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MCDM problem under single-valued neutrosophic numbers based on a novel similarity measure

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Abstract. One of the challenges in MCDM with a neutrosophic environment is the lack of an efficient analysis tool to rank the alternatives. The effective ranking of alternatives in MCDM under a neutrosophic environment presents a significant challenge due to the absence of robust analytical tools. Bridging this gap is essential for enhancing decision-making processes across various practical applications. Aggregative operators, ranking methods, distance measures, and similarity measures are among the most essential tools for sorting fuzzy data and their extensions, including Neutrosophic numbers. However, existing methods often exhibit significant shortcomings, such as contradictory results under certain conditions, and computational inefficiency. For example, Deli and Subas's approach and Ye's approach may yield contradicting results for specific instances of single-valued trapezoidal neutrosophic (SVTriN) numbers. The novel similarity measure proposed in this study introduces a new mathematical formulation that enhances the meaningfulness and conceptual soundness of the analysis by effectively capturing the intrinsic indeterminacy of neutrosophic numbers. Unlike existing methods, it provides consistent results across various scenarios, eliminating contradictory outcomes. This innovative approach deepens the understanding of relationships between alternatives, making it highly suitable for practical implementation in MCDM problems. On the other hand, very few papers have used the similarity measure to sort the alternatives in the MCDM problem under trapezoidal neutrosophic numbers. Thus, the main objective of this study is to introduce a novel similarity measure for sorting alternatives. For this purpose, we first highlight some shortcomings related to the existing ranking approach for neutrosophic numbers. After introducing the novel similarity measure, several critical properties of the suggested similarity measure between neutrosophic numbers are investigated. The proposed similarity measure offers a proper way to apply the TOPSIS method in the context of neutrosophic information. The efficiency and utility of the proposed similarity measure are illustrated by a comparative example.

AMS Subject Classification 2020: 90B50; 90C70
Keywords and Phrases: Neutrosophic set, Single-valued Neutrosophic number, Ranking methods, Similarity measure, Multi-criteria decision making.

1 Introduction

The concept of neutrosophic set (NS) as a generalization of the fuzzy set (FS) is introduced by Smarandache [20] to develop the single membership function of FS into three independent forms, namely, truth, indeterminacy, and falsity membership functions in NS. The NS can explain the uncertain and imprecise perceptions of objects more accurately and comprehensively than FS. This generalization of fuzzy set not only involves various theoretical research fields [8, 9] but also plays a prominent role in practical modeling such as mathematical

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optimization and decision-making science [3, 6, 10, 13, 14, 15, 25]. Similar to fuzzy sets, Neutrosophic sets can be divided into discrete and continuous categories. For example, in continuous construction, Wang [22] introduced single-valued trapezoidal neutrostrophic numbers as an extension of trapezoidal fuzzy numbers.

Today, with the increasing expansion of decision-making knowledge in various personal, social, economic, and technical fields, the attention of many researchers has been drawn to this field. The evaluation of each option in each criterion is done by the decision-maker with quantitative or qualitative data. Since neutrosophic sets give more flexible information than fuzzy sets, it is natural that the decision maker prefers to evaluate the options based on neutrosophic data in order to adapt the data to the existing situations logically. The ultimate goal of decision-making is to determine the best option and achieve an ordered list of alternatives based on existing criteria. For this, we should provide appropriate calculation and comparison tools. Ranking methods, methods based on agglomerative operators, distance, and similarity are among the most critical and prominent strategies. Developing new similarity measures that can effectively handle continuous neutrosophic numbers is crucial for advancing decision-making methodologies and improving the accuracy of evaluations in various practical scenarios.

