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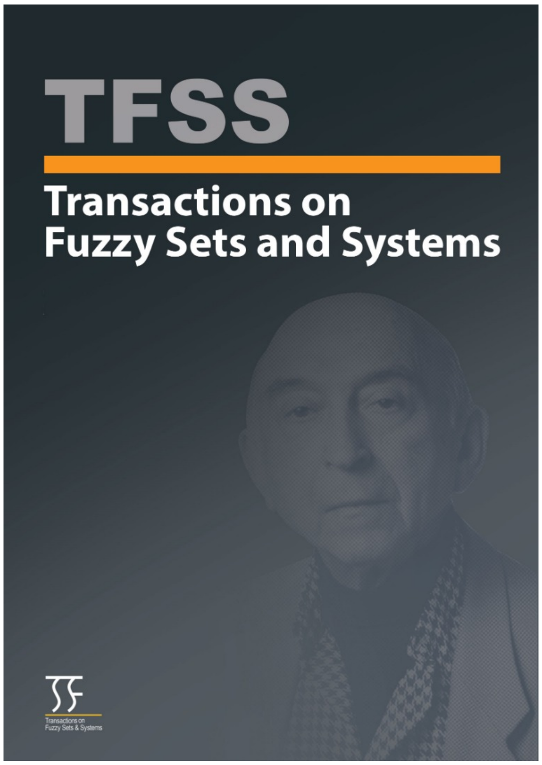

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MCGDM Using Generalized-Spherical Fuzzy VIKOR Technique for Sustainable and Optimal Foundation Crop Seed Selection in Agriculture

Haridas Mondal, **Totan Garai**, **Tipu Sultan Haque**, **Shariful Alam***

Abstract. In recent years, the VIKOR Multi-Criteria Decision-Making (MCDM) methodology has become one of the most popular techniques in the field of decision making, used to address various real-life problems. In this article, we extend this MCDM technique under the Generalized Spherical Fuzzy (GSF) realm, which provides an effective and robust framework for handling real-life uncertainty. Nowadays, sustainable agricultural practices have become essential for ensuring long-term food security, particularly in the context of increasing population pressures and growing environmental challenges. Therefore, this article aims to select the optimum high-yielding foundation crop seeds using the GSF-VIKOR MCDM technique, through dealing with real-world uncertainties and conflicting decision factors. A novel score and accuracy function have been proposed and employed with the MCDM technique in a fully neutrosophic approach to effectively manage real-life uncertainties encountered in the decision-making process. Several aggregation operators are used to handle and consolidate large data sets. The GSF-Dombi Weighted Arithmetic Aggregation(GSF-DWAA), GSF-Dombi Weighted Geometric Aggregation(GSF-DWGA), GSF-WAM, and GSF-WGM operators are incorporated into the selection process to properly manage the decision expert ratings for the decision-making. Additionally, the validity and superiority of the extended method have been verified and compared with the well-established TOPSIS MCDM techniques. The experiments have been conducted with the help of a constructed data set due to the lack of a real data set. The sensitivity and compatibility of the proposed technique have subsequently been investigated. Moreover, the advantages and limitations of the proposed method have been highlighted. Finally, the robustness of the technique has been investigated for large sets of uncertain data, and the reliability of outcomes has been compared with other existing methodologies.

AMS Subject Classification 2020: 90B50; 90C70
Keywords and Phrases: Multi-criteria group decision making (MCGDM), Generalized-spherical fuzzy (GSF), Agricultural breeder seed selection, GSF-VIKOR MCDM methodology.

1 Introduction

Today, agricultural thoughts and systems have significantly transformed due to the market entry of optimum quality, high-yielding agricultural breeder seed, advanced agricultural innovations, and modern agricultural technologies. The lesson of the Green Revolution has invaluable insight for developing genetically super-modified high-breed foundation seed for agriculture. It was a pivotal phenomenon of the 20^{th} century that profoundly transformed agricultural practices worldwide. It played a critical role in shaping the trajectory of global agriculture and food systems as well as social, economic, and environmental spheres, leaving a lasting legacy that continues to shape agricultural development today. The larger Green Revolution was initiated by Borlaug [1] in 1940 in Mexico and produced dwarf varieties, like IR8 rice and Mexican wheat

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strains, and new disease-resistant high-yield varieties (HYV) of wheat crops. The conduct of hybridization experiments on garden peas and the creation of new hybridized varieties with high characteristics was the first step of the Green Revolution, organized and established in $1856 - 1863$. After the establishment of the concept of genetically modified plants and cross-breeding plants by Mendel [2], scientists began to experiment with special techniques to enhance the breeding quality of plants. Nowadays, agriculture around the world has transformed into a modern industrial system through the adoption of technologies such as mechanized farm equipment, irrigation systems, pesticides, fertilizers, and high-yielding varieties of seeds and crops. Additionally, society has experienced significant increases in agricultural productivity and food production, and as a result, an economic boost has been observed.

Several research studies have been done in the context of the hybridization of crops, genetic improvement of plants, advanced technology innovations, and breeder crops data analysis in the agriculture sector. Several agricultural institutes across the world focused on developing certified, high-yielding crop seeds to address food security, poverty, and environmental sustainability through innovation and collaboration. Agricultural research is responsible for overseeing the production of certified breeder seeds. Additionally, the State Seed Corporation, agricultural farms, cooperatives, etc, are the organizations that organize the production of certified agricultural breeder seeds. In this regard, Hasibuan et al. [3] have shown the role of certification, risk, and time preferences in promoting climate-resilient citrus varieties in agriculture. Additionally, Kim and Zhang [4] have shown molecular control of male fertility for crop hybrid breeding for better production, and Han et al. [5] discussed a hybrid crop breeding information management system based on combining ability analysis. Also in this field, Liao et al. [6] have shown how a third-generation hybrid seed innovation enhanced rice productivity in agriculture. Additionally, Clapp and Ruder [7] discussed technological innovation in hybrid rice and corn production. After that, Divya et al. [8] discussed several models on agriculture and its impact on the small and large-scale stakeholders, as well as the agronomic development of the society. Walkowiak et al. [9] have shown the result of a field experiment with multiple wheat genomes to reveal global variation in modern breeding. Additionally, Shah and Wu [10] discussed soil and crop management strategies for achieving sustainable, higher crop production in agriculture. The practice and adoption of advanced agriculture play a critical role in society, as they enhance food security, improve resource efficiency, and contribute to environmental sustainability.

The adoption of high-yielding crop varieties is now widespread, but selecting a suitable certified, high-yielding seed according to soil conditions and other external factors is most important in agricultural management. Proper certified breeder seed selection plays a vital role in boosting crop productivity, promoting sustainable farming, and supporting economic growth. Farmers are drawn to certified seeds for their superior yields and economic returns, but the key factors in choosing hybrid foundation seeds include genetic purity, tolerance to adverse conditions, pest and disease resistance, adaptability, and market value, which are most important to gain maximum crop production. In this context, Deepa and Ganesan [11] have introduced a decision-making tool for crop selection for agricultural development. Additionally, Mishra et al. [12] implemented a crop selection technique in agriculture, and Gunawan et al. [13] showed an MCDM support system for sustainable agriculture, and optimization in crop selection for optimum crop production. After that, Mokarram et al. [14] presented a suitable location to cultivate grapes based on disease infestation using MCDM and remote sensing, and Biswas et al. [15] presented the application of the MCDM methods to the management of agricultural systems for conservation agriculture practice in rice and wheat cropping. Therefore, it has been observed that the application of MCDM techniques in various aspects of agricultural management is remarkable for proper seed selection, evaluation, and the improvement of certified seed varieties. There are some widely recognized MCDM techniques, including the Analytic Hierarchy Process (AHP), TOPSIS, EDAS, CDAS, VIKOR, and MULTIMOORA, which are extensively applied in various contexts for effective decision-making.

Real-life decision-making involves multiple conflicting criteria, and uncertainties occur in the real-life data.

Real-life imprecise data handling and decision-making using MCDM techniques have been widely applied under various uncertain environments. The MCDM under uncertainty provides a structured framework to select the best alternative considering multiple conflicting criteria involved in decision-making. It offers a powerful and flexible approach under uncertainty, allowing decision-makers to effectively navigate complex decision problems, incorporate subjective preferences, and derive meaningful insights from imprecise information. Several research works on decision-making have been done under different uncertain environments, from Zadeh's fuzzy environment [16], Atanassov's intuitionistic environment [17], to Smarandache's neutrosophic environment [18]. Various MCDM problems have been studied under different uncertain environments through incorporating different generalized fuzzy contexts, such as spherical fuzzy [19], and generalized spherical fuzzy extended by Haque et al. [20]. In this context of studying MCDM problems across several decision-making fields, we found that various complex problem studies have been done. Yazdani et al. [21] proposed agricultural decision-making under an uncertain environment for supply chain risk management, and Mohanta and Sharanappa [22] showed the MCDM under an intuitionistic environment and its application in the Indian agriculture sector. Additionally, Ashraf et al. [23] used the neutropenic MCDM concept in agricultural land selection under the neutrosophic environment, and Nguyen [24] interpreted supply chain risk management in agriculture under spherical fuzzy Analytic Hierarchy Process(AHP). Also, some special MCDM methods were extended under different uncertain environments, such as alik [25], who extended the TOPSIS method under Pythagorean fuzzy and implemented it for agricultural selection, and Luo et al. [26], who manifested the VIKOR method under neurosophic fuzzy, showed the application for sustainable supplier selection. Additionally, Kutlu Gundogdu and Kahraman [27] have extended the spherical fuzzy WASPAS, EDAS, and the MULTIMOORA [28] in the DM field, under the spherical as well as neutrosophic environment, and have applied it to several real-life decision-making problems.

