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Research Paper

# Effects of Material Properties and Dimensions on Buckling Behavior of Thin-Walled Cylindrical Shell Under Hydrostatic Pressure and Axial Force Using an Abaqus Developed Plugin

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**ABSTRACT**

Underwater thin-walled shells are widely used for underwater robots and are prone to buckling due to external hydrostatic pressure. These shells are produced by welding two circular plates to a cylinder. Various materials can be selected for shell fabrication in various sizes. The current study aims to study the effect of material properties and design parameters on the buckling behavior of cylindrical thin-walled shells. To this end, a plugin was developed in Abaqus to simulate a cylindrical shell according to defined properties and geometries. The plugin enables the operator to model a cylinder and pressurizing process with inserting the dimensions. The modeling process automatically is done by the codes. In this step, cylindrical shells with various material properties and various cylinder geometries were modeled, and the pressurizing process was simulated using the Riks method. Results showed that the buckling resistance directly increases by yield stress increase, and ultimate stress, yield strain, and plastic strain do not affect the buckling behavior. The study on geometrical parameters showed that the shell length in this case study does not change significantly the critical pressure. However, by adding the shell diameter, the buckling resistance decreases. On the contrary, the higher thickness led to higher resistance. Finally, modeling reliability is validated by an experimental approach. Here, a few cylindrical shells were manufactured and tested. Differences between results were around 5 percent, meaning the model is acceptable and reliable for future design by adding a safety factor.

***Keywords:*** Cylindrical shells; Thin-walled structures; Hydrostatic pressure; Buckling.

## 1 INTRODUCTION

CYLINDRICAL designs are employed to fabricate underwater vehicles, robots, and other devices due to their simplicity in design and fabrication [1, 2]. These structures are always under hydrostatic pressures, and buckling

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is a common failure mode. Hence, achieving an explicit knowledge of practical design parameters and reaching higher buckling resistances is essential.

Many efforts have been made to evaluate the buckling resistance of shells. In this field, Donner's studies led to a simple formula for critical buckling in a cylinder under torsion and axial force [3]. The buckling behavior of cylindrical shells under lateral pressure and various boundary conditions was also theoretically studied by Vodenitcharova et al. [4]. There have also been various studies on shell thickness as a geometrical effective parameter [5, 6]. The thickness effect on buckling was first studied by Koiter [7] in cylindrical shells under axial loads. Their result showed that a slight change in thickness can exponentially change the buckling resistance. Gusic [8] et al. studied the influence of circumferential thickness variations using FE bifurcation analysis. Non-symmetric thickness variation in the laterally pressurized cylindrical shell was also studied analytically by Yang [9] et.al. In Xue's study [10], the thickness to Radius ratio was employed to investigate the geometrical effects on the buckling behavior of a long cylinder. Further to these studies, there were other studies in which the effect of stepwise wall cylindrical shells was employed as the study subject [11, 12]. In stepwise designed shells, which are constantly employed for tanks and silos and are subjected to axial forces, the shell thickness discontinuously changes along.

Steel is mainly used to fabricate shells; various materials were also studied to achieve higher buckling resistance. For example, Guan [13] et al. numerically and experimentally studied the cylindrical shells' buckling behavior made from carbon fiber reinforcement polymers under external pressures. In some cases, the shell is made from steel and is covered by composite materials for reinforcement. However, this method is time-consuming, more effortful, and expensive. Cylindrical shells made from temperature-dependent material were another case studied [14]. In this case, thermal buckling behavior was focused, and results showed that these materials have a negative effect on shell collapse. Studies on functionally graded materials (FGM) in this field have recently focused. Mohammadzadeh [15] et al. studied cylindrical shells made from 2D-FG material. They investigated the effect of mechanical properties variations in the radial and the axial direction on the buckling behavior of the shell. Variation of FG material properties along the thickness was focused on by Khazaeinejad [16] et al., resulting in the weakness of FGM material under buckling compared to the homogenous materials. Variation of material properties in thickness direction in homogeneous material was also analytically investigated, suggesting a formula for buckling evaluation [17].

Underwater cylindrical shells are designed and manufactured in various sizes with various materials for particular purposes. Most of the studies were done to evaluate buckling in designed shells and safety issues, and some were done to study various influential parameters in buckling behavior for future design [18, 19, 20]. A few research in the second category focused on buckling under lateral pressures or axial force. Depending on purposes, cylinders in various geometries and sizes were focused. Nevertheless, shells are designed and manufactured for various purposes, and for each one, there should be an explicit knowledge of buckling behavior.

