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An Improved Atomic Orbital Search Algorithm Utilizing Firefly Algorithm for Optimization Problems

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Abstract

The Atomic Orbital Search inspiration from the rules of quantum mechanics and Firefly Algorithm is a metaheuristic technique which is widely used for solving the optimization problems. Both algorithms collectively improved the performance of search. This article goals to optimize engineering design problems utilizing a new hybrid optimizer; AOS-FA (Atomic Orbital Search-Firefly Algorithm). Incorporating the FA methodology into the basic AOS framework has successfully addressed the issue of local optima trap and significantly enhanced the quality of solutions generated by the algorithm. The FA algorithm is work on the combinatorial optimization and utilized as application of AOS algorithm. Hence, we merge these two algorithms and make a hybrid algorithm. The purpose of the suggested hybridization method was to promote the improvement of the exploration-exploitation manners of the AOS search. To analyse the viability of the suggested hybridized algorithm in real-world usages, it is studied for five constrained engineering design issues, and the performance was determined with other outstanding metaheuristics extracted from the publications.

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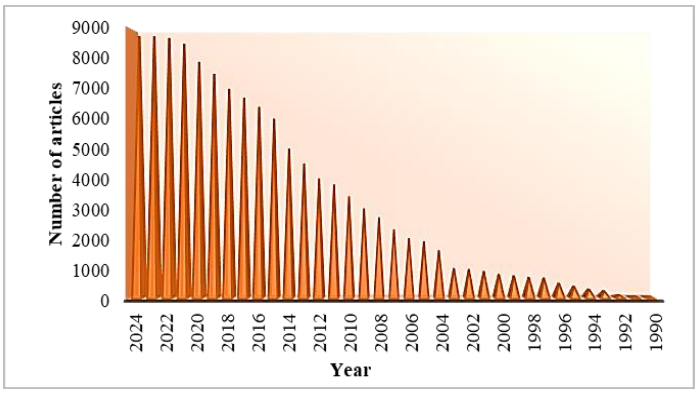
INTRODUCTION

Optimization is an essential part of engineering design, and hence in numerous real-world challenging problems with various frameworks, Meta-Heuristics (MHs) have become increasingly fascinating as a robust instrument for optimization. Engineering regulation can achieve stable and efficient mechanisms by using well-designed optimum models. These models are developed based on mathematical theorems and approaches. Although optimization methods were used by historical figures such as Newton, Lagrange, and Cauchyeski for smaller-scale issues, modern engineers rely on improved and hybrid versions of these algorithms to effectively solve more extensive and more complex engineering design problems(Ghaemifard & Ghannadiasl, 2024c). Over the past two decades, the rise of environmental and global phenomena due to techno-logical advancements and population growth has made complicated engineering designs more challenging. As a solution, metaheuristic optimization algorithms have become a popular choice for achieving reasonable solutions in less time (Ghaemifard & Ghannadiasl, 2024b; Houssein, Mahdy, Shebl, & Mohamed, 2021). Numerous metaheuristic optimization algorithms have been developed and proven effective in improving optimization processes beyond their predecessors, despite their unique processes and textures. To tackle global optimization problems, meta-heuristic algorithms are a frequently employed solution. The optimal solution is primarily achieved by simulating of both nature and human intelligence. By conducting a global search, they can identify an approximate solution that closely approximates the optimal solution to some degree. Exploration and exploitation are the fundamental principles of MHs. Exploration is crucial in order to thoroughly search the entire space and locate the optimal solution, which could potentially be located anywhere within it. To maximize the use of valuable information, it is essential to engage in effective exploitation. Optimal solutions are generally correlated in specific ways. Utilize these correlations to regulate gradually and search slowly from the initial answer to get the optimal solution. MHs strive to achieve a harmonious balance between exploration and exploitation. MHs have gained significant attention from scholars in recent years due to their numerous advantages, including their simple and intuitive operation, as well as their fast-running speed (Fazli, Khiabani, & Daneshian, 2022; Ghaemifard & Ghannadiasl, 2024c; Ghannadiasl & Ghaemifard, 2024a; Shahebrahimi, Lork, Shayegan, & Amir). There have been numerous proposals for meta-heuristic algorithms, totalling in the hundreds. MHs can be categorized into four groups based on various design inspirations: evolutionary, physical, swarm-based, and human-based algorithms. Swarm-based algorithms are a powerful tool in optimization, and computational intelligence has made great strides in recent years. These algorithms include Ant Colony Optimization (Dorigo, Birattari, & Stutzle, 2007), Artificial Bee Colony (Karaboga, 2010), Particle Swarm Optimization (Eberhart & Kennedy, 1995), Remora Optimization Algorithm (Jia, Peng, & Lang, 2021), Slap Swarm Algorithm (Hussien, 2022), Ant Lion Optimizer (Assiri, Hussien, & Amin, 2020), Grey Wolf Optimization (Mirjalili, Mirjalili, & Lewis, 2014), Bat Algorithm (X. S. Yang & Hossein Gandomi, 2012), Krill Herd (X. S. Yang & Hossein Gandomi, 2012), and Whale Optimization Algorithm (Mirjalili & Lewis, 2016). Each of these algorithms has its strengths and weaknesses, and researchers continue to explore new variations and combinations to push the boundaries of what is possible in optimization. Whether you are working in engineering, finance, or any other field where optimization is critical, these swarm-based algorithms offer robust solutions that can help you achieve your goals. There are several types of evolutionary algorithms available for use, including Genetic Algorithm (GA) (Holland, 1992), Evolution Strategy (ES) (Beyer & Schwefel, 2002), Genetic Programming (GP) (Banzhaf, Koza, Ryan, Spector, & Jacob, 2000), Differential Evolution (DE) (Price, 2013), Virulence Optimization Algorithm (VOA) (Jaderyan & Khotanlou, 2016), Black Hole Algorithm (BH) (Hatamlou, 2013), Evolutionary Programming (EP) (Sinha, Chakrabarti, & Chattopadhyay, 2003), Gravitational Search Algorithm (GSA) (Rashedi, Nezamabadi-Pour, & Saryazdi, 2009). Several physical-based algorithms have been developed to address optimization problems. These include Simulated Annealing, Flood algorithm (FLA) (Ghasemi et al., 2024), Thermal Exchange Optimization (TEO) (Ali Kaveh & Dadras, 2017) and Ray Optimization (RO) (A Kaveh & Khayatazad, 2012). Harmony Search (HS) (Geem, Kim, & Loganathan, 2001), and Exchanged Market Algorithm (EMA) (Ghorbani & Babaei, 2014) are categorized as human- based algorithms. MHs can be significantly optimized through the use of these algorithms. Researchers have proposed various methods to enhance the convergence performance and efficiency of metaheuristic algorithms. To achieve this goal, improved versions such as those developed by (Ghannadiasl & Ghaemifard, 2024b; Hakli & Ortacay, 2019; Kannan & Kramer, 1994) have been expressed, as well as hybrid versions that combine multiple algorithms such as those developed (Abouhabaga, Gadallah, Kouta, & Zaghloul, 2021; Chen & Zheng, 2024; Euchi & Sadok, 2021; Fasina, Sawyerr, Abdullahi, & Oke, 2023; Ghajarnia, Bozorg Haddad, & Mariño, 2011; Ghannadiasl & Ghaemifard, 2022; Hemagowri & Selvan, 2023; Khorram & Bahrami, 2020). These approaches have shown promise in producing solutions with fewer iterations. Fig.1a, displays a comparison graph of the number and percentage of studies conducted on hybrid optimization algorithms over the years. Over the past two decades, there has been a significant rise in the utilization of hybrid metaheuristic optimization algorithms. Fig.1b presents the findings of a study that analysed the distribution of hybrid optimization studies across various fields using data obtained from the Web of Science database. While approximately 50% of studies fall outside of the fields represented in the Fig.1, it is clear that hybrid optimization algorithms are widely utilized in multidisciplinary engineering research. The purpose of this paper is to introduce an innovative algorithm called AOS-FA (Atomic Orbital Search-Firefly optimization). The purpose of developing this algorithm was to test its effectiveness in achieving global optimum solutions and enhancing overall performance. In Fig.2, the main objective and general process of the paper are presented. There are five sections to the remainder of the study. We describe the procedures in Sect. 2. In Section 3, numerical examples are shown, and the effectiveness of recommendation algorithms is assessed. Conclusions and upcoming projects are discussed in Sect. 4.