In this study, we have demonstrated a realistic way to select a high-yielding breeder paddy seed, considering multiple criteria with real-life uncertainty for sustainable agricultural management. Real-life decision-making for sustainable and high-yielding breeder seed selection involves imprecision in data, both in the field and the laboratory. There are many potential inaccuracies that can occur in the evaluation and selection of certified seed varieties on various aspects, such as seed quality assessment, field inspection, environmental risk factor management, and resource allocation in real-world conditions. To address this, the decision-making problem for selecting foundation seeds in the agricultural sector has been demonstrated under the generalized spherical fuzzy (GSF) environment, through the VIKOR MCDM methodology. The VIKOR methodology has been implemented and extended in various decision fields under different imprecise environments. The method was implemented for industrial supplier selection under fuzzy arena [29], and the technique was also extended under spherical spherical fuzzy Dombi operator and applied for multiple selection problems. Thereafter, Salamai [30] has shown the application of VIKOR for the green supply chain management practices under the neutrosophic number. Additionally, Khan et al. [31] showed the novelty of the Dombi aggregation operator in managing and averaging the membership values in decision-making. In this work, we incorporated the VIKOR methodology and the Dombi aggregation operator under the GSF fuzzy environment, which has been fruitfully implemented in the selection of high-yielding agricultural foundation seed for sustainable agriculture practice. Additionally, multiple operators have been incorporated with the GSF-VIKOR MCDM method for properly handling and analyzing initial data associated with the decision-making, like the GSF-WAM, GSF-WGM, GSF-Dombi Weighted Arithmetic Aggregation(GSF-DWAA), and the GSF-Dombi Weighted Geometric Aggregation(GSF-DWGA) operators under the generalized-spherical fuzzy to ensure reliability and robustness in computation and analysis.

1.1 Motivation

After analyzing the existing literature, we observed several important gaps and limitations that compelled us to design a more flexible and reliable decision-making tool capable of addressing real-life selection problems involving various uncertainties. The motivational factors that inspired and guided our study are outlined below.

Firstly, the literature indicates the presence of several fuzzy-based decision-making approaches; however, these methods remain largely inadequate, as they struggle to capture and manage the multifaceted complexities and ambiguities of decision-making. Particularly when it comes to selection under a real-life uncertain environment, which motivated the extension of a reliable MCDM tool in a suitable uncertain environment to deal with real-life vagueness occurring in the agricultural system, managing diverse and conflicting criteria and uncertainties in the decision-making field.

Secondly, the generalized spherical fuzzy(GSF) environment has been considered to manage uncertain real-life ambiguities involved in decision-making for its flexibility in accepting and managing all types of decision expert ratings, compared to other uncertain environments. There are some drawbacks to handling decision expert ratings, for spherical fuzzy and different environments. We can easily understand the broad nature of the GSF environment through an example; the fuzzy expert rating, $\langle 0.8, 0.9, 0.7\rangle$ or $\langle 0.5, 0.7, 0.8\rangle$ does not satisfy the main property of spherical fuzzy, i.e., the square sum of three membership values is less than or equal to 1^2, although these expert ratings may occur in the real-life, which is a valid fuzzy ratings for the decision expert in the decision-making. To overcome these drawbacks, we consider the GSF environment in our study, where the square sum of three membership values of the said expert rating is less than or equal to $\sqrt{3}^2$, and which can effectively handle any real-life neutrosophic fuzzy ratings more precisely for decision-making.

Thirdly, we have seen that several agricultural selection problems have been studied in the literature for agricultural management, but only a few works have been done for the best crop selection in agriculture, which motivated us to study agricultural management and hybrid crop selection for the optimum productivity in agriculture. As we know, the agricultural seed sector plays a pivotal role in ensuring food security and driving the economic growth of countries by contributing to GDP, generating employment, supplying raw materials, reducing income inequality, and creating market linkages.

Fourthly, we incorporate several operators, such as GSF-DWAA, GSF-DWGA, GSF-WAM, and GSF-WGM aggregation operators under the GSF domain, and in the MCDM technique for data analysis, for more flexibility in handling large amounts of data involved with the decision-making. The GSF-DWAA and GSF-DWGA operators offer unique advantages in handling uncertainty and imprecision in the real-world crop selection in agriculture. Sustainable agricultural crop selection promotes sustainable farming practices and ensures long-term agricultural development. Therefore, the identification of high-yielding crop varieties, regular assessment, and ongoing research are crucial for achieving sustained progress in the agricultural sector.

1.2 Novelty

This study focused on real-world decision-making and effective agricultural management through the application of an appropriate Multi-Criteria Decision-Making (MCDM) technique. The research emphasizes the implementation of a flexible MCDM tool to promote sustainable agricultural practices, focusing on the selection of high-yielding foundation seeds to ensure food security and stimulate economic growth in society. The main objectives of this study are outlined below.

(i) To develop a reliable MCDM methodology in the GSF environment and implement it in the agricultural sector for hybrid agricultural foundation seed selection, addressing the uncertainties commonly faced in real life. Also, the aim of the study is to promote sustainable agriculture management from different perspectives, including environmental, social, and economic.

(ii) A novel score function and accuracy function have been constructed, and incorporated in the decision-making process to find fuzzy best value and worst value under the MCDM technique.

(iii) Advanced aggregation operators implemented for large set data handling involved in the decision-making. The GSF-DWAA and GSF-DWGA, GSF-WAM, and GSF-WGM aggregation operators have been imple-

mented in the MCDM technique for data analysis. Using GSF-DWAA, GSF-DWGA, GSF-WAM, and GSF-WGM operators uniquely handle uncertainty and imprecision within the generalized-spherical fuzzy framework. By implementing advanced aggregation techniques, decision-makers can achieve more accurate and robust results when dealing with large-scale, multi-dimensional datasets.
(iv) To offer a unique approach to green selection by considering all necessary criteria with a realistic approach and solving it using the GSF VIKOR methodology. This study integrates both qualitative and quantitative factors, ensuring a comprehensive assessment of agriculture management. The proposed framework enhances decision reliability by addressing uncertainties and conflicting criteria effectively.
(v) To incorporate multiple crucial decision factors in the selection process, such as yield potential, disease resistance, adaptability to different environments, nutritional content, and market demand, for proper assessment of crop varieties and the selection of the best variety. Also, aim for involvement of multiple decision experts in the selection process.
(vi) To conduct a comparison study with other existing methods for data sensitivity, and effectiveness of the GSF-VIKOR technique, and highlight the advantages and limitations of the proposed approach.
(vii) To provide a detailed algorithm for step-by-step understanding of the proposed method, along with a flowchart that outlines each step, and a visualization of the decision-making process.

1.3 Structure of the Paper

After the introduction, motivation, and novelty, the manuscript is designed as follows. Some preliminary definitions of fuzzy numbers and their operations and properties are discussed in Section 2, and in Section 3, we displayed the Dombi aggregation operators in the GSF environment. In Section 4, we extended the VIKOR methodology in the GSF arena, and in Section 5, a suitable MCDM problem in the agriculture sector has been illustrated and solved by implementing the GSF-VIKOR technique. The data sensitivity analysis, comparative study, advantages, and limitations of the proposed approach are shown in Section 6. In section 7, the conclusion of the study has been drawn, and at the end, a reference list has been added.

2 Preliminaries

In this section, we have discussed some basic definitions of different fuzzy sets, different types of fuzzy numbers, the Dombi operator, Dombi weighted aggregation Operators of GSFNs, and their basic properties and operations.

Definition 2.1. *Let* $\boldsymbol{R}$ *be a universal set, and* c *be a generic element of that set, then the intuitionistic fuzzy set(IFS)* $\boldsymbol{F}$ *of* $\boldsymbol{R}$ *is defined by,*

$$\mathbf{F} = \{\langle c,\ \eta_F(c),\ \zeta_F(c)\rangle : 0 \leq \eta_F + \zeta_F \leq 1, \forall\ c \in R\}.$$

Where η_F*: R* $\rightarrow$ *[0,1] and* ζ_F*:R*$\rightarrow$ *[0,1] are the positive and the negative membership functions of* $\boldsymbol{F}$.

Definition 2.2. *Let* $\boldsymbol{R}$ *be a universal set, and* c *be a generic element of that set, then the neutrosophic fuzzy set(NFS)* $\boldsymbol{F}$ *of* $\boldsymbol{R}$ *is defined by,*

$$\mathbf{F} = \{\langle c,\ \eta_F(c),\ \xi_F(c),\ \zeta(c)\rangle : 0 \leq \eta_F + \xi_F + \zeta_F \leq 3, \forall\ c \in R\}.$$

Where η_F*: R* $\rightarrow$ *[0,1],* ξ_F*: R* $\rightarrow$ *[0,1], and* ζ_F*:R*$\rightarrow$ *[0,1] are, respectively, positive, indeterminacy (satisfaction to some extent), and negative membership functions of* $\boldsymbol{F}$.

Definition 2.3. *Let $\boldsymbol{R}$ be a universal set, and c be a generic element of that set, then the spherical fuzzy set(SFS) $\boldsymbol{F}$ of $\boldsymbol{R}$ is defined by,*

$$\mathbf{F} = \{\langle c,\ \eta_F(c),\ \xi_F(c),\ \zeta_F(c)\rangle : 0 \leq {\eta_F}^2 + {\xi_F}^2 + {\zeta_F}^2 \leq 1, \forall\ c \in R\}.$$

Where η_F: $R \to$ [0,1], ξ_F: $R \to$ [0,1], and ζ_F: $R\to$ [0,1] are positive, indeterminacy(satisfaction to some extent), and negative membership functions of $\boldsymbol{F}$ respectively. The spherical fuzzy number(SFN) can be written in a triplet form as $\langle\eta_F(c),\ \xi_F(c),\ \zeta_F(c)\rangle$, that satisfy the condition; $0 \leq {\eta_F}^2 + {\xi_F}^2 + {\zeta_F}^2 \leq 1, \forall\ c \in R$.