The main aim of the present study is to reach a light path for material selection and shell size to design an underwater robot with an imaging mission. The cylindrical geometry is the best for this purpose due to its simple design and manufacturing. However, the material selection and the determination of a suitable size are two topics that should be considered for future design. The present study aims to investigate some geometrical parameters and material effects and their correlation to the buckling behavior of cylindrical shells when designing underwater robots.

## 2 THEORY AND METHOD SHELLS

Cylindrical shells are widely used for various purposes due to their simple design and manufacturing processes. To manufacture these structures, a thin pipe is cut, and then two circular plates are welded to its two ends. Fig.1(a) shows the cylindrical shell schematics studied in this paper. In the image, the $L$ is the pipe length, $D$ is the pipe diameter, and the $t$ is its thickness.

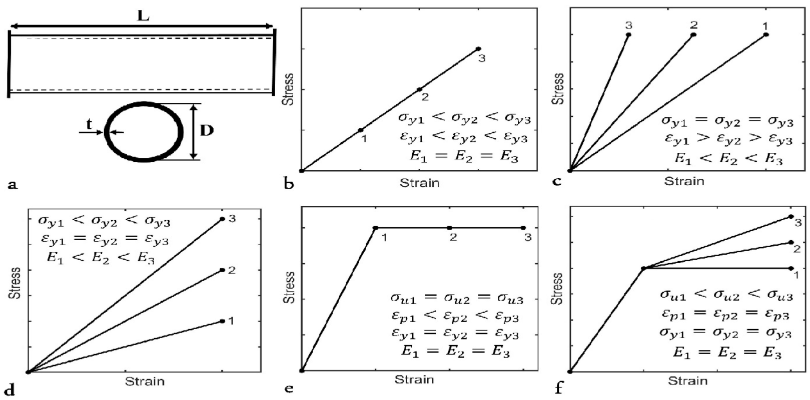


**Fig. 1**
a) Schematic of a cylindrical shell and geometrical parameters. b) Materials' stress-strain curves with different yield stress. c) Materials' curves with different elasticity modulus'. d) Materials' curves with various elasticity modulus and the same yield strain. e) Materials' curves with the same stress yields and various plastic strains. f) Materials' curves with similar yield stresses and plastic strains and various ultimate stresses.

## 3 NUMERICAL SIMULATION

Abaqus software is employed for simulation, and a plugin was developed to accelerate the simulation process. Various cylinders with various shell diameters, thicknesses, and lengths can be modeled using the plugin. Moreover, various materials with various elasticity modulus, yield stress, ultimate stress, yield strain, and plastic strain can be applied. The plugin, then, starts the process by creating a model in Abaqus according to material properties and geometries defined by the operator. Then, the software employes the Riks method to solve the problem. The modified Riks, or the arc length method, is a method that is used to analyze the instability of structures. The method is a powerful numerical technique for solving nonlinear equations. In this method, nonlinear Cauchy strains are used. Also, in addition to nonlinear geometry, the problem's nonlinear factors such as the material's non-elastic properties, are considered. Static problems that are geometrically nonlinear sometimes experience buckling or collapse. This behavior is observed when the load-displacement response shows a negative stiffness and the structure is forced to release strain energy to stay balanced. In this study, the effect of material and geometrical parameters were coincidentally studied to determine how they affect each other and how we can manipulate shell geometry by changing material properties and vice versa. Fig.2 and Fig.3 illustrate the plugin flowchart and GUI environment.