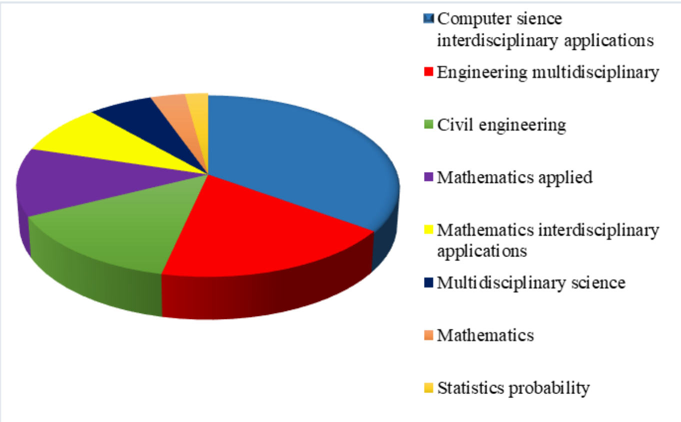
METHODS

The proposed AOS-FA method

A good meta-heuristic algorithm balances its exploration and exploitation functions to achieve optimal performance (Eiben & Schippers, 1998). The Atomic Orbital Search method boasts robust global optimization, adaptability, and robustness (Mahdi Azizi, 2021). The Firefly algorithm has strong local search abilities and fast convergence, but it often converges to a local optimum instead of a global optimal solution (X.-S. Yang, 2009).



(a)



(b)

Fig. 1. Web of Science citation report studies: (a) Number of published hybrid optimization studies, (b) Distribution of hybrid optimization studies according to fields

This section presents the proposed algorithm hybrid AOS-FA, which combines the benefits of two metaheuristic algorithms: AOS and FA. FA has strong exploration capabilities, allowing it to visit all local and global modes and find suitable solutions, while AOS has high exploitation capabilities. The AOS-FA algorithm is based on three principles. Hybrid algorithms can supplement strengths and weaknesses. By establishing new populations that share the best individuals from both groups, this mixture can protect against early convergence while retaining helpful qualities from AOS and FA. Ultimately, the AOS-FA algorithm uses only the parameters from the original AOS and FA algorithms. Fig. 3 illustrated the pseudo-code of AOS-FA. In this hybrid algorithm first the AOS algorithm is run.

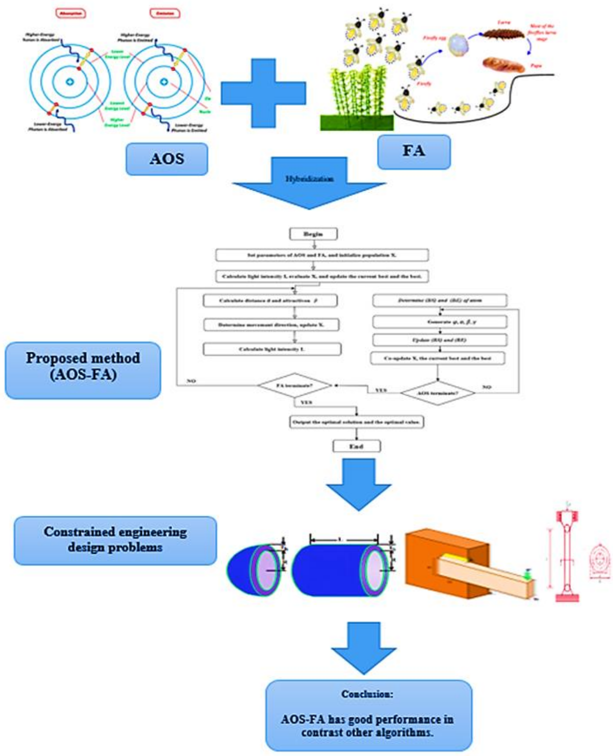


Fig.2. General process of the paper

To improve optimization algorithms, in the quantum-based atomic model, the AOS algorithm suggests using solution candidates (X) as electrons. The solution candidates (X_i) represent each electron, and decision variables ($X_{i,j}$) are used to deter in the search space. In this hybrid algorithm first the AOS algorithm is run. To improve optimization algorithms, in the quantum-based atomic model, the AOS algorithm suggests using solution candidates (X) as electrons. In the search space, solution candidates (X_i) represent each electron, and decision variables ($X_{i,j}$) are used to determine their positions.