In 2014, Deli and Subas [4] defined the single-valued trapezoidal neutrosophic weighted aggregation operator (SVTNWAO) and score values to solve personal selection problems. Later, they [5] gave a parametric ranking approach based on θ-weighted value and θ-weighted ambiguity of TrNN to solve multi-attribute decision-making problems. Later, Pramanik et al. [16, 23] presented the TOPSIS method for MADM problems with single-valued neutrosophic soft set. Ye [28] established two operators, namely, the weighted arithmetic averaging operator and the weighted geometric averaging operator for solving the investment selection in a company with trapezoidal neutrosophic. Also, they investigated the properties of these operators. In another research, Pranab et al. [1] extended the TOPSIS method for medical representative selection problems under trapezoidal neutrosophic information. Liang et al. [11] investigated a comparison method based on three functions using center of gravity (COG) to deal with green supplier selection problems with SVTNNs. Furthermore, Liang et al. [12] designed an integrated methodology based on the SVTN-DEMATEL module to analyze e-commerce website evaluation.

Meanwhile, Pramanik and Mallick [17] extended vIsekriterijumska optimizacija I kompromisno resenje (VIKOR) strategy to solve the MCGDM problem under TrNN data. In the following, Shouzhen Zeng et al. [30] introduce a novel TOPSIS method based on correlation for MADM problems under a single-valued neutrosophic environment. Broumi et al. [19] expressed a novel distance measure to distinguish the difference between two trapezoidal fuzzy neutrosophic numbers to solve the software selection process. Wang et al. [23] gave the possibility-degree and power weighted aggregation operators and application to multi-criteria group decision-making problems in which the evaluation ratings are proposed with single-valued trapezoidal neutrosophic numbers.

Moreover, Garai et al. [7] organized a new ranking methodology based on a ratio of possibility mean and standard deviation. Then, they utilized this strategy to evaluate the risk of Flood Disaster (FD) under the single-valued triangular neutrosophic numbers. Researchers such Xu and Peng [24], inspired by the advantages of TOPSIS and TODIM methods, proposed a new hybrid method under Multi-Valued Neutrosophic Sets. Their combined method not only has concise computation like the TOPSIS strategy but also considers the decision makers risk-aversion attitude. Then, the application of their method is examined for teaching evaluation in universities and personnel selection in food companies. In the same year, Ridvan et al. [18] established the novel concept of the TOPSIS method based on divergence, projection, and likelihood, which was named DPLTOPSIS method to choose the company that produces the most suitable and applicable mask type among the producer companies to deal with the COVID19 epidemic. Similarity measure is an essential tool for data analysis, which its applications are not limited to use in decision-making methods. Some researchers [2, 27, 29] are among the pioneer researchers in introducing several new similarity measures under neutrosophic data and expanding its applications such as investment selection, disease diagnosis, and other decision-making fields. Most of these papers have only used operators, distances, and similarities between neutrosophic sets in the finite (countable) universe of discourses for solving the multi-criteria decision-making problem. In contrast, continuous numbers like single-valued triangular neutrosophic numbers can illustrate more flexibility in expressing the decision maker thoughts.

The limitation of resources related to similarity measures on continuous numbers and their wide applications is our primary motivation to introduce a logical and novel similarity measure in this study. The proposed similarity measure can be practically applied in various fields, including investment selection, disease diagnosis, and other decision-making scenarios. This research endeavors to bridge the existing gap in the literature by offering a thorough methodology for handling continuous neutrosophic numbers.

Traditional decision-making methods, including the original TOPSIS approach, often struggle with the uncertainty and inconsistencies in neutrosophic information. By incorporating our proposed similarity measure into the TOPSIS framework, we effectively tackle these issues, leading to a more precise and robust evaluation of alternatives. Precisely, this innovative similarity measure adeptly captures the levels of truth, indeterminacy, and falsity within the data, facilitating a more nuanced analysis that traditional approaches might overlook.

This integration also enhances the discriminative ability of the TOPSIS method in neutrosophic contexts, resulting in more dependable and meaningful rankings of alternatives. It not only addresses the contradictions found in existing methods but also refines the overall decision-making process by accommodating the uncertainty and vagueness standard in real-world situations.

Thus, the primary contributions of this study are:

- Developing a new similarity measure designed for single-valued triangular neutrosophic numbers (SVTriNs).
- Integrating the proposed similarity measure with the TOPSIS method to enhance decision-making accuracy.
- Demonstrating the effectiveness of this integrated approach through a comparative example.

