $D2$, and $D3$.

$$\begin{cases} \text{Max} \quad \tilde{Z}^1(\tilde{y}) & = & -0.00107y_{11}^2 + ([80,86][90,100][110,125]; 0.8, 0.2, 0.1)y_{11} - 0.00102y_{12}^2 \\ & & +([125,135][138,148][150,160]; 0.9, 0.3, 0.1)y_{12} - 0.00110y_{13}^2 \\ & & +([112,120][125,135][140,150]; 0.8, 0.1, 0.2)y_{13} - 0.00109y_{21}^2 \\ & & +([100,110][115,125][130,140]; 0.9, 0.2, 0.2)y_{21} - 0.00115y_{22}^2 \\ & & +([87,95][99,120][125,140]; 0.8, 0.2, 0.2)y_{22} - 0.00113y_{23}^2 \\ & & +([72,80][85,95][100,120]; 0.8, 0.1, 0.2)y_{23}. \\ \text{Min} \quad \tilde{Z}^2(\tilde{y}) & = & ([60,70][75,85][86,96]; 0.9, 0.1, 0.1)y_{11} + ([70,75][80,90][95,110]; 0.8, 0.3, 0.2)y_{12} \\ & & +([55,65][70,80][85,95]; 0.9, 0.2, 01)y_{13} + ([80,90][85,95][100,110]; 0.8, 0.1, 0.2)y_{21} \\ & & +([50,60][65,75][80,90]; 0.7, 0.2, 0.1)y_{22} + ([60,65][69,80][85,95]; 0.8, 0.3, 0.2)y_{23}. \\ \text{subject to} & & \\ \quad y_{11} + y_{21} & \geq & ([200,210][215,225][230,250]; 0.7, 0.3, 0.3), \\ \quad y_{12} + y_{22} & \geq & ([220,230][235,245][250,260]; 0.7, 0.3, 0.3), \\ \quad y_{13} + y_{23} & \geq & ([240,250][260,270][280,290]; 0.7, 0.3, 0.3). \\ \quad y_{11} + y_{12} + y_{13} & \leq & ([300,310][320,330][340,350]; 0.7, 0.3, 0.3), \\ \quad y_{21} + y_{22} + y_{23} & \leq & ([360,380][390,410][420,450]; 0.7, 0.3, 0.3), \\ \quad y_{ij} & \geq & 0, \quad \forall i, j. \end{cases} \tag{8}$$

Now, using the ranking function in equation 1, convert NFR-MOQTP in equation 8 into crisp MOQTP to get the following problem:

$$\begin{cases} \text{Max} \quad \tilde{Z}^1(\tilde{y}) & = & -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} \\ & & -0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \\ \text{Min} \quad \tilde{Z}^2(\tilde{y}) & = & 79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} \\ \text{subject to} & & \\ \quad y_{11} + y_{21} & \geq & 174.56, \\ \quad y_{12} + y_{22} & \geq & 189, \\ \quad y_{13} + y_{23} & \geq & 208.69. \\ \quad y_{11} + y_{12} + y_{13} & \leq & 255.94, \\ \quad y_{21} + y_{22} + y_{23} & \leq & 316.31, \\ \quad y_{ij} & \geq & 0, \forall i, j. \end{cases} \tag{9}$$

Breaking the model into two sub problems as follows:

$$\begin{cases} \text{Max} \quad \tilde{Z}^1(\tilde{y}) & = & -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} \\ & & -0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \\ \text{subject to} & & \\ \quad y_{11} + y_{21} & \geq & 174.56, \\ \quad y_{12} + y_{22} & \geq & 189, \\ \quad y_{13} + y_{23} & \geq & 208.69. \\ \quad y_{11} + y_{12} + y_{13} & \leq & 255.94, \\ \quad y_{21} + y_{22} + y_{23} & \leq & 316.31, \\ \quad y_{ij} & \geq & 0, \forall i, j. \\ & & \\ \text{Min} \quad \tilde{Z}^2(\tilde{y}) & = & 79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} \\ \text{subject to} & & \\ \quad y_{11} + y_{21} & \geq & 174.56, \\ \quad y_{12} + y_{22} & \geq & 189, \\ \quad y_{13} + y_{23} & \geq & 208.69. \\ \quad y_{11} + y_{12} + y_{13} & \leq & 255.94, \\ \quad y_{21} + y_{22} + y_{23} & \leq & 316.31, \\ \quad y_{ij} & \geq & 0, \forall i, j. \end{cases}$$

The solution vectors of equation 9 are
$Y_1 = (174.56, 81.38, 0, 0, 107.62, 208.69)$ and $Y_2 = (174.56, 0, 81.38, 0, 189, 127.31)$, respectively. Now, we compute both the objective functions at each of the Y_1 and Y_2 to form pay-off Table 4.