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & \vdots & & \vdots & \ddots & \vdots \\ x_m^1 & x_m^2 & \cdots & x_m^j & \cdots & x_m^d \end{bmatrix} \quad \begin{cases} i = 1,2,\ldots,m \\ j = 1,2,\ldots,d \end{cases} \tag{1}$$

To describe the location of solution volunteers within the probe zone, the problem dimension is defined via d. Parameter m is the presenter of the number of solution candidates. Each electron has an energy state, according to the quantum atomic model. The objective subordinate is this energy state.

$$\mathbf{E}=\begin{bmatrix}\mathbf{E_1}\\ \mathbf{E_2}\\ \vdots\\ \mathbf{E_i}\\ \vdots\\ \mathbf{E_m}\end{bmatrix} \qquad \{\mathbf{i=1,2,\ldots,m} \tag{2}$$

To determine the objective function values, refer to vector E. Energy level E_i pertains to the ith solution volunteers. At the same time, the number of electrons in the probe area is represented by m. The Probability Density Function is a mathematical model utilized to define the station of electrons around the nucleus in the quantum atomic model. According to probability theory, Probability Density Function expresses the probability of a variable happening within a special scope. The Probability Density Function analysis reveals that the solution candidates are distributed among the imaginary layers created to determine the position of electrons. In the following, the vectors of location and objective subordinate values of answer volunteers in imaginary layers are expressed as Eq3 and Eq4 respectively.

$$\mathbf{X^K}=\begin{bmatrix}\mathbf{X_1^k}\\ \mathbf{X_2^k}\\ \vdots\\ \mathbf{X_i^k}\\ \vdots\\ \mathbf{X_p^n}\end{bmatrix}=\begin{bmatrix}\mathbf{X_1^1} & \mathbf{X_1^2} & \cdots & \mathbf{X_1^j} & \cdots & \mathbf{X_1^d}\\ \mathbf{X_2^1} & \mathbf{X_2^2} & \cdots & \mathbf{X_2^j} & \cdots & \mathbf{X_2^d}\\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots\\ \mathbf{X_i^1} & \mathbf{X_i^2} & & \mathbf{X_i^j} & \cdots & \mathbf{X_i^d}\\ \vdots & \vdots & \cdots & \vdots & \ddots & \vdots\\ \mathbf{X_p^1} & \mathbf{X_p^2} & & \mathbf{X_p^j} & \cdots & \mathbf{X_p^d}\end{bmatrix} \quad \begin{cases}\mathbf{i=1,2,\ldots,p}\\ \mathbf{j=1,2,\ldots,d}\\ \mathbf{K=1,2,\ldots,n}\end{cases} \tag{3}$$

$$\mathbf{E^k}=\begin{bmatrix}\mathbf{E_1^k}\\ \mathbf{E_2^k}\\ \vdots\\ \mathbf{E_i^K}\\ \vdots\\ \mathbf{E_p^n}\end{bmatrix} \qquad \begin{cases}\mathbf{i=1,2,\ldots,p}\\ \mathbf{k=1,2,\ldots,n}\end{cases} \tag{4}$$

The electrons located near to the nucleus are supposed to be in the base mode of energy. The mathematical model utilizes the place and purpose subordinate amounts of solution candidates in each layer to define the binding energy essential for drawing an electron from its cover. To define the binding condition and binding energy of solution candidates in each imaginary layer, the situations and purpose subordinate amounts of all candidates in the layer are averaged. To achieve the intended purpose, the following mathematical equations are provided:

$$\mathbf{BS}=\frac{\sum_{i=1}^{m}\mathbf{X_i}}{\mathbf{m}} \qquad \{\mathbf{i=1,2,\ldots,m} \tag{5}$$

$$\mathbf{BE}=\frac{\sum_{i=1}^{m}\mathbf{E_i}}{\mathbf{m}} \qquad \{\mathbf{i=1,2,\ldots,m} \tag{6}$$

In the kth layer, BS^k and BE^k represent the condition and energy of binding, respectively. The location and objective subordinate amount of ith solution candidates are denoted by X_i^k and E_i^k, while m represents the whole number of solution candidates in the probe area. To accurately assess the state and strength of binding of an atom, one must take into account the average positions and objective function values of all potential solutions in the search space. The location (X_i) and purpose subordinate amount (E_i) of each solution candidate within the atom can be analyzed to determine the atom's binding state BS and binding energy BE. Use the following mathematical equations to update the candidate positions:

$$\mathbf{X_{i+1}^k}=\mathbf{X_i^k}+\frac{\alpha_i}{\mathbf{k}}(\beta_i\times\mathbf{LE}-\gamma_i\times\mathbf{BS}) \qquad \begin{cases}\mathbf{i=1,2,\ldots,p}\\ \mathbf{k=1,2,\ldots,n}\end{cases} \tag{7}$$