By addressing the shortcomings of current methods and strengthening the TOPSIS framework, our research represents a significant advancement in multi-criteria decision-making (MCDM) within neutrosophic environments. The remainder of this manuscript is provided as follows. Some essential preliminaries related to neutrosophic numbers are reviewed in section 2. Some of the existing methodologies (on SVTriN-Numbers) to rank alternatives and several counterexamples are considered in section 3. The construction of the suggested similarity measure on SVTriN-numbers is mentioned in section 4. Then, to prove and research the correctness of the proposed similarity measure, some of its features are given in section 5. Also, a hybrid algorithm based on the proposed similarity measure in this section, and the TOPSIS method is designed to solve the SVTriN-MCDM problem in Section 6. The effectiveness of the hybrid algorithm suggested in section 6 for solving an example from the literature of the research topic, along with a comparative analysis, is given in section 7. The final summary of the results and some suggestions are mentioned in section 8.

2 Basic Preliminaries

This section involves some essential notions related to neutrosophic numbers which are necessitated in other parts of this study.

Definition 2.1 ([20]). Let $\mathcal{M}$ be a non-empty and fixed set. Then, a neutrosophic set $\tilde{\mathcal{N}}$ over $\mathcal{M}$ is given by

$$\tilde{\mathcal{N}} = \{\langle x, (\mathcal{T}_{\tilde{\mathcal{N}}}(x), \mathcal{I}_{\tilde{\mathcal{N}}}(x), \mathcal{F}_{\tilde{\mathcal{N}}}(x))\rangle | x \in \mathcal{M}, \mathcal{T}_{\tilde{\mathcal{N}}}(x), \mathcal{I}_{\tilde{\mathcal{N}}}(x), \mathcal{F}_{\tilde{\mathcal{N}}}(x) \in [^{-}0, 1^{+}]\}, \tag{1}$$

It involves three characterizations named by the truth-membership function $\mathcal{T}_{\tilde{\mathcal{N}}}(x)$, indeterminacy-membership function $\mathcal{I}_{\tilde{\mathcal{N}}}(x)$, and falsity-membership function $\mathcal{F}_{\tilde{\mathcal{N}}}(x)$ such that they satisfy the condition $^{-}0 \leq \mathcal{T}_{\tilde{\mathcal{N}}}(x) + \mathcal{I}_{\tilde{\mathcal{N}}}(x) + \mathcal{F}_{\tilde{\mathcal{N}}}(x) \leq 3^{+}$.

Definition 2.2 ([4])**.** Let $\mathcal{M}$ be a non-empty and fixed set. Then, a single-valued neutrosophic set $\tilde{\mathcal{N}}$ over $\mathcal{M}$ is given by

$$\tilde{\mathcal{N}} = \langle x, (\mathcal{T}_{\tilde{\mathcal{N}}}(x), \mathcal{I}_{\tilde{\mathcal{N}}}(x), \mathcal{F}_{\tilde{\mathcal{N}}}(x)) \rangle | x \in M, \mathcal{T}_{\tilde{\mathcal{N}}}(x), \mathcal{I}_{\tilde{\mathcal{N}}}(x), \mathcal{F}_{\tilde{\mathcal{N}}}(x) \in [0,1], \tag{2}$$

It involves three characterizations named by the truth-membership function $\mathcal{T}_{\tilde{\mathcal{N}}}(x)$, indeterminacy-membership function $\mathcal{I}_{\tilde{\mathcal{N}}}(x)$, and falsity-membership function $\mathcal{F}_{\tilde{\mathcal{N}}}(x)$ such that they satisfy the condition $0 \leq \mathcal{T}_{\tilde{\mathcal{N}}}(x) + \mathcal{I}_{\tilde{\mathcal{N}}}(x) + \mathcal{F}_{\tilde{\mathcal{N}}}(x) \leq 3$.