Table 4: Payoff Matrix

Solution	Z_1	Z_2
Y	$Z_1(Y_1)$	$Z_2(Y_2)$
$(0, 47.25, 208.69, 174.56, 141.75, 0)$	65511.84	42997.69
$(174.56, 0, 81.38, 0, 189, 127.31)$	55652.30	40386.03

It further yields $U1 = 65511.84, L1 = 55652.30, U2 = 42997.69, L2 = 40386.03$. The optimal solution for the minimization type objective function is $(174.56, 81.38, 0, 0, 107.62, 208.69)$ with objective value 40386.03, and the optimal solution for the maximization type objective function is $(0, 47.25, 208.69, 174.56, 141.75, 0)$ with objective value 65511.84.

Considering the differences between the two solutions, we want an ideal solution vector that compromises the two solutions to a given degree of satisfaction. This is known as a compromise solution vector. Different shape functions may be used by decision makers to accomplish this purpose. Different curves may be followed by the solution vector as it moves from the optimal solution to the compromise solution. Here, a few of these routes are covered.

If both objective functions are used with linear membership functions, which are provided as

$$\mu_{\mathrm{U}_l}(\mathrm{Z}^k) = \begin{cases} 0, & \mathrm{Z}^k \leq \mathrm{L}_k, \\ \dfrac{\begin{pmatrix} -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} - \\ 0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \end{pmatrix}^{-55652.30}}{9859.54}, & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 1, & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

$$\mu_{\mathrm{L}_l}(\mathrm{Z}^k) = \begin{cases} 1, & \mathrm{Z}^k \leq \mathrm{L}_k, \\ \frac{42997.69-(79.65y_{11}+74.75y_{12}+73.13y_{13}+87.5y_{21}+63y_{22}+65.26y_{23})}{2611.66}, & \mathrm{L}_k \leq \mathrm{Z}^k \leq \mathrm{U}_k, \\ 0, & \mathrm{Z}^k \geq \mathrm{U}_k. \end{cases}$$

Our problem becomes a goal programming problem as under

$$\begin{cases} \text{Max } \theta \\ \text{subject to} \\ \quad \begin{pmatrix} -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} - \\ 0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} - 55652.30 \end{pmatrix} \geq \theta(9859.54), \\ \quad 42997.69 - 79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} \geq \theta(2611.66), \\ \quad y_{11} + y_{21} \geq 174.56, \\ \quad y_{12} + y_{22} \geq 189, \\ \quad y_{13} + y_{23} \geq 208.69. \\ \quad y_{11} + y_{12} + y_{13} \leq 255.94, \\ \quad y_{21} + y_{22} + y_{23} \leq 316.31, \\ \quad 0 \leq \theta \leq 1. \end{cases}$$

When traversing the parabolic trajectory from the optimal solution to the compromise solution, the previously stated parabolic membership function may be used. If a parabolic membership function is used for both objective functions, which are specified as

$$\mu_{U_p}(Z^k) = \begin{cases} 0, & Z^k \leq L_k, \\ 1 - \left(\dfrac{65511.84 - \begin{pmatrix} -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} - \\ 0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \end{pmatrix}}{9859.54} \right)^2, & L_k \leq Z^k \leq U_k, \\ 1, & Z^k \geq U_k. \end{cases}$$

$$\mu_{L_p}(Z^k) = \begin{cases} 1, & Z^k \leq L_k, \\ 1 - \left(\frac{79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} - 40386.03}{2611.66} \right)^2, & L_k \leq Z^k \leq U_k, \\ 0, & Z^k \geq U_k. \end{cases}$$