To determine the amount of emitted energy, vectors α_i, β_i, and γ_i are randomly generated with a uniform distribution between (0,1). X_i^k and X_{i+1}^k represent the current and upcoming positions, while LE refers to the candidate with the lowest energy level in the atom. Additionally, BS denotes the binding state of the atom. photon absorption happened when the energy level of a solution candidate in a special layer is lower than the layer's binding energy. This procedure involves solution candidates absorbing a photon with energy levels β and γ to efficiently get both the binding situation of the layer and the lowest energy level condition of the electron within the layer. The mathematical equation to update the location of solution candidates is (Mahdi Azizi, 2021):

$$\mathbf{X_{i+1}^{k}=X_{i}^{k}+\alpha_i\times(\beta_i\times LE^{k}-\gamma_i\times BS^{k})}\quad,\begin{cases}\mathbf{i=1,2,\dots,p}\\ \mathbf{k=1,2,\dots,n}\end{cases}\tag{8}$$

After running the AOS algorithm, it is time to run FA algorithm so that firstly, rank the population in FA algorithm. The attractiveness of a Firefly is shown in as Eq 9.

$$\mathbf{I(r)=\frac{I_s}{r^2}}\tag{9}$$

Let's consider a scenario with n fireflies, where xi represents the solution for each individual Firefly. The brightness of a firefly, denoted as i, is closely linked to the objective function $f(x_i)$. The objective function expressed in Eq 10, showed the brightness I of a firefly.

$$\mathbf{I_i=f(x_i)}\tag{10}$$

The dimmer Firefly is absorbed and moves towards the shining one, and parameter β expressed the specific level of attractiveness of each Firefly. β, is related to the spacing between Fireflies. The attractiveness function of the Firefly is showed as Eq11.

$$\mathbf{\beta(r)=\beta_0 e^{-\gamma r^2}}\tag{11}$$

β_0 represents the attractiveness of the Firefly when it is at r = 0, while γ represents the light absorption coefficient of the media. Firefly at location xi moves towards a brighter Firefly at location xj using Eq12 (X.-S. Yang, 2009).

$$\mathbf{X_i(t+1)=x_i(t)+\beta_0 e^{-\gamma r^2}(x_i-x_j)\alpha\varepsilon_i}\tag{12}$$

When the Firefly xj is attracted, $\beta_0 e^{-\gamma r^2}(x_i-x_j)$ affects the movement, while $\alpha\varepsilon_i$ is a randomization parameter. If $\beta_0 = 0$, the movement is random The algorithm compares the Firefly's new location to the past one to define its fascination. If the new position seems more attractive, the Firefly will move; if not, it will stay in its current position. The stopping criteria for the FA are set by a pre-defined number of iterations or a fitness value deemed appropriate. According to Eq13, the Firefly that shines the brightest moves in a random pattern.

$$\mathbf{X_i(t+1)=x_i(t)+\alpha\varepsilon_i}\tag{13}$$

NUMERICAL EXAMPLES

The results illustrated in this part can be used to compare the efficiency of the proposed algorithm investigated in this article. An Intel i5 (2.4 GHz) system with 8 GB of RAM was utilized for all of the simutlations. To validate and compare the algorithms detailed in this article with other algorithms, these results are compared with the results of some studies. In Table 1, the control parameters are defined for estimating different plans of the suggested algorithm. The control parameters have been set to assess the various processes of the suggested algorithm based on the standard range of each algorithm which stated in other articles.

Table 1**:** Control parameters of algorithms

Optimization parameters	Value
Gamma parameter of FA	1
Beta$_0$ parameter of FA	2
Alpha parameter of FA	0.2
FotonRate parameter of AOS	0.1
LayerNumber parameter of AOS	5

In this section, we will delve into a comprehensive analysis of the prevailing engineering design issues. It is noteworthy to emphasize that the ensuing discussion focuses exclusively on the most renowned engineering design problems encountered in practical applications. Efficacious resolution of these problems typically necessitates a proactive methodology aimed at determining the optimal parameters for the most ideal design.

Procedure Atomic Orbital Search- Firefly Algorithm
Objective function $f(x)$, $x = (x_1, x_2, \ldots, x_d)^T$
Determine initial positions of solution candidates (X_i) *in the search space with m candidates*
Evaluate fitness values (E_i) *for initial solution candidates*
Determine the binding state(BS) and binding energy (BE) of the atom
Determine the candidate with the lowest energy level in the atom (LE)
While Iteration < Maximum number of iterations
Generate n as the number of imaginary layers
Create imaginary layers
Sort solution candidates in an ascending or descending order
Distribute solution candidates in the imaginary layers by PDF
For k=1: n
Determine the binding state (BS^k) *and binding energy* (BE^k) *of the kth layer*
Determine the candidate with the lowest energy level in the kth layer (LE^k)
For i=1: p
Generate $\varphi, \alpha, \beta, \gamma$
Determine PR
If $\varphi \geq PR$
If $E_i^K \geq BE^k$
$$X_{i+1}^k = X_i^k + \frac{\alpha_i \times (\beta_i \times LE - \gamma_i \times BS)}{k}$$
Else if $E_i^K < BE^k$
$$X_{i+1}^k = X_i^k + \alpha_i \times (\beta_i \times LE - \gamma_i \times BS^k)$$
end
Else if $\varphi < PR$
$$X_{i+1}^k = X_i^k + r_i$$
End
End
End
Update binding state(BS) and binding energy (BE) OF ATOM
Update candidate with the lowest energy level in the atom (LE)
End while
Rank the population $\boldsymbol{X}_i$, *and update the current best.*
Initialize a population of fireflies X_i (i=1, 2,…, n)
Calculate the fitness value $f(Xi)$ *to determine the light intensity* I_i *at* X_i
Define light absorption coefficient γ
while (t<$MaxGeneration\_FA$)
for i=1:n=1: *all n fireflies*
for j=1:n=1: *all n fireflies*
if (I_j>I_i)
Move firefly i towards j in all d-dimensions via Lévy flight.
end if.
Attractiveness varies with distance r via $-e{-}\gamma r^2$.
Evaluate new solutions and update light intensity.
end for j
end for i
Rank the fireflies and find the current best.
end while
Output the best solution.
End procedure

Fig.3. Pseudo-code of AOS-FA Algorithm

Optimization of truss size

In this section, the size optimization of a 10-bar truss (Fig. 4) is studied. This truss has been numerically investigated by several researchers like Schmit Jr and Farshi (1974), Farshi and Alinia-Ziazi (2010). In analysing the 10-bar plane truss, displacement, and stress constraints are utilized with each other. The translations of nodes 5 and 6, located on the left, are constrained in the x and y directions. The two free nodes of the lower bars (2 and 4) obtain vertical loads (y-direction).