Definition 2.3 ([21])**.** Let α, β, γ and $\delta \in \mathbb{R}$ such that $\alpha \leq \beta \leq \gamma \leq \delta$ and $w_{\tilde{n}}, u_{\tilde{n}}$, and $v_{\tilde{n}} \in [0,1]$. A single-valued trapezoidal neutrosophic (SVTN) number $\tilde{n} = \langle (\alpha, \beta, \gamma, \delta); w_{\tilde{n}}, u_{\tilde{n}}, v_{\tilde{n}} \rangle$ is a particular neutrosophic set on the real set $\mathbb{R}$ that is included by truth-membership, indeterminacy membership and falsity membership functions are given, respectively, as follows,

$$\mu_{\tilde{n}}(x) = \begin{cases} \frac{(x-\alpha)}{(\beta-\alpha)} w_{\tilde{n}}, & \alpha \leq x \leq \beta \\ w_{\tilde{n}}, & \beta \leq x \leq \gamma \\ \frac{(\delta-x)}{(\delta-\gamma)} w_{\tilde{n}}, & \gamma \leq x \leq \delta \\ 0, & otherwise, \end{cases}, \tag{3}$$

$$\pi_{\tilde{n}}(x) = \begin{cases} \frac{(\beta-x+u_{\tilde{n}}(x-\alpha))}{(\beta-\alpha)}, & \alpha \leq x \leq \beta \\ u_{\tilde{n}}, & \beta \leq x \leq c \\ \frac{(x-\gamma+u_{\tilde{n}}(\delta-x))}{(\delta-\gamma)}, & \gamma \leq x \leq \delta \\ 1, & otherwise, \end{cases} \tag{4}$$

and,

$$\eta_{\tilde{n}}(x) = \begin{cases} \frac{(\beta-x+v_{\tilde{n}}(x-\alpha))}{(\beta-\alpha)}, & \alpha \leq x \leq \beta \\ v_{\tilde{n}}, & \beta \leq x \leq \gamma \\ \frac{(x-\gamma+v_{\tilde{n}}(\delta-x))}{(\delta-\gamma)}, & \gamma \leq x \leq \delta \\ 1, & otherwise, \end{cases} \tag{5}$$

Remark 2.4. According to definition 2.3, we can conclude

a1) If $\alpha \geq 0$, then $\tilde{n}$ is a positive SVTN number.

a2) If $\delta \leq 0$, then $\tilde{n}$ is a negative SVTN number.

a3) If $0 \leq \alpha \leq \beta \leq \gamma \leq \delta \leq 1$ and $\delta \neq 0$, then $\tilde{n}$ is a normalized SVTN number.

a4) If $\beta = \delta$, then $\tilde{n}$ is a single-valued triangular neutrosophic (SVTriN) number.

Remark 2.5. Complementary functions related to the indeterminacy-membership function and falsity-membership function can be obtained by following equations, respectively

$$\pi'_{\tilde{n}}(x) = 1 - \tau_{\tilde{n}}(x) \tag{6}$$
$$\eta'_{\tilde{n}}(x) = 1 - \eta_{\tilde{n}}(x) \tag{7}$$

where $\pi'_{\tilde{n}} : \mathbb{R} \to [0, 1 - u_{\tilde{n}}]$ and $\eta'_{\tilde{n}} : \mathbb{R} \to [0, 1 - v_{\tilde{n}}]$.

Definition 2.6 ([21]). Let $\tilde{n}_1 = \langle(\alpha_1, \beta_1, \gamma_1, \delta_1); w_{\tilde{n}_1}, u_{\tilde{n}_1}, v_{\tilde{n}_1}\rangle$ and $\tilde{n}_2 = \langle(\alpha_2, \beta_2, \gamma_2, \delta_2); w_{\tilde{n}_2}, u_{\tilde{n}_2}, v_{\tilde{n}_2}\rangle$, are two SVTN numbers and c is an arbitrary real number. Then

$$\text{1)} \quad \tilde{n}_1 \oplus \tilde{n}_2 = \langle(\alpha_1+\alpha_2, \beta_1+\beta_2, \gamma_1+\gamma_2, \delta_1+\delta_2); w_{\tilde{n}_1} \wedge w_{\tilde{n}_2}, u_{\tilde{n}_1} \vee u_{\tilde{n}_2}, v_{\tilde{n}_1} \vee v_{\tilde{n}_2}\rangle, \tag{8}$$