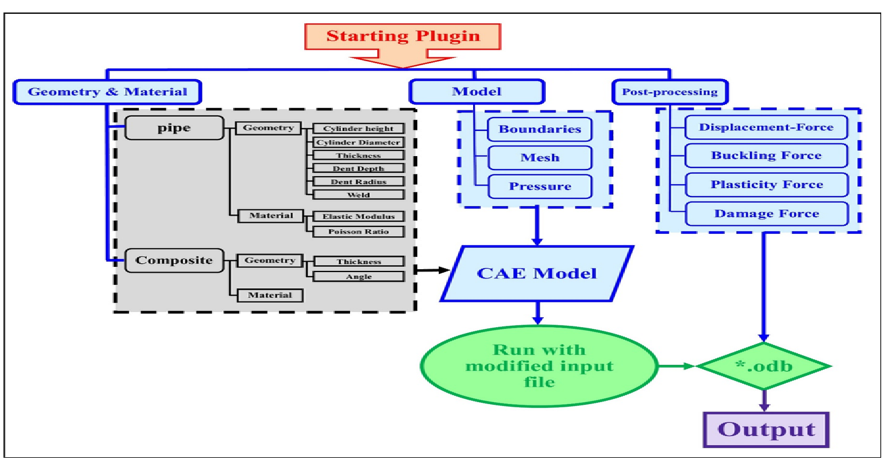


**Fig. 2**
Plugin flowchart showing the inserting parameters, modeling process, post-processing, and outputs.

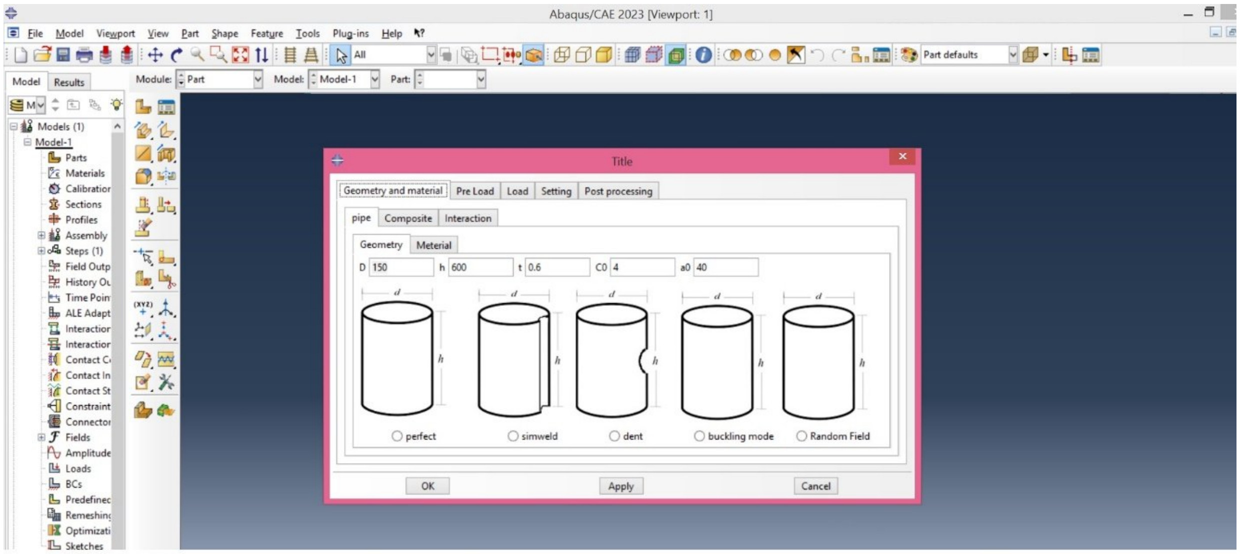


**Fig. 3**
Plugin GUI setting for inserting material properties and modeling the cylinder.

To simulate the shell, the Riks method in Abaqus, which is commonly used for unstable collapse and post-buckling behavior, was employed to investigate the effects of nonlinearities on buckling. The Riks method employs the load magnitude as an unknown and simultaneously solves the problem of loads and displacements. Here, the Abaqus standard applies the arc length method to remove nonlinearities. Thus, the software solves the problem regardless of instabilities in response.

For simulation, a cylinder is modeled with two loads applied. The first is the lateral uniform pressure due to hydrostatic pressures. The second is the axial load applied to the shell from circular plates welded to two shell ends. The axial load is proportional to the lateral pressure. This force equals the lateral pressure multiplied by the circular plate area. This load proportionally increases by the lateral pressure increase. Fig.4 shows the loads on models. The structural mesh is employed to solve the problem.