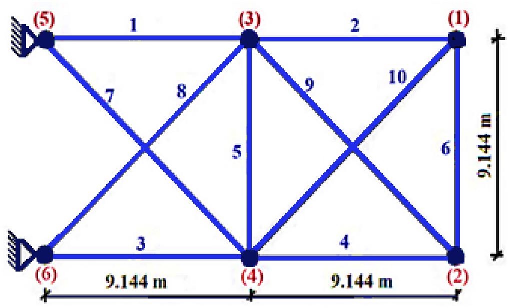


Fig.4. Ten-bar truss

All bars, except number 9, have the same tension limit for traction and compression. Nodes 1 through 4 have the same displacement limit in the y-direction. The cross-sectional areas of the ten elements are considered as continuous design variables. In Table 2, the mechanical properties, loading, stresses and displacements, and design variables of the truss are presented in Table 3, while Table 4 details the decision-making criteria and constraints to arrive at the best option. Table 4 shows that the FA algorithm with an objective function value of 2298.77 provided the best solution for truss size optimization, outperforming AOS and AOS-FA. All optimization algorithms had 100 research agents and 300 iterations. Fig.6 presents the computational time and standard deviation values for the minimum mass obtained after four independent executions of each algorithm. Based on Table 4 and Fig.5, it is clear that the AOS-FA algorithm, despite its shorter runtime, was not effective in optimizing the truss size. It is noticed that in this problem, numerical method has better results against to AOS-FA although it has not good results against to other algorithms that mentioned.

Table 2: Mechanical properties of the considered truss

Material	Aluminium
Density, ρ	2767.99 kg/m3
Young's modulus, E	68.95×109 N/m 2

Table 3: Nodal loading components and constraints for 10 bar plane truss

Loading		
No.	X-direction	Y-direction
2 and 4	0	-444.82 KN
Tension restrictions		
Bar	Value	
9	±517.11 MPa	
Displacement		
NO.	Value	
1,2,3,4	±50.8 mm (Y direction)	
The range of design variables		
64.5 mm² ≤ $A_{i\leq}$ 20000 mm²		

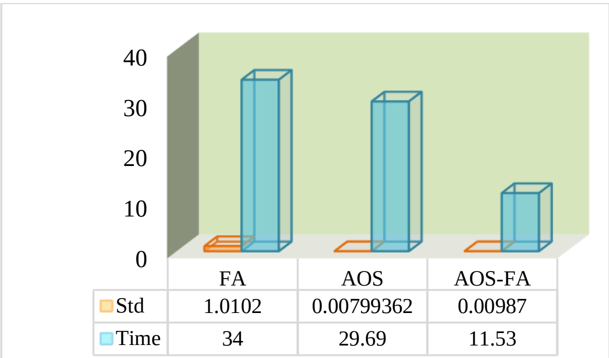


Fig.5. Standard deviation and computational time (s) for ten-bar plane truss

Design of I-shaped beams

In the I-beam design problem (Fig. 7), the goal is to minimize the vertical deflection, while satisfying the cross-sectional area and stress constraints under given loads. The variables of this problem are the width of the flange b (= x_1), the height of section h (= x_2), the thickness of the web t_w (= x_3), and the thickness of the flange t_f (= x_4). The maximum vertical deflection of the beam is $f(x) = \frac{PL^3}{48EI}$. The objective function when the modulus of elasticity is 523.104 kN/cm^2 and L=5200 cm, is formulated as follows:

Minimize:

$$F(X) = \frac{5000}{\frac{1}{12}x_3(x_2-2x_4)^3 + 2bx_4(x_2-\frac{x_4}{2})^2 + \frac{1}{6}(x_1x_4^3)} \quad (14)$$

Subject to:

$$g_1(x) = 2(\frac{1}{2}x_1x_3 + x_2x_4 - x_3x_4) \le 300 \quad (15)$$

$$g_2(x) = 15 \times 10^3 x_1 \frac{1}{x_3(x_2-2x_4)x_3 + 2x_1^3} + 18 \times 10^4 x_2 \frac{1}{x_3(1-2x_4)^3 + 2x_1(4x_4^2 - 3x_2^2)} \le 56 \quad (16)$$

In this problem, the best statistical results were achieved after four independent implementations using 100 search agents and 300 iterations by algorithms mentioned in Tables 5 and 6, respectively. From Table 6, it can be understood that AOS obtained the shortest computational execution time. According to Table 5 can say that metaheuristic algorithms that used for this problem, has good performances and the obtained results against to ARSM method is good. This subject shows that metaheuristic algorithms have good results than numerical methods. Also, it is determined that AOS-FA is top from AOS.