$$\text{2)} \quad \tilde{n}_1 \ominus \tilde{n}_2 = \langle(\alpha_1-\delta_2, \beta_1-\gamma_2, \gamma_1-\beta_2, \delta_1-\alpha_2); w_{\tilde{n}_1} \wedge w_{\tilde{n}_2}, u_{\tilde{n}_1} \vee u_{\tilde{n}_2}, v_{\tilde{n}_1} \vee v_{\tilde{n}_2}\rangle, \tag{9}$$

$$\text{3)} \quad c\tilde{n}_1 = \begin{cases} \langle(c\alpha_1, c\beta_1, c\gamma_1, c\delta_1); w_{\tilde{n}_1}, u_{\tilde{n}_1}, v_{\tilde{n}_1}\rangle, & \text{If } c \geq 0, \\ \langle(c\alpha_1, c\gamma_1, c\beta_1, c\alpha_1); w_{\tilde{n}_1}, u_{\tilde{n}_1}, v_{\tilde{n}_1}\rangle, & \text{If } c \leq 0. \end{cases} \tag{10}$$

Where $\wedge = \min, \vee = \max$.

3 Some of the existing methodology

Various tools such as arithmetic and geometric operators, distance measures, and similarity measures have been used to rank options in problem-solving methods. Most of them have some advantages, disadvantages, limitations, and applications. Some contradictions of these approaches are mentioned below. First of all, let $\tilde{n}_1 = \langle(\alpha_1, \beta_1, \delta_1); w_{\tilde{n}_1}, u_{\tilde{n}_1}, v_{\tilde{n}_1}\rangle$ and $\tilde{n}_2 = \langle(\alpha_2, \beta_2, \delta_2); w_{\tilde{n}_2}, u_{\tilde{n}_2}, v_{\tilde{n}_2}\rangle$ are two SVTriN numbers.

- Deli & Subass approach [4]
  According to their approach, if the following equalities hold for two SVTriN numbers

  $$S_{DS}(\tilde{n}_1) = \frac{1}{16}(\alpha_1 + 2\beta_1 + \delta_1)(2 + w_{\tilde{n}_1} - u_{\tilde{n}_1} - v_{\tilde{n}_1}) = \frac{1}{16}(\alpha_2 + 2\beta_2 + \delta_2)(2 + w_{\tilde{n}_2} - u_{\tilde{n}_2} - v_{\tilde{n}_2}) = S_{DS}(\tilde{n}_2)$$
  $$H_{DS}(\tilde{n}_1) = \frac{1}{16}(\alpha_1 + 2\beta_1 + \delta_1)(2 + w_{\tilde{n}_1} - u_{\tilde{n}_1} + v_{\tilde{n}_1}) = \frac{1}{16}(\alpha_2 + 2\beta_2 + \delta_2)(2 + w_{\tilde{n}_2} - u_{\tilde{n}_2} + v_{\tilde{n}_2}) = H_{DS}(\tilde{n}_2)$$

  Then, $\tilde{n}_1 = \tilde{n}_2$, whereas contradicting reults obtain for $\tilde{n}_1 = \langle(0.5, 0.7, 0.8); 0.2, 0.3, 0.4\rangle$ and $\tilde{n}_2 = \langle(0.4, 0.7, 0.9); 0.2, 0.3, 0.4\rangle$.

- Yes approach [16]
  According to Yes approach, if the following equality holds for two SVTriN numbers

  $$S_{Y}(\tilde{n}_1) = \frac{1}{12}(\alpha_1 + 2\beta_1 + \delta_1)(2 + w_{\tilde{n}_1} - u_{\tilde{n}_1} - v_{\tilde{n}_1}) = \frac{1}{12}(\alpha_2 + 2\beta_2 + \delta_2)(2 + w_{\tilde{n}_2} - u_{\tilde{n}_2} - v_{\tilde{n}_2}) = S_{Y}(\tilde{n}_2)$$

  Then, $\tilde{n}_1 = \tilde{n}_2$, whereas contradicting result obtaines for $\tilde{n}_1 = \langle(0.4, 0.5, 0.9); 0.2, 0.3, 0.5\rangle$ and $\tilde{n}_2 = \langle(0.3, 0.6, 0.8); 0.2, 0.5, 0.3\rangle$.