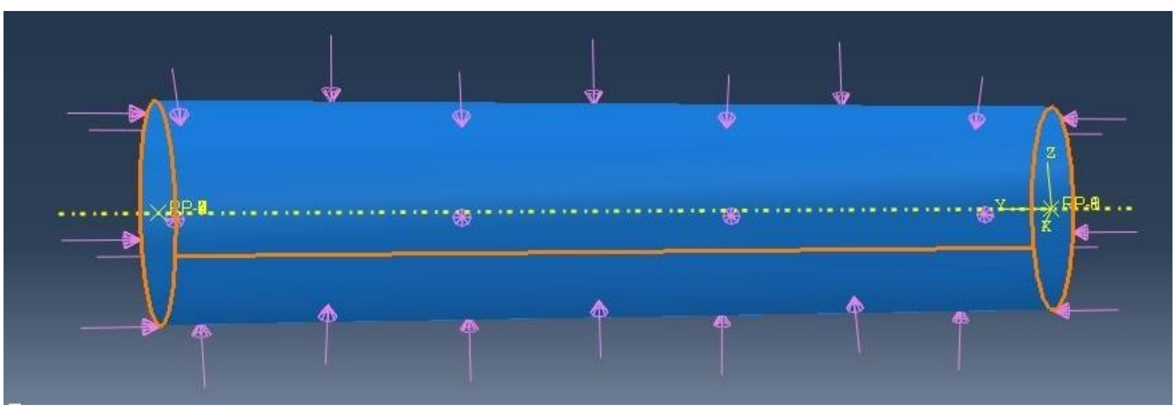

**Fig. 4**
Loads applied to the cylinder modeled in Abaqus.

## 4 GEOMETRICAL AND MATERIAL PARAMETERS

In the study, the effect of material parameters was studied step by step by inputting various material parameters into models with various geometrical parameters. The geometrical parameters are 1, 1.5, and 2 *mm* for thickness and

100, 200, 300, and 400 *mm* for diameter, meaning that for each material properties, 12 shells were modeled. These lengths are selected according to experiences and possible underwater robot geometries. Shell length was 1000 *mm* for all cases to study the material parameter effect. Additionally, the effect of shell length on buckling behavior was finally studied, modeling shells with four different lengths containing 500, 750, 1000, and 1250 *mm*, which matches natural shells that are suitable shell sizes for underwater robots.

Yield stress on buckling behavior was first investigated to investigate the effects of material parameters. Fig.1(b) shows the stress-strain curves of three different materials in which their elasticity modulus' equals 100 *GPa* for all, and yield stresses are 100 *Mpa*, 200 *MPa*, and 300 *MPa*. Thirty-six shells with three thicknesses and four diameters were modeled by these three yield stresses ($3 \times 4 \times 3$). This procedure was repeated for the next steps. In the second step, the elasticity modulus changes and yield stresses are the same (Fig.1(c)). Repeatedly, 36 shells were modeled with three thicknesses, four diameters, and three elasticity modulus' that were 100, 200, and 300 *GPa* ($3 \times 4 \times 3$). In the third step, Fig.1(d), the elasticity modulus' were 100, 200, and 300 *GPa*, but in this step, the yield stresses were 100, 200, and 300 *MPa* respectively, and yield strains were equal. The fourth and fifth steps investigated the plastic strain and the ultimate stress effect. Fig.1(e) shows three materials' properties with similar elasticity modulus (200 *GPa*), yield stress (200 *MPa*), and ultimate stress (200 *MPa*). In this category, the plastic strain varies from zero to 0.1 and 0.2 to investigate the plastic strain effect on buckling behavior. Fig.1(f) shows the materials' curves in which elasticity modulus (200 *GPa*), yield stresses (200 *MPa*), and plastic strains (0.2) are equal, and the ultimate strains vary. In the six*th* step, the shell length effect was investigated. To this end, four lengths containing 500, 750, 1000, and 1250 *mm* were modeled with various thicknesses and diameters, as in previous studies on the effects of materials.

## 5 EXPERIMANTAL VALIDATION

Numerical studies are always associated with experimental validation to figure out how the numerical results are acceptable and how much the difference is between tests and ideal simulations. In this study, cylinders were fabricated from steel materials with 250 *mm* diameters, 0.7 *mm* thickness, and 600 *mm* length. Three similar shells were fabricated to check the repeatability. Like simulations, stainless steel pipes were cut, and two circular plates were welded to each pipe weld using the Tungsten Inert Gas (TIG) method. This method uses tungsten electrode to create an arc and melt metal. The quality of the welding was checked by leakage tests. The quality of the weld was checked by visual inspections. These shells were then slowly pressurized in a chamber with water. The pressure was slowly increased by a rate of 0.1 *bar/min* and the pressure number was recorded continuously by an electrical recorded designed for this purpose. At the buckling point, the water pressure dropped and was recorded to compare with the numerical result. Fig.5 shows one of the fabricated shells. The material is galvanized steel produced by Mobarakeh Steel Company at Esfahan city of Iran with 240 *MPa* yield stress, 360 *MPa* ultimate stress, 0.14% elongation at failure, and 0.3 Poisson ratio. The material properties are given from the manufacturer.