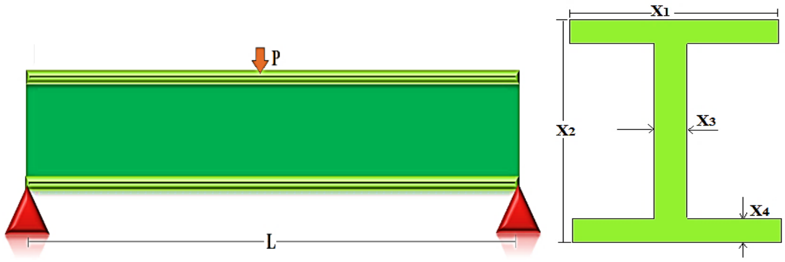


Fig.7. Beam design problem

Table 4**:** Optimum design of cross-sections (cm^2) for ten-bar plane truss

Present paper				Schmit Jr and Farshi (1974)	Borges (2013)		Ghaemifard and Ghannadiasl (2024a)
Member	FA	AOS	AOS-FA	Numerical method	PSO	HS	GWO
1	193.699	164.245	197.142	215.676	180.150	196.180	198.94
2	0.645	5.673	0.645	0.645	0.645	1.128	0.71
3	159.320	192.922	164.340	156.5158	151.150	144.560	156.27
4	93.244	187.862	106.074	91.9998	99.269	103.170	95.80
5	0.645	0.645	0.645	0.645	0.645	0.645	0.65
6	3.517	0.645	0.645	0.645	3.726	3.560	0.64
`7	47.892	61.004	52.977	54.1289	47.710	48.884	54.72
8	140.162	167.049	139.323	133.8061	139.810	138.040	133.59
9	134.560	99.228	121.686	127.032	146.780	138.020	134.10
10	0.645	0.645	0.645	0.645	0.645	0.673	0.74
Mass (kg)	2298.77	2700.16	2314.5	2308.3315	2301.41	2302.60	2303.41

Pressure vessel design

The goal of designing pressure vessels is to meet production needs while reducing container costs. The key design variables are head thickness (T_h), shell thickness (T_s), container length (L), and inner radius (R). in this problem, T_s and T_h are integers of 0.625, while R and L are continuous variables. Fig.8 shows the optimal structure design schematic.

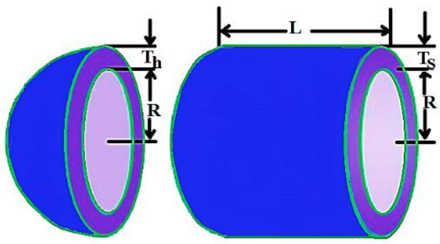


Fig.8**.** Schematic of the pressure vessel

Mathematical formulation for this problem is:
Consider:

$$\bar{X} = [X_1 X_2 X_3 X_4] = [T_s T_h R\ \ L] \tag{17}$$

Minimize:

$$f(\bar{X}) = 0.6224[(X_3 X_1 (X_4 + 31.876 X_1)) + (2.856 X_2 X_3^2 + 5.086 X_1^2 X_4)] \tag{18}$$

Subject to:

$$g_1(\bar{X}) = -[\left(-\frac{193}{10000} X_3\right) + X_1] \le 0 \tag{19}$$

$$g_2(\bar{X}) = \frac{954}{100000} X_3 - X_2 \le 0 \tag{20}$$

$$g_3(\bar{X}) = -\pi X_3^2 (\frac{4}{3} X_3 + X_4) + 1296000 \le 0 \tag{21}$$

$$g_4(\bar{X}) = -(-X_4 + 240) \le 0 \tag{22}$$

Variable Range:

$$\begin{gathered} 0 \le X_1 \le 99, \\ 0 \le X_2 \le 99, \\ 10 \le X_3 \le 200, \\ 10 \le X_4 \le 200 \end{gathered} \tag{23}$$

Table 5: Best results for the optimal design of I-Shaped Beam

Present work					Another research		
		FA	AOS	AOS-FA	SOS (Cheng & Prayogo, 2014)	SNS (Bayzidi, Talatahari, Saraee, & Lamarche, 2021b)	ARSM (Wang, 2003)
Variables	x_1	80	80	80	80	80	80
	x_2	50	50	50	50	50	37.05
	x_3	0.9	0.9	0.9	0.9	0.9	1.71
	x_4	2.32	2.32	2.32	2.3217	2.3217	2.31
Constraints	g1	0.0766	-21.89	0.07668	-0.000222	0	---
	g_2	4.4285	5.9667	4.4285	-1.57	-1.5702	---
F_{Cost}		0.0131	0.0130	0.0130	0.0130	0.0130	0.0157

Table 6: The statistical results of each algorithm

Method	Standard deviation	Mean	Max	Min	Time(s)
FA	11493.9	1.96171	1.96171	1.96171	35.04
AOS	33.6697	31.5419	80	0.9	0.0009688
AOS-FA	0	1.9622	1.9622	1.9622	3.0606
Ghaemifard and Ghannadiasl (2024a)	0	1.99466e+16	1.99466e+16	1.99466e+16	11.862

Table 7, presents the results of pressure vessel design issues. AOS-FA is a cost-effective algorithm that produces excellent results. The statistical results of the Pressure Vessel Design problem are presented in Table 8. It shows that the AOS-FA has the shortest computational execution time. According to the Table, it is determined that AOS obtained optimal result contrast to AOS-FA and FA and FA has better result from AOS-FA. Although AOS-FA algorithm has optimal result contrast to NLP method which shows hybrid algorithm has good performance.

Tubular column design

In this section, the design of a tubular column that is uniform in shape and can withstand the pressure at minimum cost was presented. The optimization variables for this problem are the average diameter of the column d (x_1) and the thickness of the tube t (x_2). The object has a yield stress of 500 kgf/cm^2, modulus of elasticity of 0.85 × 106 kgf/cm^2, and density of 0.0025 kgf/cm^3. The formula for this problem is:

$$f(X) = X_1(2 + 9.8X_2) \tag{24}$$

Table 7: Variables design of problem

	Algorithm	X_1	X_2	X_3	X_4	Optimal cost
Present study	FA	0.7967	0.3938	41.281408	187.031440	5979.983
	AOS	0.7782	0.3933	40.714312	195.311948	5888.6
	AOS-FA	0.9950	0.4933	51.2635	178.6489	6404.9
Other research	CS (Amir Hossein Gandomi, Yang, & Alavi, 2013)	0.8125	0.4375	42.0984456	176.6365	6059.714
	ABC (Akay & Karaboga, 2012)	0.8125	0.4375	42.098446	176.6365	6059.714
	NLP (Sandgren, 1990)	1.125	0.625	48.97	106.72	7982.5

Table 8: Statistical results of the Pressure vessel design problem

	FA	AOS	AOS-FA
Std	98.8673	17.5656	0
Min	5917.84	0.493384	6443.12
Max	6093.99	185.38	6446.12
Mean	5979.98	62.3957	6443.12
Time	3.34	1.97	1.32