Definition 2.4. *Let $\boldsymbol{R}$ be a universal set, and c be a generic element of that set, then the Generalized-spherical fuzzy set(G-SFS) $\boldsymbol{F}$ of $\boldsymbol{R}$ is defined by,*

$$\mathbf{F} = \{\langle c,\ \eta_F(c),\ \xi_F(c),\ \zeta_F(c)\rangle : 0 \leq {\eta_F}^2 + {\xi_F}^2 + {\zeta_F}^2 \leq 3, \forall\ c \in R\}.$$

Where η_F: $R \to$ [0,1], ξ_F: $R \to$ [0,1], and ζ_F:$R\to$ [0,1] are positive, indeterminacy, and negative membership functions of $\boldsymbol{F}$ respectively. The Generalized-spherical fuzzy number(GSFN) can be written in a triplet form as $\langle\eta_F(c),\ \xi_F(c),\ \zeta_F(c)\rangle$, that satisfy the condition; $0 \leq {\eta_F}^2 + {\xi_F}^2 + {\zeta_F}^2 \leq 3, \forall\ c \in R$.

2.1 Properties and Operations of GSFNs

Let $\mathbf{m_i} = \langle\eta_{m_i}(c),\ \xi_{m_i}(c),\ \zeta_{m_i}(c)\rangle$ and $\mathbf{m_j} = \langle\eta_{m_j}(c),\ \xi_{m_j}(c),\ \zeta_{m_j}(c)\rangle\ \forall\ c \in \mathbf{R}$, be two GSFNs, then union, intersection, complement, addition, and multiplication are described as follows,

(a) $\mathbf{m_i} \subseteq \mathbf{m_j}$ if and only if $\eta_{m_i} \leq \eta_{m_j}$, $\xi_{m_i} \leq \xi_{m_j}$ and $\zeta_{m_i} \geq \zeta_{m_i}$

(b) $\mathbf{m_i} = \mathbf{m_j}$ if and only if $\mathbf{m_i} \subseteq \mathbf{m_j}$ and $\mathbf{m_j} \supseteq \mathbf{m_i}$

(c) $\mathbf{m_i} \cup \mathbf{m_j} = \left\langle max(\eta_{m_i},\ \eta_{m_j}),\ min(\xi_{m_j},\ \xi_{m_j}), min\ (\zeta_{m_i},\ \zeta_{m_i}\right\rangle$

(d) $\mathbf{m_i} \cap \mathbf{m_j} = \left\langle min(\eta_{m_i},\ \eta_{m_j}), min(\xi_{m_i},\ \xi_{m_j}),\ max(\zeta_{m_i},\ \zeta_{m_i})\right\rangle$

(e) $\mathbf{m_i^c} = \langle\zeta_{m_i},\ \xi_{m_i},\ \eta_{m_i}\rangle$

(f) $\lambda\mathbf{m_i} = \left\langle\sqrt{1-(1-\eta_{m_i}^2)^\lambda}, \xi_{m_i}^\lambda,\ \zeta_{m_i}^\lambda\right\rangle$, where $\lambda \geq 0$

(g) $\mathbf{m_i} \oplus \mathbf{m_j} = \left\langle\sqrt{\eta_{m_i}^2 + \eta_{m_J}^2 - \eta_{m_i}^2\eta_{m_J}^2},\ \xi_{m_i}\xi_{m_j},\ \zeta_{m_i}\zeta_{m_i}\right\rangle$

(h) $\mathbf{m_i} \otimes \mathbf{m_j} = \left\langle\eta_{m_i}\eta_{m_j},\ \xi_{m_i}\xi_{m_j},\ \sqrt{\zeta_{m_i}^2 + \zeta_{m_j}^2 - \zeta_{m_i}^2\zeta_{m_j}^2}\right\rangle$

(i) $\mathbf{m_i}^\lambda = \left\langle\eta_{m_i}^\lambda,\ \xi_{m_i}^\lambda, \sqrt{1-(1-\zeta_{m_i}^2)^\lambda}\right\rangle$, where $\lambda \geq 0$

2.2 Some Identities of GSFs Numbers

Let $\mathbf{m_i} = \langle\eta_{m_i}(c),\ \xi_{m_i}(c),\ \zeta_{m_i}(c)\rangle$ and $\mathbf{m_j} = \langle\eta_{m_j}(c),\ \xi_{m_j}(c),\ \zeta_{m_j}(c)\rangle\ \forall\ c \in \mathbf{R}$, be two GSFNs, then the following are identical.
(a) $\mathbf{m_i} \oplus \mathbf{m_j} = \mathbf{m_j} \oplus \mathbf{m_i}$
(b) $\mathbf{m_i} \otimes \mathbf{m_j} = \mathbf{m_j} \otimes \mathbf{m_i}$
(c) $\lambda(\mathbf{m_i} \oplus \mathbf{m_j}) = \lambda\mathbf{m_j} \oplus \lambda\mathbf{m_i}$, where $\lambda \geq 0$
(d) $\lambda_1\mathbf{m_i} \oplus \lambda_2\mathbf{m_i} = (\lambda_1 + \lambda_2)\mathbf{m_i}$, where $\lambda_1, \lambda_2 \geq 0$

(e) $(\mathbf{m_i} \oplus \mathbf{m_j})^{\lambda} = \mathbf{m_j}^{\lambda} \oplus \mathbf{m_i}^{\lambda}$
(f) $\mathbf{m_i}^{\lambda_1} \otimes \mathbf{m_i}^{\lambda_2} = \mathbf{m_i}^{\lambda_1+\lambda_2}$

The proofs of the above identities are similar to those in [19].

Theorem 2.5. GSF-WAM Operator: *If $\mathbf{m_1}, \mathbf{m_2}, \ldots, \mathbf{m_n}$ are GSFNs and $w_1, w_2, w_3, \ldots, w_n$ be the corresponding weight vectors, then Generalized-Spherical Weighted Arithmetic Mean(GSF-WAM) aggregation operator is defined by,*

$$G - SFAM_w(\mathbf{m_1}, \mathbf{m_2}, \mathbf{m_3}, \ldots, \mathbf{m_n}) = w_1\mathbf{m_1} \oplus w_2\mathbf{m_2} \oplus w_3\mathbf{m_3} \oplus \cdots \oplus w_n\mathbf{m_n}$$

$$= \left\langle \sqrt{1 - \prod_{i=1}^{n}(1-\eta_{m_i}^2)^{w_i}}, \prod_{i=1}^{n}(\xi_{m_i})^{w_i}, \prod_{i=1}^{n}(\zeta_{m_i})^{w_i} \right\rangle. \quad (1)$$

Where $w_i \in$ [0, 1], along with $\sum_{i=1}^{n} w_i = 1$.

Proof. The proof is similar to that in [19]. □

Theorem 2.6. GSF-WGM Operator: *Let $\mathbf{m_1}, \mathbf{m_2}, \ldots, \mathbf{m_n}$ are GSFNs and $w_1, w_2, w_3, \ldots, w_n$ be the corresponding weight vectors, then Generalized-Spherical Weighted Geometric Mean(GSF-WGM) aggregation operator is defined by,*

$$G - SFGM_w(\mathbf{m_1}, \mathbf{m_2}, \mathbf{m_3}, \ldots, \mathbf{m_n}) = \mathbf{m_1}^{w_1} \otimes \mathbf{m_2}^{w_2} \otimes \mathbf{m_3}^{w_3} \otimes \cdots \otimes \mathbf{m_n}^{w_n}$$

$$= \left\langle \prod_{i=1}^{n}(\eta_{m_i})^{w_i}, \prod_{i=1}^{n}(\xi_{m_i})^{w_i}, \sqrt{1 - \prod_{i=1}^{n}(1-\zeta_{m_i}^2)^{w_i}} \right\rangle. \quad (2)$$

Where $w_i \in$ [0, 1], along with $\sum_{i=1}^{n} w_i = 1$.

Proof. The proof is the same as in [19]. □

3 Dombi Aggregation Operator

Let ϕ and ψ be two real numbers; then the Dombi triangular norm and the Dombi triangular co-norm are defined by,

$$Dom(\phi,\psi) = \frac{1}{1 + \left\{\left(\frac{1-\phi}{\phi}\right)^{\rho} + \left(\frac{1-\psi}{\psi}\right)^{\rho}\right\}^{\frac{1}{\rho}}}; \quad \text{and} \quad Dom_c(\phi,\psi) = 1 - \frac{1}{1 + \left\{\left(\frac{\phi}{1-\phi}\right)^{\rho} + \left(\frac{\psi}{1-\psi}\right)^{\rho}\right\}^{\frac{1}{\rho}}}$$

Where $\rho \geq 1$, and $(\phi,\psi)\in[0,1]\times[0,1]$.