Fig.6 shows the test setup. Obviously, there is a large, cylindrical, thick chamber with a lid. The cylinder shells are put in this chamber from the right side, and after that, the lid plate is joined by pitches. A valve in the large chamber injects the water with a pump with a high head and a low flow. The pump injects the water slowly. To record the pressure, a pressure hose joins the chamber to a pressure gauge that can report pressure with 0.01 bar accuracy. Thin shells are put in the chamber, and the water is pumped. The pressure increases continuously until a drop reveals the buckling. It is acceptable that equipment has accuracies that limits the studies. In this setup, accuracy in the pressure recording devices is a possible source of error. However, its accuracy and calibration is checked and the error is less than 1 percent.


**Fig. 5**
A sample fabricated cylindrical shells for experimental validation made from galvanized steel.

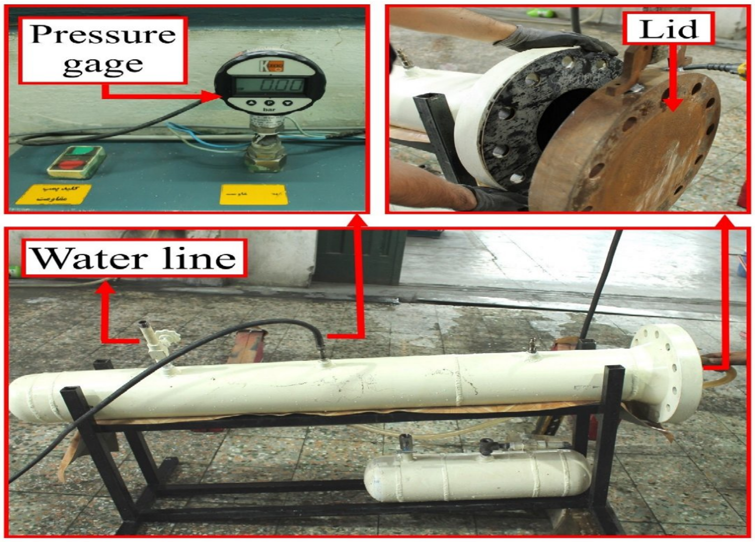


**Fig. 6**
Test setup containing the main chamber for pressuring and its lid with pressure gage for recording the pressure.

## 6 RESULTS AND DISCUSSION
### *6.1 Numerical simulation results*

Materials with 100, 200, and 300 *MPa* yield stresses were modeled to investigate the yield stress effect on cylindrical shell buckling behavior. Fig.7 shows the shell geometry after buckling. Results for various shell diameters and thicknesses are shown in Fig.8. Clearly, in all cases, the critical pressure decreases where the shell diameter increases. There is a similar trend in all cases. The buckling resistance in 100mm diameter is approximately twice that of the 200 *mm* diameter. And the number continues to decline moderately in higher diameters. Focusing on the thickness, a linear trend is evident in all three figures and curves. The buckling resistance in 2 *mm* and 1.5 *mm* thicknesses are twice and 1.5 times that of the 1mm thickness. Nevertheless, the shell length does not change the buckling resistance. It is because the shell cross area is large, meaning that the moment of inertia is high and bending because of the axial forces is not possible before the failure due to the yielding.

Comparing three images showing critical pressure for three various yield stresses, the pressure resistance linearly increases with the yield stress increase. In 2 *mm* thickness and 100 diameters, for example, the critical pressure is less than 5 *MPa* where the yield stress is 100 *MPa*, and the number is less than ten where the yield stress is 200 *MPa*. The number increased to less than 15 where the yield stress is 300 *MPa*. Similar trends are obviously seen in other points.