- Liang et al.s approach [11]
  Liang et al. [11] defined the score, accuracy, and certainty functions of $\tilde{n}_1$, respectively, as:

  $$S_L(\tilde{n}_1) = COG(\tilde{n}_1) * \Big(\frac{(2 + w_{\tilde{n}_1} - u_{\tilde{n}_1} - {}_{\tilde{n}_1})}{3}\Big)$$
  $$A_L(\tilde{n}_1) = COG(\tilde{n}_1) * (w_{\tilde{n}_1} - u_{\tilde{n}_1})$$
  $$C_L(\tilde{n}_1) = COG(\tilde{n}_1) * w_{\tilde{n}_1}$$

  Where,

  $$COG(\tilde{n}_1) = \begin{cases} \alpha_1, & \alpha_1 = \beta_1 = \delta_1 \\ (\frac{1}{3})\{\alpha_1 + 2\beta_1 + \delta_1 - \frac{\beta_1\delta_1 - \alpha_1\beta_1}{\delta_1 - \alpha_1}\}, & otherwise \end{cases}$$

  Based on the above functions, they analyzed when $S_L(\tilde{n}_1) = S_L(\tilde{n}_2), A_L(\tilde{n}_1) = A_L(\tilde{n}_2)$ and, $C_L(\tilde{n}_1) = C_L(\tilde{n}_1)$, meaning that $\tilde{n}_1$ is indifferent to $\tilde{n}_2$. Meanwhile contradicting results obtains for $\tilde{n}_1 = \langle(0, 0, 0.5); 0.2, 0.3, 0.4\rangle$ and $\tilde{n}_2 = \langle(0, 0.2, 0.3); 0.2, 0.3, 0.4\rangle$.

- Garai et al.s approach [7]
  According to their approach, if the following equalities hold for two SVTriN numbers

$$
\begin{aligned}
&1)\quad S_\mu(\tilde{n}_1)=\frac{2w_{\tilde{n}_1}(\alpha_1+4\beta_1+\delta_1)}{\sqrt{6}(\delta_1-\alpha_1)}=\frac{2w_{\tilde{n}_2}(\alpha_2+4\beta_2+\delta_2)}{\sqrt{6}(\delta_2-\alpha_2)}=S_\mu(\tilde{n}_2).\\
&2)\quad S_u(\tilde{n}_1)=\frac{2(2(1+2u_{\tilde{n}_1})\beta_1+(2+u_{\tilde{n}_1})(\alpha_1+\delta_1)(\sqrt{1-u_{\tilde{n}_1}})))}{\sqrt{6(3+u_{\tilde{n}_1})}(\delta_1-\alpha_1))}\\
&\qquad\qquad\quad=\frac{2(2(1+2u_{\tilde{n}_2})\beta_2+(2+u_{\tilde{n}_2})(\alpha_2+\delta_2)(\sqrt{1-u_{\tilde{n}_2}}))}{\sqrt{6(3+u_{\tilde{n}_2})}(\delta_2-\alpha_2)}=S_u(\tilde{n}_2).\\
&3)\quad S_\nu(\tilde{n}_1)=\frac{2(2(1+2_{\tilde{n}_1})\beta_1+(2+_{\tilde{n}_1})(\alpha_1+\delta_1)(\sqrt{1-_{\tilde{n}_1}}))}{\sqrt{6(3+_{\tilde{n}_1})}(\delta_1-\alpha_1)}\\
&\qquad\qquad\quad=\frac{2(2(1+2_{\tilde{n}_2})\beta_2+(2+_{\tilde{n}_2})(\alpha_2+\delta_2)(\sqrt{1-_{\tilde{n}_2}}))}{\sqrt{6(3+_{\tilde{n}_2})}(\delta_2-\alpha_2)}=S_\nu(\tilde{n}_2).
\end{aligned}
$$

  Then, $\tilde{n}_1=\tilde{n}_2$, whereas contradicting results obtaines for $\tilde{n}_1=\langle(0.1,0.2,0.3);0.2,0.3,0.4\rangle$ and $\tilde{n}_2=\langle(0.2,0.4,0.6);0.2,0.3,0.4\rangle$.