3.1 Generalized-spherical Dombi Operations

Let $\mathbf{m_i} = \langle \eta_{m_i}(c),\ \xi_{m_i}(c),\ \zeta_{m_i}(c)\rangle$ and $\mathbf{m_j} = \langle \eta_{m_j}(c),\ \xi_{m_j}(c),\ \zeta_{m_j}(c)\rangle, \forall\ c \in \mathbf{R}$ be two GSFNs and $\rho \geq 1$, $\lambda > 0$, then the following operations according to the Dombi t-norm and the Dombi t-conorm under the GSF environment are defined as,

1. $$\mathbf{m}_i \oplus \mathbf{m}_j = \left\langle \sqrt{1 - \frac{1}{1+\left\{\left(\frac{1-\eta_{m_i}^2}{\eta_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\eta_{m_j}^2}{\eta_{m_j}^2}\right)^{2\rho}\right\}^{1/\rho}}}, \sqrt{\frac{1}{1+\left\{\left(\frac{1-\xi_{m_i}^2}{\xi_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\xi_{m_j}^2}{\xi_{m_j}^2}\right)^{2\rho}\right\}^{1/\rho}}}, \sqrt{\frac{1}{1+\left\{\left(\frac{1-\zeta_{m_i}^2}{\zeta_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\zeta_{m_j}^2}{\zeta_{m_j}^2}\right)^{2\rho}\right\}^{1/\rho}}} \right\rangle$$

2. $$\mathbf{m_i} \otimes \mathbf{m_j} = \left\langle \sqrt{\frac{1}{1+\left\{\left(\frac{1-\eta_{m_i}^2}{\eta_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\eta_{m_j}^2}{\eta_{m_j}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\left(\frac{1-\xi_{m_i}^2}{\xi_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\xi_{m_j}^2}{\xi_{m_j}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{1 - \frac{1}{1+\left\{\left(\frac{1-\zeta_{m_i}^2}{\zeta_{m_i}^2}\right)^{2\rho}+\left(\frac{1-\zeta_{m_j}^2}{\zeta_{m_j}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}} \right\rangle$$

3. $$\lambda\mathbf{m_i} = \left\langle \sqrt{1 - \frac{1}{1+\left\{\lambda\left(\frac{1-\eta_{m_i}^2}{\eta_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-\xi_{m_i}^2}{\xi_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-\zeta_{m_i}^2}{\zeta_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}} \right\rangle$$

4. $$\mathbf{m_i}^{\lambda} = \left\langle \sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-\eta_{m_i}^2}{\eta_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\lambda\left(\frac{1-\xi_{m_i}^2}{\xi_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{1 - \frac{1}{1+\left\{\lambda\left(\frac{1-\zeta_{m_i}^2}{\zeta_{m_i}^2}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}} \right\rangle$$

3.2 Dombi Weighted Aggregation Operators of GSFNs

We define the GSF-Dombi Weighted Arithmetic Aggregation (GSF-DWAA) and the GSF-Dombi Weighted Geometric Aggregation (GSF-DWGA) operators of GSFNs and investigate their properties as follows.

Theorem 3.1. *Let $\mathbf{m_1}, \mathbf{m_2}, \ldots, \mathbf{m_n}$ be a set of GSFNs, and $\mathbf{w} = (w_1, w_2, \cdots, w_n)$, $w_i \in [0, 1]$, with $\sum_{i=1}^{n} w_i$=1 be the set of weight vector for $\mathbf{m_i}$, then the aggregated value of the GSFNs obtained through GSF-DWAA also be a GSFN, which can be computed using the formula defined as follows,*

$$GSF-DWAA_w(\mathbf{m_1}, \mathbf{m_2}, \ldots, \mathbf{m_n}) = w_1\mathbf{m_1} \oplus w_2\mathbf{m_2} \oplus w_3\mathbf{m_3} \oplus \cdots \oplus w_n\mathbf{m_n}$$

$$= \left\langle \sqrt{1 - \frac{1}{1+\left\{\sum_{i=1}^{n} w_i\left(\frac{1-\eta_{m_i}}{\eta_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} w_i\left(\frac{1-\xi_{m_i}}{\xi_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}, \sqrt{\frac{1}{1+\left\{\sum_{i=1}^{n} w_i\left(\frac{1-\zeta_{m_i}}{\zeta_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}} \right\rangle$$

Where $\rho \geq 1$.

Proof. By mathematical induction, we can easily prove the Theorem. See the proof in [32]. □

Theorem 3.2. *Let $\mathbf{m_1}, \mathbf{m_2}, \ldots, \mathbf{m_n}$ be a set of GSFNs, and $\mathbf{w} = (w_1, w_2, \cdots, w_n)$, $w_i \in [0, 1]$, with $\sum_{i=1}^{n} w_i$=1 be the set of weight vector for $\mathbf{m_i}$, then the aggregated value of the GSFNs obtained through GSF-DWGA also be a GSFN, which can be computed using the formula defined as follows,*

$$GSF-DWGA_w(\mathbf{m_1},\mathbf{m_2},\ldots,\mathbf{m_n})=\mathbf{m_1}^{w_1}\otimes\mathbf{m_2}^{w_2}\otimes\mathbf{m_3}^{w_3}\otimes\cdots\otimes\mathbf{m_n}^{w_n}$$
$$=\left\langle\sqrt{\frac{1}{1+\left\{\sum_{i=1}^n w_i\left(\frac{1-\eta_{m_i}}{\eta_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}},\sqrt{\frac{1}{1+\left\{\sum_{i=1}^n w_i\left(\frac{1-\xi_{m_i}}{\xi_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}},\sqrt{1-\frac{1}{1+\left\{\sum_{i=1}^n w_i\left(\frac{1-\zeta_{m_i}}{\zeta_{m_i}}\right)^{2\rho}\right\}^{\frac{1}{\rho}}}}\right\rangle.$$

Where $\rho\geq 1$.

Proof. The proof is similar to that in [32]. □

3.3 The Score and Accuracy Function

The score function is a crucial tool in the fuzzy environment for measuring the degree of membership of a fuzzy number in a crisp value. The score value of fuzzy numbers can easily discriminate between fuzzy numbers in ascending or descending order. The score and the accuracy function assist in decision-making by evaluating the crisp values of fuzzy numbers in order to provide a structured approach to handling the uncertainty of fuzzy data. The accuracy function plays an important role in the defuzzification process; it provides accuracy values to fuzzy numbers. There might be many fuzzy numbers where the score function fails to make a proper comparison of those fuzzy numbers; then, the accuracy values should be calculated using the accuracy function to find the discrimination of the fuzzy numbers. Here, we define the score and the accuracy function as follows.
If $m=\langle\eta_m(c),\ \xi_m(c),\ \zeta_m(c)\rangle$ be a GSFN, then the score value and accuracy value of the fuzzy number in the crisp is denoted by the function $\Theta(m)$, and $\Omega(m)$, and is denoted by,

$$\Theta(m)=\frac{3\eta_m^2+\xi_m^2-4\zeta_m^2}{4}\ \in[-1,1];\quad\text{and }\Omega(m)=\frac{3\eta_m^2+\xi_m^2}{4}\ \in[0,1]. \tag{3}$$

Note that; $\mathbf{m_i}\leq\mathbf{m_j}$ if and only if
(*i*) $\Theta(m_i)\leq\Theta(m_j)$
(*ii*) $\Theta(m_i)=\Theta(m_j)$; and, $\Omega(m_i)\leq\Omega(m_j)$

We now verify both the functions as follows,
m_1=$\langle 1,1,0\rangle$, $\Theta(m_1)$=1; m_2=$\langle 0,0,1\rangle$, $\Theta(m_2)$)=-1;
m_3=$\langle 0.5,0.5,0\rangle$, $\Theta(m_3)$=0.25; m_4=$\langle 0,0,0.5\rangle$, $\Theta(m_4)$=-0.25
On the other hand, for m_5=$\langle 1,1,1\rangle$, m_6=$\langle 0.5,0.5,0.5\rangle$, and m_7=$\langle 0,0,0\rangle$, the score function provides the same value, which is 0, but the accuracy function provides $\Omega(m_5)$=1, $\Omega(m_6)$=0.25, and $\Omega(m_7)$=0, which can clearly arrange fuzzy numbers in ascending or descending order.

The main significance of this score function is to emphasize the role of indeterminacy (satisfaction to a certain extent) and falsity degrees equally, along with the truthiness degrees of the GSFNs. The score function $\Theta(m)$ provides the maximum score value in crisp to the fuzzy number if the number contains the maximum truthiness value along with maximum indeterminacy and minimum falsity value. On the other hand, $\Theta(m)$ provides the minimum score in crisp if the fuzzy number has the minimum truthiness value along with minimum indeterminacy and maximum falsity.

4 Extension of VIKOR MCDM Methodology Under the GSF Environment

The VIKOR method is a well-recognized MCDM methodology widely used in several fields of decision-making. The method was originally developed by Opricovic et al. [33] to manage decision-making problems with conflicting and non-commensurable criteria, assuming that compromise is acceptable for conflict resolution. The method provides a compromise solution that is closest to the ideal solution. Here, we define the VIKOR methodology step-by-step under the GSF environment as follows.

Step-1: Consider a set of alternatives $I_j(i = 1, 2, 3, \cdots, m)$, and a set of criteria or factors $F_i(i = 1, 2, 3, \cdots, n)$ of those alternatives, and then collect data through a group of decision experts in this field.

Step-2: Identify the appropriate linguistic variable for the importance weight of the criteria and for the ratings of the alternatives with respect to the criteria, and allocate appropriate GSF numbers for the values of linguistic variables and expert ratings are shown in Table 1 and Table 2.

Step-3: Initial decision matrix construction corresponding to criteria and alternatives, where decision-makers E_i assign the initial fuzzy decision matrices M_i. Combining all initial decision matrices, an aggregated decision matrix M has been formulated using GSF-DWAA (3.1) and GSF-DWGA(3.2) operators, also decision-maker's weights have been aggregated $\mathbf{w}$.

$$M = [g_{ij}]_{n\times m} = \begin{array}{c} \\ F_1 \\ F_2 \\ \vdots \\ F_n \end{array} \begin{array}{c} \begin{array}{cccc} \quad I_1 \quad & \quad I_2 \quad & \cdots & \quad I_m \quad \end{array} \\ \begin{pmatrix} \langle \eta_{11}, \xi_{11}, \zeta_{11} \rangle & \langle \eta_{12}, \xi_{12}, \zeta_{12} \rangle & \cdots & \langle \eta_{1m}, \xi_{1m}, \zeta_{1m} \rangle \\ \langle \eta_{21}, \xi_{21}, \zeta_{21} \rangle & \langle \eta_{22}, \xi_{22}, \zeta_{22} \rangle & \cdots & \langle \eta_{2m}, \xi_{2m}, \zeta_{2m} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \eta_{n1}, \xi_{n1}, \zeta_{n1} \rangle & \langle \eta_{n2}, \xi_{n2}, \zeta_{n2} \rangle & \cdots & \langle \eta_{nm}, \xi_{nm}, \zeta_{nm} \rangle \end{pmatrix} \end{array}$$

Where g_{ij} be the fuzzy ratings to the alternatives I_j corresponding to the criteria F_i, and $\mathbf{w} = (w_1, w_2, \cdots, w_n)$, $\forall\ w_i \in [0, 1]$; with $\sum_{i=1}^{n} w_i$=1 are weight vectors.