Our problem becomes a goal programming problem as under

$$\begin{cases} \text{Max } \theta \\ \text{subject to} \\ \left(65511.84 - \begin{pmatrix} -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} \\ -0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \end{pmatrix} \right)^2 + (9859.54)^2\theta \leq (9859.54)^2, \\ (79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} - 40386.03)^2 + (2611.66)^2\theta \leq (2611.66)^2, \\ y_{11} + y_{21} \geq 174.56, \\ y_{12} + y_{22} \geq 189, \\ y_{13} + y_{23} \geq 208.69. \\ y_{11} + y_{12} + y_{13} \leq 255.94, \\ y_{21} + y_{22} + y_{23} \leq 316.31, \\ 0 \leq \theta \leq 1. \end{cases}$$

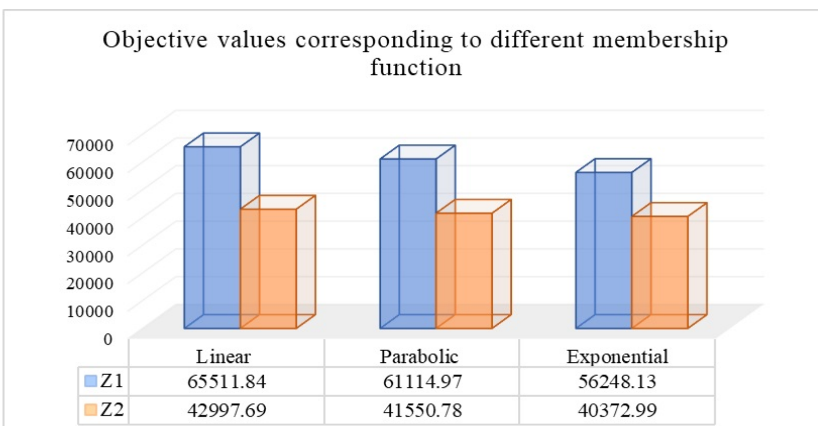

Figure 4: Objective function with respect to different membership function

By progressing along the exponential trajectory from the optimal solution to the compromise solution, the previously stated exponential membership function may be used. An exponential membership function of the maximization/minimization type for the shape parameter $\mathcal{S}_k$ is defined as

$$\mu_{U_e}(Z^k) = \begin{cases} 0, & Z^k \leq L_k, \\ \dfrac{e^{\left\{ -\mathcal{S}_k \left(65511.84 - \begin{pmatrix} -0.00107y_{11}^2 + 92.34y_{11} - 0.00102y_{12}^2 + 133.75y_{12} - 0.00110y_{13}^2 + 122.19y_{13} \\ -0.00109y_{21}^2 + 112.5y_{21} - 0.00115y_{22}^2 + 99.9y_{22} - 0.00113y_{23}^2 + 86.25y_{23} \end{pmatrix} / 9859.54 \right) \right\}} - e^{\{-\mathcal{S}_k\}}}{1 - e^{\{-\mathcal{S}_k\}}}, & L_k \leq Z^k \leq U_k, \\ 1, & Z^k \geq U_k. \end{cases}$$

$$\mu_{L_e}(Z^k) = \begin{cases} 1, & Z^k \leq L_k, \\ \left(e^{\{-\mathcal{S}_k \left(\frac{79.65y_{11} + 74.75y_{12} + 73.13y_{13} + 87.5y_{21} + 63y_{22} + 65.26y_{23} - 40386.03}{2611.66} \right)\}} - e^{\{-\mathcal{S}_k\}} / 1 - e^{\{-\mathcal{S}_k\}} \right), & L_k \leq Z^k \leq U_k, \\ 0, & Z^k \geq U_k. \end{cases}$$

Our problem becomes a goal programming problem as under

$$\begin{cases} \text{Max} \quad \theta \\ \text{subject to} \\ \quad e^{\left\{ \frac{\left(-s_1\left(65511.84-\left(\begin{array}{c}-0.00107y_{11}^2+92.34y_{11}-0.00102y_{12}^2+133.75y_{12}-0.00110y_{13}^2+122.19y_{13}\\ -0.00109y_{21}^2+112.5y_{21}-0.00115y_{22}^2+99.9y_{22}-0.00113y_{23}^2+86.25y_{23}\end{array}\right)\right)\right)}{9859.54} \right\}} + (1-e^{-s_1})\theta \geq e^{-s_1}, \\ \quad e^{\left\{ -s_2\left(\frac{(79.65y_{11}+74.75y_{12}+73.13y_{13}+87.5y_{21}+63y_{22}+65.26y_{23}-40386.03)}{(2611.66)}\right) \right\}} + (1-e^{-s_2})\theta \geq e^{-s_2}, \\ \quad y_{11}+y_{21} \geq 174.56, \\ \quad y_{12}+y_{22} \geq 189, \\ \quad y_{13}+y_{23} \geq 208.69. \\ \quad y_{11}+y_{12}+y_{13} \leq 255.94, \\ \quad y_{21}+y_{22}+y_{23} \leq 316.31, \\ \quad 0 \leq \theta \leq 1. \end{cases}$$