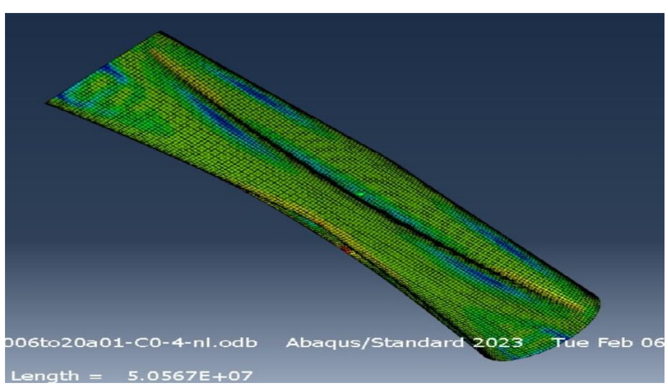


**Fig. 7**
A sample of a simulated cylinder after buckling.

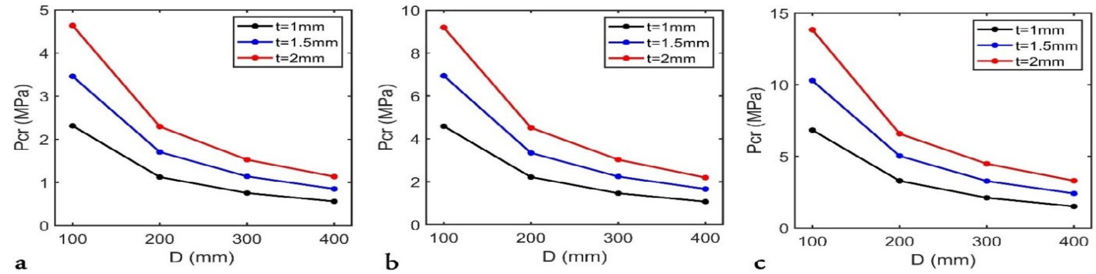


**Fig. 8**
Numerical results showing the buckling pressure in three materials with three different yield stresses and different yield strains: a) 100 *MPa* yield stress b) 200 *MPa* yield stress c) 300 *MPa* yield stress.

In the second step, elasticity modulus with similar yield stresses were investigated. Results showed that the elasticity modulus does not affect buckling behavior in cylindrical shells. This is because the elasticity modulus involves with the deformation and does not have any effect of the failure. Due to similar numbers, results in this section are shown in Table 1. In the table, the third, fourth, and fifth columns illustrate buckling pressure in 100, 200, and 300 *GPa* elasticity modulus. A glance at these columns reveals no changes where the elasticity modulus increases from 100 to 200 and 300 *GPa*, meaning that the elasticity modulus does not have any role in buckling pressure. Notably, similar to Fig.1, the buckling pressure linearly increases with an increase in shell thickness. Trends are the same as in the previous section.

**Table1**
Buckling pressure in various elasticity modulus.

| *D* (*mm*) | *t* (*mm*) | $E_1$ = 100 *GPa* | $E_2$ = 200 *GPa* | $E_3$ = 300 *GPa* |
|---|---|---|---|---|
| 100 | 1 | 2.3109 | 2.3034 | 2.3069 |
| 200 | 1 | 1.1237 | 1.1450 | 1.1468 |
| 300 | 1 | 0.7540 | 0.7523 | 0.7646 |
| 400 | 1 | 0.5593 | 0.5622 | 0.5669 |
| 100 | 1.5 | 3.4587 | 3.4862 | 3.4779 |
| 200 | 1.5 | 1.7046 | 1.7250 | 1.7283 |
| 300 | 1.5 | 1.1389 | 1.1448 | 1.1524 |
| 400 | 1.5 | 0.8496 | 0.8521 | 0.8607 |
| 100 | 2 | 4.6370 | 4.6216 | 4.6233 |
| 200 | 2 | 2.2928 | 2.2818 | 2.3061 |
| 300 | 2 | 1.5268 | 1.5312 | 1.5347 |
| 400 | 2 | 1.1341 | 1.1436 | 1.1491 |

The third section focused on the effect of various elasticity modulus and yield stress in equal yield strains. The results are precisely similar to the first section. Fig.9 shows the results of this study. Similar to the first step, the buckling pressures in 200 and 300 *MPa* yield stresses are triple and twice that of the 100 *MPa* yield stress in all cases. The effect of thickness and diameters are precisely the same as in the first step. This means the yield stress is the only effective parameter, and yield strain and elasticity modulus do not have any role in buckling behavior.