Step-4: Weighted normalized the fuzzy decision matrix calculated N by multiplying the weights of the decision-makers. The weighted normalized matrix $N = [\bar{m}_{ij}]_{n\times m}$, where $\bar{m}_{ij}$=$w_i \otimes m_{ij}$. For the normalization, the GSF-DWAA (3.1) and GSF-DWGA (3.2) operators have been used.

Step-5: Determine the fuzzy best value $\tilde{m}^+$ and fuzzy worst value $\bar{m}^-$ of each alternative based on the score value of the fuzzy numbers. Here, we have used the score and accuracy function [3] to calculate the score value in the crisp of the GSFNs, where the best score refers to the best fuzzy value and the worst score refers to the worst fuzzy value.

$$\bar{m}_i^+ = \max_j \{\bar{m}_{i1}, \bar{m}_{i2}, \cdots, \bar{m}_{im}\} \tag{4}$$

and,

$$\bar{m}_i^- = \min_j \{\bar{m}_{i1}, \bar{m}_{i2}, \cdots, \bar{m}_{im}\} \tag{5}$$

Step-6: Compute the value S_j, and R_j by the relations,

$$S_j = \sum_{i=1}^{n} w_i.\frac{d(\bar{m_i}^+ - \bar{m_{ij}})}{d(\bar{m_i}^+ - \bar{m_i}^-)} = \sum_{i=1}^{n}(w_i.D) \tag{6}$$

and,

$$R_j = \max_i \left(w_i.\frac{(d\bar{m_i}^+ - \bar{m_{ij}})}{d(\bar{m_i}^+ - \bar{m_i}^-)} \right) = \max_i (w_i.D) \tag{7}$$

Where D is a crisp value and $w_i.D = \left\langle 1 - \frac{1}{1+\left\{\lambda\left(\frac{\eta_{m_i}}{1-\eta_{m_i}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{\xi_{m_i}}{1-\xi_{m_i}}\right)^{\rho}\right\}^{\frac{1}{\rho}}}, \frac{1}{1+\left\{\lambda\left(\frac{\zeta_{m_i}}{1-\zeta_{m_i}}\right)^{\rho}\right\}^{\frac{1}{\rho}}} \right\rangle$;

with $\rho \geq 1$, and $\lambda \geq 0$.

The full fuzzy approach has been used here to measure the distance between two GSFNs, $d(\bar{m_i}^+ - \bar{m_{ij}})$ and $d(\bar{m_i}^- - \bar{m_{ij}})$. The Spherical distance approach has been utilized to find the distance as follows.

$$d(m_i - m_j) = \sqrt{3}cos^{-1}\left[1 - \frac{1}{2.\{\sqrt{3}\}^2}\{(\eta_{m_i} - \eta_{m_j})^2 + (\xi_{m_i} - \xi_{m_j})^2 + (\zeta_{m_i} - \zeta_{m_j})^2\}\right] \in \left[0, \frac{\sqrt{3}\pi}{2}\right] \quad (8)$$

Here, we obtain the distance value in the interval $\left[0, \frac{\sqrt{3}\pi}{2}\right]$. To map this into the fuzzy interval $[0, 1]$, we simply multiply the value by $\frac{2}{\sqrt{3}\pi}$.

Step-7: Calculate the values $\bar{S}^+$, $\bar{S}^-$, $\bar{R}^+$, $\bar{R}^-$, and Q_j.

$$\bar{S}^+ = \min_j S_j, \quad \bar{S}^- = \max_j S_j \quad (9)$$

$$\bar{R}^+ = \min_j R_j, \quad \bar{R}^+ = \max_j R_j \quad (10)$$

$$Q_j = v\frac{(S_j - \bar{S}^+)}{(\bar{S}^- - \bar{S}^+)} + (1-v)\frac{(R_j - \bar{R}^+)}{(\bar{R}^- - \bar{R}^+)} \quad (11)$$

Here, v is introduced as the weight of the strategy of the maximum group utility, usually v=0.5. The index $\min S_j$, and $\min R_j$ refers to a maximum majority rule and a minimum individual regret of an opponent strategy, respectively.

Step-8: The best alternative is with the minimum Q_j, and according to the values of Q_j we get the ranking list of the alternatives.

4.1 Algorithm for GSF-VIKOR MCDM Method

In this sub-section, we have presented a stepwise algorithm for the proposed GSF-VIKOR methodology. Additionally, a graphical flowchart is provided in Fig. 1 to clearly illustrate the logical flow of the method and facilitate easy understanding of the process from the first to the final step. Here, all numerical calculations for the MCDM problem have been done by MATLAB R2024a. The algorithm of the method has been given as follows:

Algorithm:

Step 1. Set $i \to n$, $j \to m$ for alternatives I_j and criteria F_i;
Step 2. Input M_i and w_i corresponding to E_i, get $M = [g_{ij}]$, and $\mathbf{w}$;
Step 3. Compute $N = [\bar{m}_{ij}]$, where $\bar{m}_{ij}$=$w_i \otimes m_{ij}$;
Step 4. Find $\bar{m}_i^+$ and $\bar{m}_i^-$;
Step 5. Calculate $S_j = \sum_{i=1}^{n} w_i.\frac{d(\bar{m_i}^+ - \bar{m_{ij}})}{d(\bar{m_i}^+ - \bar{m_i}^-)} = \sum_{i=1}^{n}(w_i.D)$, and using (7), (8)
$R_j = \max_i \left(w_i.\frac{(d\bar{m_i}^+ - \bar{m_{ij}})}{d(\bar{m_i}^+ - \bar{m_i}^-)}\right) = \max_i(w_i.D)$;
Step 6. Find $\bar{S}^+ = \min_j S_j$, $\bar{S}^- = \max_j S_j$, and $\bar{R}^+ = \min_j R_j$, $\bar{R}^+ = \max_j R_j$;

Step 7. Calculate $Q_j = v\frac{(S_j - \bar{S}^+)}{(\bar{S}^- - \bar{S}^+)} + (1-v)\frac{(R_j - \bar{R}^+)}{(\bar{R}^- - \bar{R}^+)}$, v=0.5;
Step 8. Final rank based on Q_j;

MCDM Methodology - visual.png MCDM Methodology - visual.bb

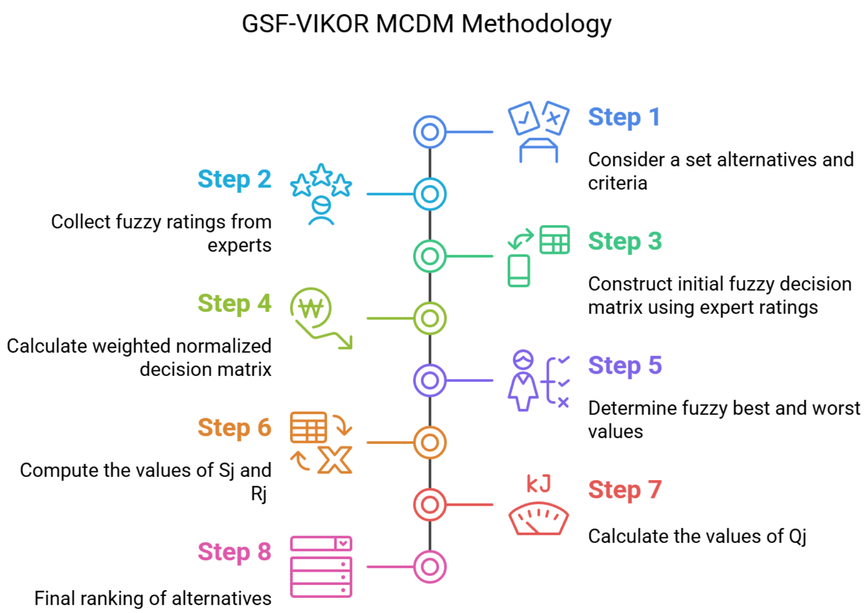

Figure 1: Flowchart for GSF-VIKOR technique

5 Optimum Breeder Seed Selection in the Agriculture Sector Using the GSF-VIKOR Method

Agricultural breeder crop seed selection is essential to meet global food demand, production, environmental sustainability, and economic development. High-yielding foundation seed selection is the most critical input for sustainable agriculture. Foundation Seeds serve as the foundation for agricultural systems, influencing factors such as crop yield, quality, resilience to environmental stresses, and, ultimately, the economic viability of farming operations. Hybrid crop production maximizes productivity, ensuring food security and supporting rural livelihoods. High-quality breeder seeds can lead to higher crop yields, improved product quality, and increased market competitiveness. Additionally, selecting seeds with specific traits tailored to market demand can enhance a farmer's income and profitability. The selection of the best crop seeds depends on multiple factors called required inputs, such as water, fertilizers, pesticides, quality, productivity, environmental sustainability, etc. Furthermore, seeds with traits such as drought tolerance, nitrogen fixation, and pest resistance promote sustainable farming practices and minimize negative impacts on soil and water resources. The selection of foundation crop seeds adapted to local climatic conditions, soil types, and farming practices is crucial for optimizing crop performance and resource utilization. Locally adapted seeds have the potential to thrive in specific zones, reducing the need for external inputs such as fertilizers and pesticides, and have maximum tolerance levels in adaptation to adverse soil and water conditions.