Consequently, Table 5 presents the solutions obtained by the application of linear, parabolic, and exponential

Table 5: The solutions using different membership functions.

Solution:	linear membership function	parabolic membership function	exponential membership function
Y	(0,47.25,208.69,174.56,141.75,0)	(80.38,0,175.56,94.18,189,33.13)	(165.66,41.96,48.32,8.90,147.04,160.37)
Z1	65511.84	61114.97	56248.13
Z2	42997.69	41550.78	40372.99

membership functions for each objective function. As the quantities of the three products cannot be fractional, they must be rounded down to the next positive integer. The MOQTP problem yields a compromise solution instead of an optimal one.

5.1 Comparative analysis

To demonstrate the reliability of the proposed methodology, let us consider an **Example 5.2** of NFRMOQTP. The problem is further reduced to crispMOQTP, which is solved by our proposed approach and Garg's approach [14] for validation.

Example 5.2. Consider NFRMOQTP

$$\begin{cases} \begin{array}{lcl} \text{Min} \ \ Z1 & = & ([9,11],[8,12],[7,13],0.8,0.2,0.2)y_{11} - 0.00109y_{11}^2 \\ & & +([9,11],[10,10],[12,17],0.8,0.1,0.2)y_{12} - 0.00111y_{12}^2 \\ & & +([8,8],[7,9],[8,12],0.8,0.3,0.2)y_{13} - 0.00113y_{13}^2 \\ & & +([11,13],[11,13],[14,17],0.7,0.2,0.1)y_{21} - 0.00108y_{21}^2 \\ & & +([7,6],[7,9],[7,9],0.7,0.3,0.3)y_{22} - 0.0011y_{22}^2 \\ & & +([6,10],[5,11],[4,12],0.6,0.4,0.3)y_{23} - 0.00112y_{23}^2. \\ \text{Min} \ \ Z2 & = & ([10,10][9,11][13,14],0.8,0.2,0.2)y_{11} + ([10,13][10,15][10,20],0.9,0.1,0.2)y_{12} \\ & & +([14,17][12,17][15,17],0.7,0.2,0.1)y_{13} + ([13,15][12,16][15,17],0.7,0.3,0.3)y_{21} \\ & & +([8,11][7,14][9,15],0.9,0.2,0.2)y_{22} + ([7,9][6,10][10,13],0.8,0.2,0.3)y_{23}. \\ \text{subject to} & & \\ y_{11}+y_{12}+y_{13} & \leq & ([13,15][12,16][14,20],0.7,0.3,0.3). \\ y_{21}+y_{22}+y_{23} & \leq & ([9,19][20,26][21,25],0.7,0.3,0.3). \\ y_{11}+y_{21} & \geq & ([5,9][10,15][12,15],0.7,0.3,0.3). \\ y_{12}+y_{22} & \geq & ([7,10][7,10][7,10],0.7,0.3,0.3). \\ y_{13}+y_{23} & \geq & ([10,15][15,17][16,20],0.7,0.3,0.3). \\ y_{ij} \geq 0. & & \end{array} \end{cases} \quad (10)$$

Solution: The NFR-MOQTP problem in equation 10 may now be transformed into a crisp MOQTP problem by utilizing the ranking function.