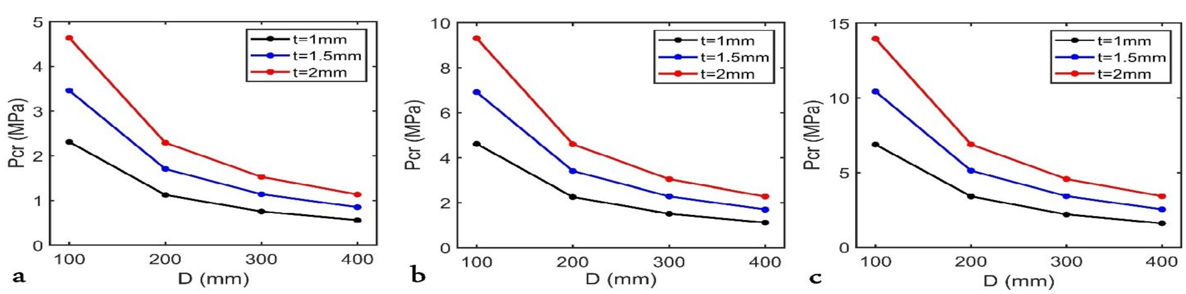


**Fig. 9**
Numerical results showing the buckling pressure in three materials with three different yield stresses and equal yield strains: a) 100 *MPa* yield stress b) 200 *Mpa* yield stress c) 300 *Mpa* yield stress.

The plastic strain and ultimate stress effects were investigated in the fourth and fifth steps. Table 2 and Table 3 show the buckling pressure for these two studies. Like Table 1, the third, fourth, and fifth columns show buckling pressure at various plastic strains and ultimate stresses. Also similar to Table 1, plastic strain and ultimate stress do not affect the buckling behavior of cylindrical shells, and the two tables are similar in each other. Because the ultimate stress affects the fracture of the cylinder and does not have any effect on the deformations that leads to buckling. This issue matches similar studies done by Allouti and coworkers [21]. They studied the dent depth effect on the critical pressure of pipes using damage models, showing that the ultimate stress does not affect the buckling pressure. The reason is that after reaching the yield stress the material experiences work-hardening effects. Here, the material resistance to the failure increase, leading to a hardening and increased resistance to buckling.

**Table 2**
Buckling pressure in various plastic strain.

| **D** (*mm*) | *t* (*mm*) | $\varepsilon p1 = 0$ | $\varepsilon p2 = 0.1$ | $\varepsilon p3 = 0.2$ |
|---|---|---|---|---|
| 100 | 1 | 4.6215 | 4.6215 | 4.6215 |
| 200 | 1 | 2.2563 | 2.2563 | 2.2563 |
| 300 | 1 | 1.5106 | 1.5106 | 1.5106 |
| 400 | 1 | 1.1198 | 1.1198 | 1.1199 |
| 100 | 1.5 | 6.9147 | 6.9147 | 6.9147 |
| 200 | 1.5 | 3.4177 | 3.4177 | 3.4177 |
| 300 | 1.5 | 2.2825 | 2.2825 | 2.2829 |
| 400 | 1.5 | 1.6994 | 1.6995 | 1.6995 |
| 100 | 2 | 9.3087 | 9.3087 | 9.3087 |
| 200 | 2 | 4.6039 | 4.6039 | 4.6039 |
| 300 | 2 | 3.0505 | 3.0505 | 3.0505 |
| 400 | 2 | 2.2758 | 2.2757 | 2.2757 |

**Table 3**
Buckling pressure in various ultimate stresses.

| *D* (*mm*) | *t* (*mm*) | $\sigma ut1 = 200\ MPa$ | $\sigma ut2 = 250\ MPa$ | $\sigma ut3 = 300\ MPa$ |
|---|---|---|---|---|
| 100 | 1 | 4.6215 | 4.5932 | 4.6311 |
| 200 | 1 | 2.2563 | 2.2647 | 2.2799 |
| 300 | 1 | 1.5106 | 1.5083 | 1.5120 |
| 400 | 1 | 1.1199 | 1.1147 | 1.1212 |
| 100 | 1.5 | 6.9147 | 7.0069 | 7.0349 |
| 200 | 1.5 | 3.4177 | 3.4202 | 3.4187 |
| 300 | 1.5 | 2.2829 | 2.2884 | 2.2945 |
| 400 | 1.5 | 1.6995 | 1.6865 | 1.6952 |
| 100 | 2 | 9.3087 | 9.3520 | 9.3657 |
| 200 | 2 | 4.6039 | 4.6023 | 4.5975 |
| 300 | 2 | 3.0505 | 3.0661 | 3.0625 |
| 400 | 2 | 2.2757 | 2.2849 | 2.2835 |