Day by day, the demand for certified foundation seed practices is increasing in the agriculture sector. Cultivators generally allocate a budget for high-quality certified foundation seeds for cultivation. Certified seeds offer higher yields, better market value, and greater adaptability to the local environment. A certified seed is the progeny of a good-quality crop seed that meets the standards of seed certification in the state or the country. High-yielding foundation seeds must meet the seed certification standards outlined in the country's minimum seed certification standards, both at the field and laboratory testing institutes. For example, the Indian Council of Agricultural Research (ICAR) is responsible for overseeing the production of certified breeder seeds. Also, the National Research Centers and All India Coordinated Research Projects of various crops, as well as State Agricultural Universities (SAUs) centers, are working on the certification process. These research institutes usually collect the data through experts from the testing field for the external factors of the crops, like soil, water, sunlight, fertilizer, and other environment-related growing conditions. Additionally, laboratory testing examines other influencing factors, like genetic purity, seed nutrient value, physical strength, and resistance to pests. Finally, after analyzing all the external and internal factors of the crops, decide on the certification label. Here, we discussed some important internal and external factors for certified high-yielding foundation crop seed selection in the agriculture sector,

1. Genetic purity:

Genetic purity is essential for meeting market demand, quality, appearance, taste, and nutritional value. Consistent and pure traits are more marketable and enhance consumer confidence. Genetic purity is important for seed certification programs that ensure seed quality and uniformity. Genetic purity contributes to the conservation of crop genetic diversity by preserving unique and valuable genetic resources within crop varieties. Conserving genetic diversity is essential for the long-term resilience of agricultural systems to environmental stresses and changing production conditions. It serves as the foundation for sustainable and productive agriculture by providing farmers with reliable and high-quality seeds for successful crop production.

2. Adaptation to local climate and growing conditions:

For optimal crop production, the crop variety must meet local climate and growing conditions, including water, nutrients, and sunlight. This ensures that farmers can utilize resources more efficiently while minimizing input costs. Where locally adapted crops need limited irrigation water or fertilizer inputs. Using local climate and growing conditions, farmers can reduce the risk of crop failure due to adverse weather events, pest and disease outbreaks, or other environmental stresses. Also, these crops are more resilient and better equipped to withstand fluctuations in temperature, rainfall, and other climatic factors. Selecting and cultivating locally adapted crops, farmers can enhance the productivity, resilience, and sustainability of agricultural systems while preserving cultural heritage and culinary diversity.

3. Tolerance to adverse soil conditions:

Tolerant crop seeds maintain better growth and productivity under adverse soil conditions, leading to crop performance and yield stability. Tolerant crop seeds promote environmentally sustainable agricultural practices by reducing the need for soil amendments and chemical inputs. These adaptations contribute to soil conservation, biodiversity preservation, and reduced environmental pollution associated with fertilizer and pesticide use. Tolerant crop seeds become increasingly important for ensuring food security and resilience in agricultural systems in an adverse climate. So, tolerance to adverse soil conditions for a crop variety is crucial for enhancing resilience, expanding agricultural land use, mitigating soil degradation, reducing input costs, improving crop performance, promoting environmental sustainability, and adapting to climate change.

4. Resistance to insects, pests, and diseases: Resistance in the inherent crop seeds helps protect plants from damage caused by pests, insects, and diseases. Resistance crop varieties contribute to higher overall productivity and improved farm profitability by reducing yield losses due to pest and disease infestations. This leads to decreased pesticide application, lower input costs, and reduced environmental pollution, promoting sustainable and environmentally friendly agricultural practices. Resistance from pests, insects, and diseases in agricultural crop seeds is crucial for protecting crop yield and quality, reducing reliance on chemical pesticides, promoting crop health and vigor, ensuring long-term sustainability, supporting food security, and adapting to climate change.

5. Market value:

The market value of crop seeds directly impacts the economic viability of an agricultural system. High-market-value seeds can command better prices in the market, leading to increased farm income and financial stability for farmers. Crop seeds with higher market value enhance the competitiveness of agricultural products in domestic and international markets. Crop variety with high market value strengthens the position of farmers in the market and supports agricultural trade and exports. A strong market value for crop seeds supports the growth and development of the agricultural seed industry, including seed production, distribution, and marketing.

Decision-making:

Here, we have shown how a fuzzy MCDM tool can help to select the best quality certified breeder seed in agriculture. The proposed GSF-VIKOR MCDM technique has been implemented for the selection process. Here we have considered all the important factors as criteria which are defined above, and consider ten different varieties of paddy seeds as alternatives. Five decision experts were assigned to collect data from the field in their respective zones. The step-by-step decision-making process using the GSF-VIKOR methodology is described as follows.

Step-1: We have chosen ten different types, but the same category paddy seeds variety as alternatives $Ps_1, Ps_2, Ps_3, Ps_4, Ps_5, Ps_6, Ps_7, Ps_8, Ps_9, Ps_{10}$ and five important factors for high-yielding crop considered as a set of decision criteria,

F_1: Genetic purity.

F_2: Adaptation to local climate and growing conditions.

F_3: Tolerance to adverse soil conditions.

F_4: Resistance to insects, pests, and diseases.

F_5: Market value.

Step-2: The fuzzy linguistic terms for the alternatives and for the rating of the criteria are identified, and corresponding to those linguistic variables, the GSF fuzzy numbers are identified as shown in Table 1.

Step-3: Fuzzy opinions collected through five assigned decision experts E_1, E_2, E_3, E_4, and E_5, where the experience levels of the five experts are $0.15, 0.25, 0.20, 0.22$, and 0.18, respectively, which are the weight vector corresponding to the rating expert. According to the values of the linguistic variables and using the expert rating, the initial decision matrices according to the decision expert are constructed as shown in Table 3 - 7.

Step-4: The aggregated decision matrix has been calculated in Table 10 and aggregated criteria weights as in Table 9. The aggregation has been done using the GSF-Dombi aggregation operator in Theorem 3.1 (GSF-DWAA operator).

Step-5: We have identified the fuzzy best value and the worst value according to the score value of the GSFNs using the score function as in Equation 3. The best value and the worst value with respect to the criteria have been chosen and shown in Table 11.

Table 1: Linguistic variables for the rating of the alternatives corresponding to the criteria

Linguistic variables	G-SFNs
Absolutely Good(AG)	$\langle 0.9, 0.9, 0.1\rangle$
Very Good(VG)	$\langle 0.8, 0.8, 0.2\rangle$
Pretty Good(PG)	$\langle 0.7, 0.7, 0.3\rangle$
Slightly Good (SG)	$\langle 0.6, 0.6, 0.4\rangle$
Moderately Good(MG)	$\langle 0.5, 0.5, 0.5\rangle$
Fair(F)	$\langle 0.4, 0.6, 0.4\rangle$
Slightly Poor(SP)	$\langle 0.3, 0.7, 0.3\rangle$
Poor(P)	$\langle 0.2, 0.8, 0.2\rangle$
Worst(W)	$\langle 0.1, 0.9, 0.1\rangle$

Table 2: Linguistic variables for the weights of the criteria

Linguistic variables	G-SFNs
Absolutely High Importance(AHI)	$\langle 0.9, 0.9, 0.1\rangle$
Very High Importance(VHI)	$\langle 0.8, 0.8, 0.2\rangle$
High Importance(HI)	$\langle 0.7, 0.7, 0.3\rangle$
Medium Importance(MI)	$\langle 0.5, 0.5, 0.5\rangle$
Low Importance(LI)	$\langle 0.4, 0.7, 0.4\rangle$
Very Low Importance(VLI)	$\langle 0.3, 0.5, 0.3\rangle$

Table 3: Decision matrix M_1 concluded by expert E_1

Criteria	Alternatives									
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	P	SP	AG	AG	VG	PG	SG	PG	MG	MG
F_2	SP	AG	MG	PG	SP	VG	AG	PG	SG	F
F_3	F	MG	VG	PG	AG	SP	VG	VG	MG	MG
F_4	MG	AG	VG	PG	MG	SP	SP	MG	SG	SG
F_5	VG	VG	MG	MG	SG	PG	AG	PG	VG	AG

Step-6: At this step, we have determined the values of S_j and R_j using Equation 6 and 7 for each alternative shown in Table 12. We determined the value of S_j and R_j of each alternative using the spherical distance measuring approach through a full fuzzy approach by Equation 8.

Step-7: In this step, we identified S^+, S^-, R^+ and R^- using the score function as in Equation 3, and the values are shown in Table 13.

Step-8: Finally, we calculate the value Q_j and get the ranking order of the alternatives as in Table 14. The minimum value of Q_j refers to the best rank, and the maximum value of Q_j refers to the worst position in the ranking list. The obtained final Ranking order for the alternatives is $Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$.