$$
\begin{cases}
\text{Min} \quad Z1 & = 9.18y_{11} - 0.00109y_{11}^2 + 10.78y_{12} - 0.00111y_{12}^2 + 7.48y_{13} - 0.00113y_{13}^2 \\
& \quad +11.85y_{21} - 0.00108y_{21}^2 + 5.91y_{22} - 0.0011y_{22}^2 + 5.7y_{23} - 0.00112y_{23}^2. \\
\text{Min} \quad Z2 & = 10.05y_{11} + 12.68y_{12} + 13.8y_{13} + 11.55y_{21} + 10y_{22} + 8.59y_{23}. \\
\text{subject to} & \\
y_{11} + y_{12} + y_{13} & \leq 11.81. \\
y_{21} + y_{22} + y_{23} & \leq 15.75. \\
y_{11} + y_{21} & \geq 8.663. \\
y_{12} + y_{22} & \geq 6.694. \\
y_{13} + y_{23} & \geq 12.21. \\
y_{ij} \geq 0. &
\end{cases}
$$

Now we consider the following two crisp problems separately:

$$
\begin{cases}
\text{Min} \quad Z1 & = 9.18y_{11} - 0.00109y_{11}^2 + 10.78y_{12} - 0.00111y_{12}^2 + 7.48y_{13} - 0.00113y_{13}^2 \\
& \quad +11.85y_{21} - 0.00108y_{21}^2 + 5.91y_{22} - 0.0011y_{22}^2 + 5.7y_{23} - 0.00112y_{23}^2. \\
\text{subject to} & \\
y_{11} + y_{12} + y_{13} & \leq 11.81 \\
y_{21} + y_{22} + y_{23} & \leq 15.75 \\
y_{11} + y_{21} & \geq 8.663 \\
y_{12} + y_{22} & \geq 6.694 \\
y_{13} + y_{23} & \geq 12.21 \\
y_{ij} >= 0. & \\
& \\
\text{Min} \quad Z2 & = 10.05y_{11} + 12.68y_{12} + 13.8y_{13} + 11.55y_{21} + 10y_{22} + 8.59y_{23} \\
\text{subject to} & \\
y_{11} + y_{12} + y_{13} & \leq 11.81 \\
y_{21} + y_{22} + y_{23} & \leq 15.75 \\
y_{11} + y_{21} & \geq 8.663 \\
y_{12} + y_{22} & \geq 6.694 \\
y_{13} + y_{23} & \geq 12.21 \\
y_{ij} \geq 0. &
\end{cases} \tag{11}
$$

The solution vector for the problems in equation 10 with linear membership function is
$Y = (5, 3.44, 4.57, 0, 3.56, 5.44)$ with the objective values as $169.23, 239.68$ for respective objective functions.

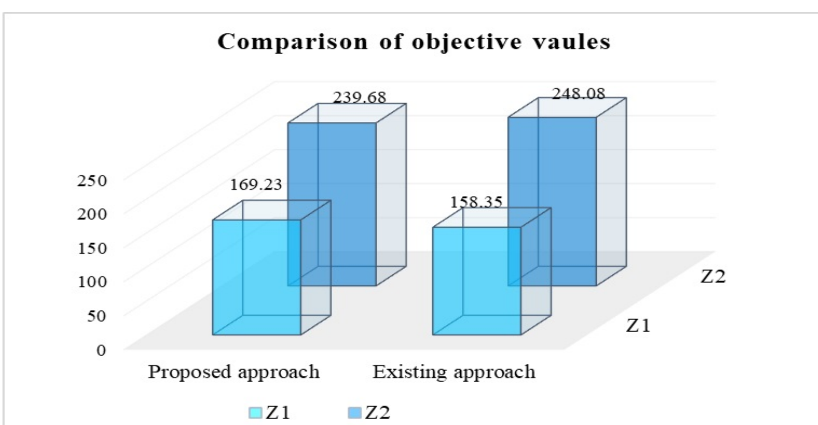

Figure 5: Comparison of objective functions

For Garg's approach [14], let us consider $w_1 = 0.5$ and $w_2 = 0.5$ to formulate the weighted sum problem to combine the two function into single function as $\mathbb{Z}^R = w_1\mathbb{Z}_1 + w_2\mathbb{Z}_2$ is presented in the equation 12.