The constraints on the stresses in the columns are:

$$g_1(X) = \frac{P}{\pi\sigma_y}\frac{1}{X_1X_2} - 1 \le 0 \tag{25}$$

$$g_2(X) = -1 + \frac{8PL^2}{\pi^3 E}\frac{1}{X_1^2X_2^2(X_1+X_2)} \le 0 \tag{26}$$

$$g_3(X) = -1 + 2\frac{1}{X_1} \le 0 \tag{27}$$

$$g_4(X) = -1 + \frac{1}{14}X_1 \le 0 \tag{28}$$

$$g_5(X) = -1 + \frac{1}{5}\frac{1}{X_2} \le 0 \tag{29}$$

$$g_6(X) = -1 + \frac{1}{8}X_2 \le 0 \tag{30}$$

Where the range of variables is:

$$\begin{gathered} 2 \le X_1 \le 14 \\ 0.2 \le X_2 \le 0.8 \end{gathered} \tag{31}$$

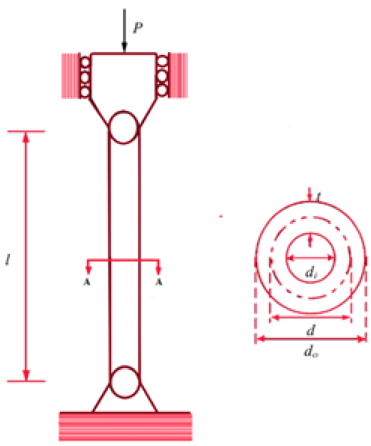


Fig.9. Model of the tubular column

We compared the optimal solution obtained by AOS-FA with other papers and found that AOS-FA had good results in this problem. The particular information is shown in Table 9 and 10 respectively.

Welded beam design

This is an engineering optimization problem involving the design of a welded beam to minimize cost, with four variables to optimize and seven constraints, as illustrated in Fig.10. The formula for this problem is:

Minimize:

$$f(X) = \frac{481}{10000}(14x_3x_4 + x_2x_3x_4 + 22.9669\, x_1^2x_2) \tag{32}$$

$$g_1(x) = -13600 + \tau(x) \le 0 \tag{33}$$

$$g_2(x) = -30000 + \sigma(x) \le 0 \tag{34}$$

$$g_3(x) = -0.25 + \delta(x) \le 0 \tag{35}$$

$$g_4(x) = -x_4 + x_1 \le 0 \tag{36}$$

$$g_5(x) = 6000 - P_c(x) \le 0 \tag{37}$$

$$g_6(x) = -x_1 + 0.125 \le 0 \tag{38}$$

$$g_7(x) = \frac{481}{10000}(14x_3x_4 + x_2x_3x_4 + 22.9669\, x_1^2) - 5.0 \le 0 \tag{39}$$

$$P_c(x) = \frac{4.013E}{6L^2}x_3x_4\sqrt{1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}} \tag{40}$$

It was denoting the thickness of the weld, height, length, and reinforcement thickness as h, l, t, and b showed as x_1, x_2, x_3, and x_4, respectively.

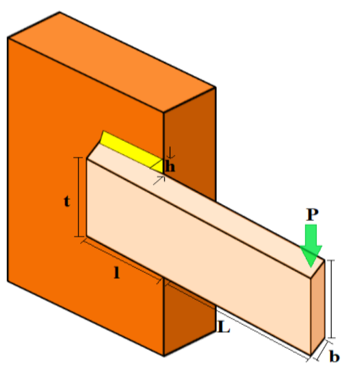


Fig. 10. Welded beam design problem

Table 9: Statistical results of the Pressure vessel design problem

			Present work			Another research	
		Exact value (Rao, 2019)	FA	AOS	AOS-FA	SNS (Bayzidi, Talatahari, Saraee, & Lamarche, 2021a)	ISA (Amir H Gandomi & Roke, 2014)
Variables	x_1	5.44	5.4521	5.4520	5.4526	5.4513	5.4511
	x_2	0.293	0.2916	0.2916	0.2916	0.2919	0.2919
Constraints	g_1	---	-9.9747e-09	-2.0701e-04	-1.2823e-04	-0.024	-2.5e-10
	g_2	---	-3.2983e-07	-0.0041	-0.0121	-0.109	-1.8e-10
	g_3	---	-0.6332	-0.6332	-0.6332	-0.633	-0.633
	g_4	---	-0.6106	-0.6106	-0.6105	-0.610	-0.6106
	g_5	---	-0.6332	-0.6332	-0.6332	-0.315	-0.3149
	g_6	---	-0.3185	-0.3185	-0.3184	-0.635	-0.635
F_{Cost}		26.53	26.4864	26.4882	26.4886	26.532	26.4994

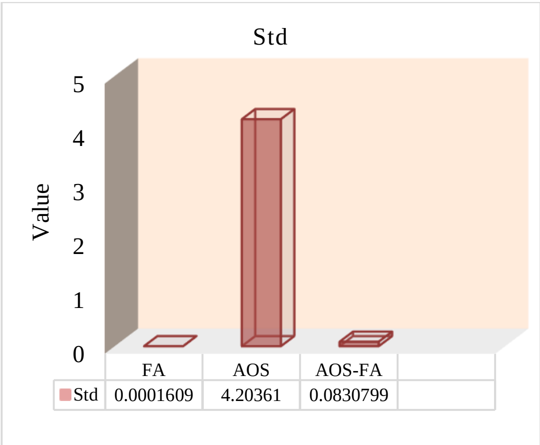
Table 10: The statistical results of each algorithm

Method	Standard deviation	Mean	Max	Min	Time(s)
FA	2.66919e-05	26.4864	26.4864	26.4864	35.154
AOS	3.64895	2.87187	5.45206	0.291671	7.2689
AOS-FA	0	26.4886	26.4886	26.4886	27.618