Table 4: Decision matrix M_2 concluded by expert E_2

Criteria	Alternatives									
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	VG	PG	SG	MG	F	SP	P	AG	SG	AG
F_2	PG	SG	VG	F	AG	PG	SG	MG	PG	SG
F_3	VG	AG	MG	F	SG	PG	PG	SP	MG	P
F_4	SP	MG	MG	W	MG	SG	PG	SG	F	AG
F_5	PG	PG	PG	SG	W	SP	MG	AG	VG	F

Table 5: Decision matrix M_3 concluded by expert E_3

Criteria	Alternatives									
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	MG	AG	AG	MG	VG	VG	PG	AG	MG	AG
F_1	SG	MG	AG	MG	VG	AG	PG	AG	MG	VG
F_1	SG	VG	VG	SG	VG	PG	SG	PG	MG	VG
F_1	PG	F	PG	SG	VG	PG	SG	PG	SG	VG
F_1	VG	F	PG	SP	PG	VG	VG	PG	PG	MG

Table 6: Decision matrix M_4 concluded by expert E_4

Criteria	Alternatives									
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	AG	AG	AG	SG	SG	PG	PG	MG	MG	AG
F_1	VG	MG	SG	MG	PG	VG	F	AG	VG	MG
F_1	VG	MG	SG	AG	PG	SG	MG	AG	PG	MG
F_1	VG	SG	SP	PG	PG	PG	MG	VG	SG	VG
F_1	VG	SG	F	PG	AG	PG	MG	VG	MG	F

6 Data Sensitivity Analysis of the GSF-VIKOR

The main purpose of sensitivity analysis is to observe the data sensitivity, how a certain change in input data affects the final result or output of the model. Our target of sensitivity analysis for the method is to check and analyze the different ranking orders as the output of the method for the different input values of the decision-maker weights. The changes in the weights of the criteria by the decision-maker can make a major impact on the final alternative ranking. In Fig. 2, we have shown the ranking order of the alternatives for ten different cases, for different weights of the criteria for the alternatives and criteria. The bar diagram displays the alternative ranking for the input of different criteria weights, and it has been observed that for most of the cases, the position of the alternatives in the ranking list remains unaltered, but in a few cases, minor ranking position alterations happened. Additionally, in Table 15, we have displayed the ranking of the alternatives for different cases for different weight vectors of decision-maker ratings. Throughout the sensitivity analysis, it has been noticed that the proposed technique efficiently provides alternative rankings for Decision-making.

6.1 A Comparative Study of GSF-VIKOR with other Methodologies

In this section, a comparative study has been conducted in the GSF environment to verify the efficiency and consistency of the GSF-VIKOR technique. The advantages and data handling accuracy of our proposed GSF-VIKOR technique have been examined comparatively with the TOPSIS (Technique for the Order Preference by the Similarity to the Ideal Solution) methodology, applying different aggregation operators. In this analysis, we calculate the ranking list for the chosen MCDM problem using the Dombi aggregation operator, as well as the GSF aggregation operator. After finding the ranking list for both MCDM techniques, it has been noticed that for different operators, GSF-VIKOR provides a consistent and stable ranking order compared to TOPSIS, as shown in Table 16. Moreover, by the experiment, it is noticed that the GSF-VIKOR MCDM method is more efficient in providing a complete ranking list and handling a large set of complex data associated with real-life decision-making.

Table 7: Decision matrix M_5 concluded by expert E_5

Criteria	Alternatives									
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	SP	VG	AG	SP	VG	MG	VG	SG	SG	VG
F_1	VG	VG	PG	SG	PG	MG	VG	SG	MG	VG
F_1	VG	PG	PG	MG	PG	VG	PG	MG	F	PG
F_1	MG	PG	SG	VG	SG	PG	VG	MG	PG	SG
F_1	SP	VG	W	MG	SG	SG	VG	MG	SG	SG

Table 8: Weights according to experts to the criteria

Criteria	Expert's ratings				
	E_1	E_2	E_3	E_4	E_5
F_1	AHI	AHI	MI	VHI	MI
F_1	AHI	VHI	AHI	VHI	AHI
F_1	AHI	VHI	AHI	VHI	AHI
F_1	HI	AHI	VHI	VHI	AHI
F_1	AHI	AHI	VHI	MI	AHI

Table 9: Aggregated criteria weights using GSF-DWAA operator

Criteria	Criteria weights
F_1	$\langle 0.851287, 0.524700, 0.879754\rangle$
F_2	$\langle 0.905347, 0.424672, 0.985121\rangle$
F_3	$\langle 0.905347, 0.424672, 0.985121\rangle$
F_4	$\langle 0.869410, 0.494091, 0.956717\rangle$
F_5	$\langle 0.885926, 0.463827, 0.904269\rangle$

Table 10: Aggregated decision matrix using GSF-DWAA operator

Criteria	For alternatives $Ps_1 Ps_5$				
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5
F_1	$\langle 0.869624, 0.482141, 0.904520\rangle$	$\langle 0.896770, 0.441064, 0.956778\rangle$	$\langle 0.916325, 0.400436, 0.938542\rangle$	$\langle 0.839467, 0.538042, 0.862034\rangle$	$\langle 0.807410, 0.586225, 0.916856\rangle$
F_2	$\langle 0.799057, 0.593168, 0.930874\rangle$	$\langle 0.850544, 0.525904, 0.865531\rangle$	$\langle 0.868106, 0.496378, 0.901014\rangle$	$\langle 0.644029, 0.745049, 0.858431\rangle$	$\langle 0.876975, 0.478284, 0.950331\rangle$
F_3	$\langle 0.820529, 0.569720, 0.925968\rangle$	$\langle 0.875453, 0.483303, 0.879058\rangle$	$\langle 0.853874, 0.520480, 0.885526\rangle$	$\langle 0.862634, 0.504285, 0.885610\rangle$	$\langle 0.856730, 0.515765, 0.927130\rangle$
F_4	$\langle 0.758623, 0.630586, 0.881160\rangle$	$\langle 0.842614, 0.536748, 0.877292\rangle$	$\langle 0.842628, 0.532301, 0.884837\rangle$	$\langle 0.763253, 0.481022, 0.932604\rangle$	$\langle 0.760142, 0.649757, 0.868080\rangle$
F_5	$\langle 0.817642, 0.568249, 0.952624\rangle$	$\langle 0.788491, 0.611158, 0.916282\rangle$	$\langle 0.695733, 0.516602, 0.897623\rangle$	$\langle 0.666900, 0.706495, 0.871797\rangle$	$\langle 0.863387, 0.438418, 0.925611\rangle$

Criteria	For alternatives $Ps_6 Ps_{10}$				
	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
F_1	$\langle 0.765057, 0.624405, 0.901731\rangle$	$\langle 0.765722, 0.587799, 0.935149\rangle$	$\langle 0.896582, 0.442877, 0.892678\rangle$	$\langle 0.622143, 0.782903, 0.845962\rangle$	$\langle 0.913658, 0.406483, 0.918767\rangle$
F_2	$\langle 0.872207, 0.489137, 0.907226\rangle$	$\langle 0.852986, 0.520308, 0.914147\rangle$	$\langle 0.893615, 0.448834, 0.888178\rangle$	$\langle 0.766605, 0.642119, 0.871122\rangle$	$\langle 0.786206, 0.614947, 0.883372\rangle$
F_3	$\langle 0.769616, 0.626931, 0.925171\rangle$	$\langle 0.762014, 0.647561, 0.887723\rangle$	$\langle 0.868130, 0.491889, 0.904545\rangle$	$\langle 0.652985, 0.743636, 0.848739\rangle$	$\langle 0.751305, 0.597053, 0.878588\rangle$
F_4	$\langle 0.727826, 0.667837, 0.920129\rangle$	$\langle 0.755926, 0.641186, 0.888038\rangle$	$\langle 0.765025, 0.644001, 0.873677\rangle$	$\langle 0.676384, 0.721196, 0.894054\rangle$	$\langle 0.880701, 0.473673, 0.928658\rangle$
F_5	$\langle 0.767936, 0.621572, 0.928122\rangle$	$\langle 0.859242, 0.511569, 0.868659\rangle$	$\langle 0.877298, 0.479945, 0.905983\rangle$	$\langle 0.797675, 0.603087, 0.891108\rangle$	$\langle 0.838917, 0.539897, 0.876141\rangle$

6.2 Advantages and Limitations of GSF-VIKOR Method

The advantages and disadvantages of the GSF-VIKOR are highlighted from the different perspectives of the decision-making process. For real-life decision-making, large-scale complex data handling ability, fuzzy decision-making efficiency, and other positive and negative impacts of the method are discussed below.

Advantages:

The proposed MCDM method offers a systematic and transparent approach for handling the complexity and uncertainty involved with the real-life decision-making problem. The methodology allows multiple decision-making criteria for the overall assessment of the alternatives, and ensures a complete ranking list. The method can easily handle a large set of alternatives as well as decision factors, and is highly efficient in managing large datasets. Additionally, the flexibility of the methods enables diverse applications in the decision-making field, including agriculture management, industry, and environmental management.