$$\begin{cases} \text{Min } \mathbb{Z} & = & 8.73y_{11} - 0.00109y_{11}^2 + 10.6y_{12} - 0.00111y_{12}^2 + 10.1y_{13} - 0.00113y_{13}^2 \\ & & +11.5y_{21} - 0.00108y_{21}^2 + 7.66y_{22} - 0.0011y_{22}^2 + 7.24y_{23} - 0.00112y_{23}^2 \\ \text{subject to} & & \\ y_{11} + y_{12} + y_{13} & \leq & 11.81 \\ y_{21} + y_{22} + y_{23} & \leq & 15.75 \\ y_{11} + y_{21} & \geq & 8.663 \\ y_{12} + y_{22} & \geq & 6.694 \\ y_{13} + y_{23} & \geq & 12.21 \\ y_{ij} \geq 0. & & \end{cases} \tag{12}$$

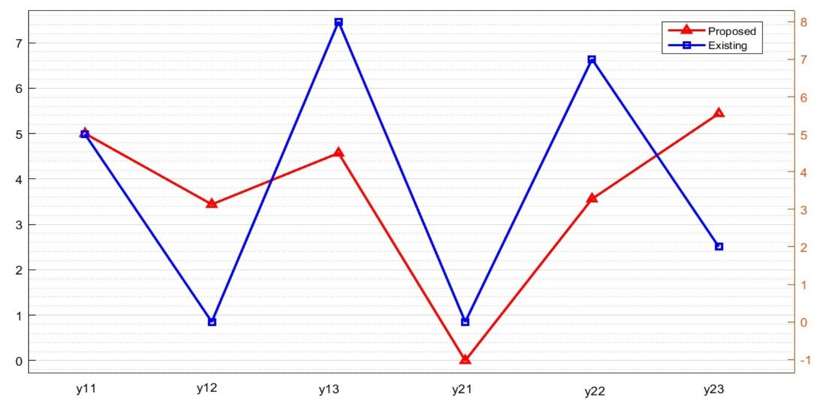

Figure 6: Comparison of solution vectors

In order to compare the outcomes, the aforementioned NFRMOQTP is reduced to a single objective function using weighted sum method and obtain a solutions vector $Y = (5, 0, 8, 0, 7, 2)$ with objective value as $158.35, 248.08$, respectively.

5.2 Results and Discussion

Section 5 employs linear, parabolic, and exponential membership functions to address a numerical problem using the proposed technique. Figure 4 and Table 5 provide the results of **Example 5.1** using various membership functions. A compromise solution was used to address the MOQTP issue, as previously mentioned. Consequently, every enhancement in one objective value necessitates a corresponding decline in another objective value. An increase in the objective value signifies an enhancement in the objective function for maximization, and conversely. On the other hand, it is inferred that the objective value is declining when an objective function of the minimization type is present.

- The suggested technique using linear membership functions yields objective values of $(169.23, 239.68)$, respectively. The resolution of the same issue using the weighted sum technique, as derived by Garg et al. [14], is $(158.35, 248.08)$; these findings are almost congruent with those of the established method.
- Our proposed method clearly generalizes the fuzzy MOQTP issues, since NFRN serves as an extension of fuzzy numbers.
- The graphical representation of the objective values and solution vectors for Example 5.1 is shown in Figures 5 and 6, respectively.

6 Conclusion

The MOQTP, a specific family of uncertain transportation problems, is optimized in this present study. The unique neutrosophic fuzzy rough environment and its arithmetic operation with its ranking function were originally described in this work. Next, we modify the NFRN within MOQTP. In order to obtain a compromise optimum solution, we further create a solution approach that converted MOQTP into single-objective problems employing linear, parabolic, and exponential membership functions. We have demonstrated the suggested approach by solving a numerical example. Furthermore, a comparison study is given to validate the proposed approach, providing a comparable outcome between the suggested technique and the existing Garg's method. In addition to expanding the application of the MOQTP with various membership functions, this work offers a novel viewpoint by handling the uncertainty of the transportation parameters using neutrosophic fuzzy rough numbers.

Limitation: Even though this study is quantitative and objective regarding to transportation problems, it does have certain limitations. Since neutrosophic fuzzy rough numbers have more complex arithmetic operations than crisp or fuzzy numbers, computational approaches that lighten the workload of experts must be developed and are limited to fuzzy rough numbers and do not apply to complex fuzzy environments.

Future work: Eventually, we could extend the scope of this research to include higher dimensional and large-scale transportation problems in the same setting.

Acknowledgements: "The work has been strengthened by the insightful comments provided by the anonymous referees and editor, for which the authors are grateful."

Conflict of Interest: "Authors states that there is no conflict of interest."

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