### *6.2 Experimental results*

For the experimental validation, three cylindrical shells were fabricated and tested. One of them failed at the first test due to a defect in the weld. Table 4 shows the experimental and numerical results for the two cylinders tested. In both cases, the numerical simulation reported more buckling resistance. This issue occurs due to material inhomogeneity, defects, manufacturing processes, and imperfections caused during transformation and testing. Nevertheless, the differences between the numbers are lower than 5 percent. This means the numerical results are reliable and can draw a good perspective from the buckling behavior of cylindrical shells. Fig.10 shows shells after the test.

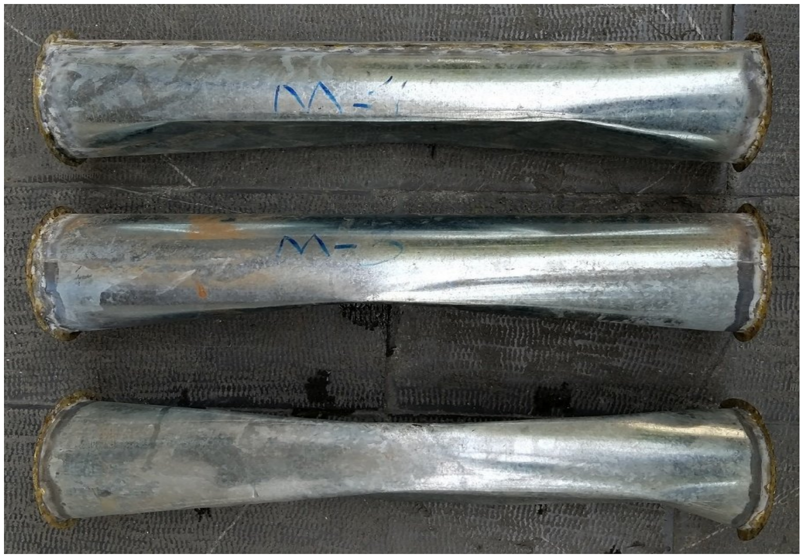

**Fig. 10**
Shells after test.

**Table 4**
Comparison for the experimental and numerical results.

| Specimen | Experimental critical pressure (*MPa*) | Numerical critical pressure (*MPa*) | Error(%) |
|---|---|---|---|
| Shell 1 | ~1.81 | ~1.91 | 5% |
| Shell 2 | ~1.77 | ~1.91 | 7% |

In total, the study showed the yield stress effect is dominant in comparison with the ultimate stress and plastic strain. This is due to the buckling mechanism that involves the deformation. In the buckling process the cylinder with ductile material experiences harsh deformation and there is no fracture. Thus, the buckling relates to the yield stress and ultimate stress does not influence that.

## 7 CONCLUSIONS

Underwater vehicles have always been designed for various purposes. The current study aimed to provide a sense of material and geometry effect of underwater shells on the buckling resistance of underwater robots. To this end, the effect of material properties and geometrical parameters on the buckling behavior of cylindrical shells was studied to draw good knowledge for future cylindrical shell design. Various geometries and various material properties were employed for numerical modeling using the Riks method in Abaqus. An Abaqus plugin was developed and employed to reduce modeling time. The plugin helps the operator by giving only a few parameters such as shell length, diameter, and mechanical properties. Materials and geometrical parameters were selected based on the needs of an underwater robot. The results are listed as follows:

1) The yield stress is the only effective material parameter affecting the buckling behavior of cylindrical shells, and elasticity modulus, yield strain, ultimate stress, and plastic strain do not affect this behavior.
2) Among geometrical parameters, the shell lengths we studied were not effective on the buckling behavior. However, the buckling pressure linearly increased with an increase in shell thickness. On the contrary, the bucking pressure dropped as the diameters increased.

Experimental validation on two fabricated shells revealed around 5 percent differences between numerical and experimental results. This level of variance is deemed acceptable, considering the influence of manufacturing weaknesses, material defects, and other unavoidable deficiencies.

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