Limitations:

Table 11: Fuzzy best values and fuzzy worst values of the alternatives according to the score function

Criteria	Best Value	Worst Value
F_1	$\langle 0.839467, 0.538042, 0.862034 \rangle$	$\langle 0.765722, 0.587799, 0.935149 \rangle$
F_2	$\langle 0.850544, 0.525904, 0.865531 \rangle$	$\langle 0.799057, 0.593168, 0.930874 \rangle$
F_3	$\langle 0.875453, 0.483303, 0.879058 \rangle$	$\langle 0.769616, 0.626931, 0.925171 \rangle$
F_4	$\langle 0.842614, 0.536748, 0.877292 \rangle$	$\langle 0.763253, 0.481022, 0.932604 \rangle$
F_5	$\langle 0.859242, 0.511569, 0.868659 \rangle$	$\langle 0.695733, 0.516602, 0.897623 \rangle$

Table 12: The values of S_j and R_j calculated by a full fuzzy approach using fuzzy spherical distance measure technique

	For alternatives $Ps_1 Ps_5$				
	Ps_1	Ps_2	Ps_3	Ps_4	Ps_5
S_j	$\langle 0.952007, 0.248749, 0.782288 \rangle$	$\langle 1, 0.319436, 0.751631 \rangle$	$\langle 0.971018, 0.272248, 0.735124 \rangle$	$\langle 1, 0.222238, 0.787900 \rangle$	$\langle 0.958209, 0.256318, 0.783641 \rangle$
R_j	$\langle 0.551740, 0.497991, 0.865021 \rangle$	$\langle 0.614735, 0.438153, 0.825636 \rangle$	$\langle 0.631348, 0.422867, 0.813921 \rangle$	$\langle 0.576529, 0.362107, 0.843668 \rangle$	$\langle 0.555800, 0.494036, 0.862703 \rangle$

	For alternatives $Ps_6 Ps_{10}$				
	Ps_6	Ps_7	Ps_8	Ps_9	Ps_{10}
S_j	$\langle 0.949692, 0.239720, 0.750162 \rangle$	$\langle 1, 0.256411, 0.784848 \rangle$	$\langle 0.967785, 0.265888, 0.774005 \rangle$	$\langle 0.937584, 0.210439, 0.691164 \rangle$	$\langle 0.957485, 0.245630, 0.756937 \rangle$
R_j	$\langle 0.595713, 0.455893, 0.838334 \rangle$	$\langle 0.591179, 0.460162, 0.841254 \rangle$	$\langle 0.591170, 0.460170, 0.841259 \rangle$	$\langle 0.689660, 0.370503, 0.767578 \rangle$	$\langle 0.623502, 0.430063, 0.819529 \rangle$

ranking.png ranking.bb

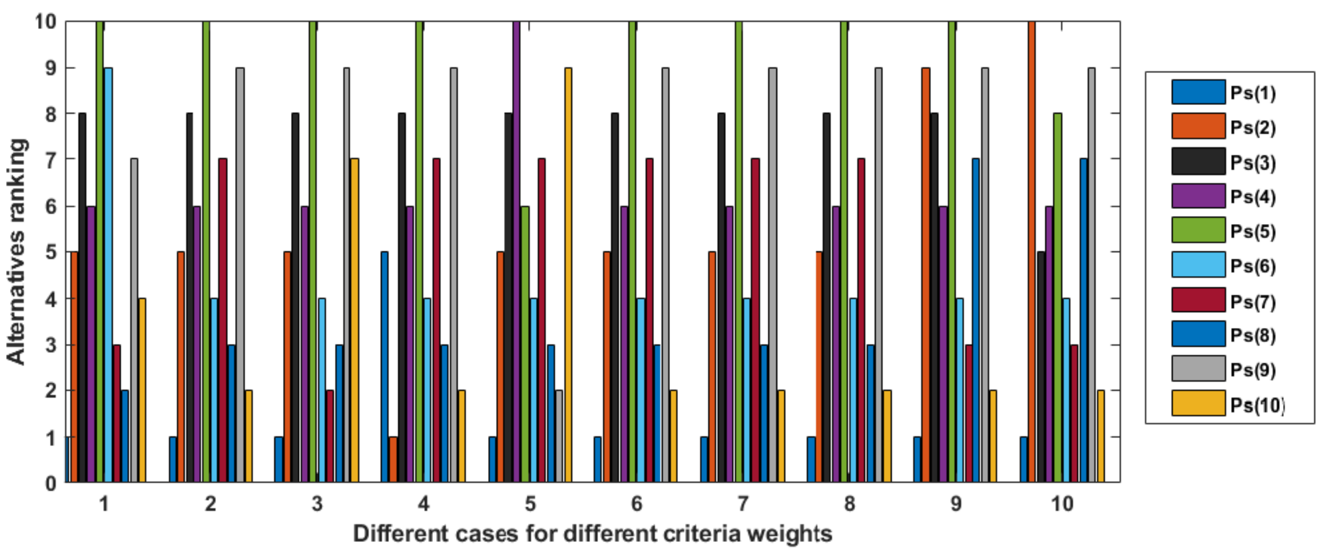

Figure 2: Ranking of the alternatives for different cases for different input criteria weights

However, the method also has some limitations. The technique is quite lengthy and requires considerable time to get final results, as it involves several complex steps. Due to the multiple steps involved, handling data manually with pen and paper is difficult; advanced data processing software is necessary.

Table 13: Values of S^+, S^-, R^+ and R^-

	S^+	S^-	R^+	R^-
Fuzzy value	$\langle 0.952007, 0.248749, 0.782288\rangle$	$\langle 1, 0.319436, 0.751631\rangle$	$\langle 0.551740, 0.497991, 0.865021\rangle$	$\langle 0.689660, 0.370503, 0.767578\rangle$
Score value	0.083231639	0.210561388	-0.457948896	-0.198135572

Table 14: Alternatives ranking based on the values of S, R, and Q

Alternatives	S	R	Q	Ranking based on S	Ranking based on R	Ranking based on Q
Ps_1	0.083231639	-0.457948896	0	1	1	1
Ps_2	0.210561388	-0.350256764	0.292750945	10	7	10
Ps_3	0.185279461	-0.318812956	0.132961259	8	9	8
Ps_4	0.141560304	-0.429706276	0.174693953	6	3	6
Ps_5	0.090953789	-0.451553107	0.018015001	2	2	2
Ps_6	0.128060240	-0.384689617	0.035049040	4	7	4
Ps_7	0.150450957	-0.392650771	0.138294091	7	5	7
Ps_8	0.121046704	-0.392666436	0.022859260	3	4	3
Ps_9	0.192661756	-0.122135572	0.070526184	9	10	9
Ps_{10}	0.129713809	-0.113822788	0.015891021	5	8	5

Table 15: Ranking of the alternatives for different input weights to the criteria

Case	Aggregated weights of the decision-makers	Ranking order of the alternatives
1	(0.20, 0.10, 0.20, 0.25, 0.25)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$
2	(0.20, 0.20, 0.20, 0.20, 0.20)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$
3	(0.15, 0.15, 0.25, 0.25, 0.20)	$Ps_5 > Ps_1 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_2 > Ps_3 > Ps_9 > Ps_7$
4	(0.30, 0.10, 0.30, 0.15, 0.15)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$
5	(0.10, 0.15, 0.30, 0.15, 0.30)	$Ps_1 > Ps_5 > Ps_8 > Ps_{10} > Ps_6 > Ps_4 > Ps_7 > Ps_3 > Ps_2 > Ps_9$
6	(0.25, 0.20, 0.15, 0.20, 0.20)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$
7	(0.15, 0.15, 0.30, 0.30, 0.10)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$
8	(0.35, 0.15, 0.15, 0.15, 0.20)	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$
9	(0.27, 0.13, 0.15, 0.20, 0.15)	$Ps_1 > Ps_5 > Ps_6 > Ps_8 > Ps_{10} > Ps_4 > Ps_3 > Ps_7 > Ps_9 > Ps_2$
10	(0.18, 0.26, 0.22, 0.20, 0.14)	$Ps_1 > Ps_{10} > Ps_5 > Ps_6 > Ps_8 > Ps_4 > Ps_7 > Ps_3 > Ps_9 > Ps_2$

Table 16: Comparative analysis between GSF-VIKOR and TOPSIS method

Comparative analysis using different operators		
Methodology	Operator used	Ranking order
GSF-VIKOR	GSF-DWAM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$
	GSF-DWGM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$
GSF-VIKOR	GSF-WAM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$
	GSF-WGM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$
TOPSIS	GSF-DWAM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_4 > Ps_2 > Ps_7 > Ps_3$
	GSF-DWGM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_3 > Ps_9 > Ps_{10} > Ps_2 > Ps_7 > Ps_4$
TOPSIS	GSF-WAM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_4 > Ps_9 > Ps_{10} > Ps_2 > Ps_7 > Ps_3$
	GSF-WGM	$Ps_1 > Ps_5 > Ps_8 > Ps_6 > Ps_{10} > Ps_9 > Ps_3 > Ps_2 > Ps_7 > Ps_4$

7 Conclusion

In this study, we have explored the extension of the VIKOR MCDM technique within the framework of Generalized Spherical Fuzzy (GSF) sets and its application in agricultural decision-making. It has been observed that the demonstrated GSF-VIKOR method offers a robust decision-making framework when dealing with the inherent vagueness and uncertainty in agricultural systems, such as fluctuating weather conditions, variable soil fertility, market price instability, and unpredictable pest and disease outbreaks. We have established that the proposed technique is more reliable for incorporating all possible important objectives, such as seed quality, genetic purity, yield potential, disease resistance capability, local adaptability, market demand, and other possible factors for selecting the best crop in agriculture. The incorporated score and accuracy function with the technique accurately support the identification of the fuzzy best value and worst value of GSFNs for decision-making. Additionally, we can conclude that the GSF-DWAA and GSF-DWGA operators, as well as GSF-WAM, GSF-WGM aggregation operators, and the spherical distance measuring approach are fruitfully utilized, which makes the technique more realistic and reliable in the decision-making field. In the sensitivity analysis, we evaluated the data sensitivity of the proposed technique and observed a high level of precision and accuracy in generating the ranking of the considered alternatives. The comparison analysis indicates that GSF-VIKOR offers a more consistent and discriminative ranking compared to TOPSIS, handling real-world uncertain data. Finally, it can be concluded that several future research directions may be proposed based on this study. Promising areas for further exploration include enhancing the application and effectiveness of the GSF-VIKOR MCDM technique in decision-making under uncertain environments. These directions involve combining GSF-VIKOR with machine learning algorithms and developing mobile-based applications that integrate the GSF-VIKOR technique to assist agricultural planners, policymakers, and farmers. Furthermore, the technique has the potential for widespread application in real-life decision-making problems and can be extended to various sectors such as business and marketing management, healthcare, public policy management, education, supply chain management, energy resource allocation, urban planning, project selection, financial investment analysis, and other related fields.

Conflict of Interest: "All co-authors have read this manuscript and declared permission to submit it to your journal. All authors declare that there are no conflicts of interest."

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