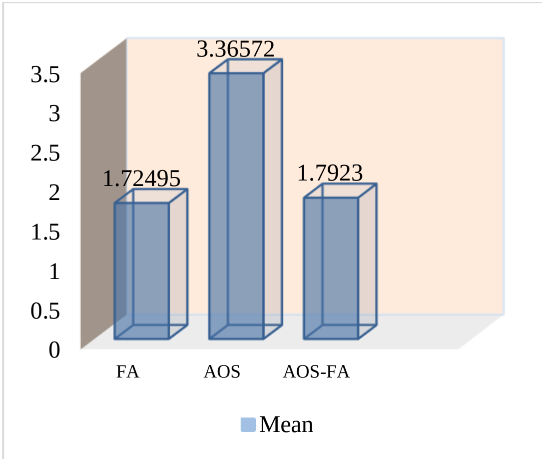
Table 11 shows the best results achieved after four independent implementations, using 100 search agents and 300 iterations with the mentioned algorithms. According to the Table 11 can say that performance of the AOS-FA algorithm is better than the FA algorithm and that is better than the AOS algorithm. Also, it is funded that metaheuristic algorithms have good result against to exact value which is showed the robust of these algorithms. Figure 11 displays the statistical results from four independent algorithm executions and their corresponding computational times. The problem is evaluated using the studied algorithms in Table 11 and compared to other literature. For instance, Sadollah, Bahreininejad, Eskandar, and Hamdi (2013) achieved the best cost value of 1.724853 when investigating MBA for this problem. In previous studies, different algorithms were evaluated for welded beam optimization. Kamalinejad, Arzani, and Kaveh (2019) and Mezura-Montes and Coello (2008) obtained 1.742706 and 1.737300 values for QEA and ES algorithms, respectively. Mirjalili, Mirjalili, and Hatamlou (2016) used MVO and achieved a result of 1.72645. The best cost values were obtained by ICO (A Kaveh & S Talatahari, 2010), CSS (A Kaveh & Siamak Talatahari, 2010), CSA (Askarzadeh, 2016), and MCSS (A Kaveh, Motie Share, & Moslehi, 2013) algorithms, which resulted in 1.724918, 1.724866, 1.7248523, and 1.724855, respectively. Additionally, FA algorithm achieved a low standard deviation value, and AOS was the fastest algorithm in computational execution time, as shown in Fig.11.

Table 11: The best result of algorithms

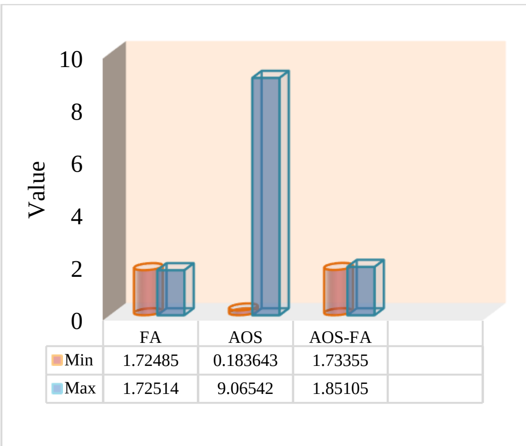
		Exact value (Rao, 2019)	FA	AOS	AOS-FA
Variables	x_1	0.2455	0.20572	0.18364	0.20157
	x_2	6.1960	3.47049	4.00818	3.54910
	x_3	8.2730	9.03662	9.06541	9.07004
	x_4	0.2455	0.20572	0.205644	0.20557
Constraints	g_1	---	-0.0081	-7.6980	0
	g_2	---	-0.0038	-0	0
	g_3	---	-9.2473e-08	-0.0220	0
	g_4	---	-3.4330	-3.3813	-3.4121
	g_5	---	-0.0807	-0.0586	-0.0717
	g_6	---	-0.2355	-0.2357	-0.2356
	g_7	---	-0.0072	-5.0511	0
F_{Cost}		2.386	1.7249	1.7645	1.7212



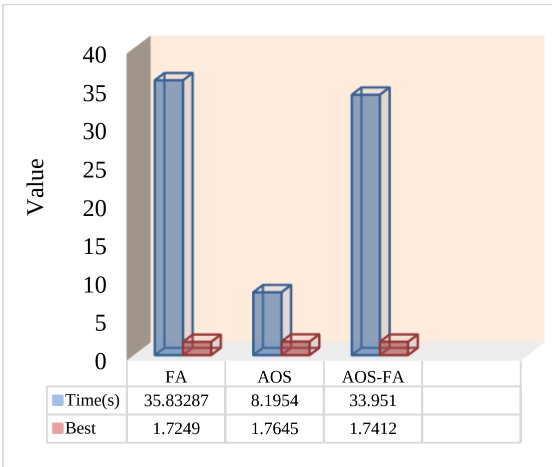
(a)



(b)



(c)



(d)

Fig.11. The statistical results of each algorithm (a) Std, (b) Mean, (c) Max-Min, (d) best value and time

CONCLUSION

The AOS algorithm outperforms other alternative meta-heuristics in converging to the global best for various mathematical test functions. It also excels in generating superior results with fewer function evaluations, showcasing its efficiency in addressing computational complexity issues. To boost the algorithm's performance, several researchers have introduced various enhancements (M. Azizi, Talatahari, Khodadadi, & Sareh, 2022; Elaziz et al., 2021). Additionally, the Firefly algorithm, inspired by the flashing behavior and bioluminescent communication of fireflies, is susceptible to premature convergence. Studies recommend adjusting constant parameters to alleviate this issue. This paper introduces a hybrid algorithm called AOS-FA for optimal engineering design, combining Atomic Orbital Search and the Firefly Algorithm based on quantum mechanics principles. The algorithm is evaluated on five well-known constrained design problems across different engineering fields, demonstrating that AOS-FA surpasses most recent meta-heuristic algorithms in the literature in terms of performance. Furthermore, the suggested AOSFA might be very useful in resolving additional challenging optimization issues like feature selection, picture segmentation, path planning, traveling salesman issues, and flow shop scheduling issues.

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