4 Suggested Similarity Measure on SVTriN-Numbers

Before introducing the suggested similarity measure, we should identify the factors influencing the formation of the proposed similarity measure structure. For this purpose, it is desired to use two structural and intuitive views simultaneously in the similarity measure. It means, from the structural point of view, the nature of the numbers used in Neutrosophic numbers. To apply an intuitive point of view, we will use the concept of the left and right halves of the area. To do so, the following steps are suggested.
Input: a set of positive SVTriN-Numbers; $\mathfrak{R}=\{\tilde{n}_i|\langle(\alpha_i,\beta_i,\delta_i);w_{\tilde{n}_i},u_{\tilde{n}_i},v_{\tilde{n}_i}\rangle,i\in\mathbb{N}\}$.
S1. Find $\theta=\max\{\alpha_i,\beta_i,\delta_i|i\in\mathbb{N}\}$.

Then, obtain normalized $\tilde{n}_i=\langle(\frac{\alpha_i}{\theta},\frac{\beta_i}{\theta},\frac{\delta_i}{\theta});w_{\tilde{n}_i},u_{\tilde{n}_i},v_{\tilde{n}_i}\rangle$ for every $i\in\mathbb{N}$. For convenience and simplicity of writing, assume that the elements of the set $\mathfrak{R}$ are normalized.
S2. Consider $\mu_{\tilde{n}}(x),\pi'_{\tilde{n}}(x)$ and $\eta'_{\tilde{n}}(x)$.
S3. Obtain the areas of the left half and right half of $\mu_{\tilde{n}}(x),\pi'_{\tilde{n}}(x)$ and $\eta'_{\tilde{n}}(x)$ to the vertical lines $\zeta=0$ respectively, as following formulas.

$$Q^{l_\mu}(\tilde{n}_i)=\int_0^{w_{\tilde{n}_i}}\Big((\alpha_i)+\frac{\zeta}{w_{\tilde{n}_i}}(\beta_i-\alpha_i)\Big)d\zeta=((\alpha_i)+\frac{1}{2}(\beta_i-\alpha_i))w_{\tilde{n}_i}\tag{11}$$

$$Q^{l_{\pi'}}(\tilde{n}_i)=\int_0^{1-u_{\tilde{n}_i}}\Big((\alpha_i)+\frac{\zeta}{(1-u_{\tilde{n}_i})}(\beta_i-\alpha_i)\Big)d\zeta=((\alpha_i)+\frac{1}{2}(\beta_i-\alpha_i))(1-u_{\tilde{n}_i})\tag{12}$$

$$Q^{l_{\eta'}}(\tilde{n}_i)=\int_0^{1-v_{\tilde{n}_i}}\Big((\alpha_i)+\frac{\zeta}{(1-v_{\tilde{n}_i})}(\beta_i-\alpha_i)\Big)d\zeta=((\alpha_i)+\frac{1}{2}(\beta_i-\alpha_i))(1-v_{\tilde{n}_i})\tag{13}$$

$$Q^{r_\mu}(\tilde{n}_i)=\int_0^{w_{\tilde{n}_i}}\Big((\delta_i)+\frac{\zeta}{w_{\tilde{n}_i}}(\beta_i-\delta_i)\Big)d\zeta=((\delta_i)+\frac{1}{2}(\beta_i-\delta_i))w_{\tilde{n}_i}\tag{14}$$

$$Q^{r_{\pi'}}(\tilde{n}_i)=\int_0^{1-u_{\tilde{n}_i}}\Big((\delta_i)+\frac{\zeta}{(1-u_{\tilde{n}_i})}(\beta_i-\delta_i)\Big)d\zeta=((\delta_i)+\frac{1}{2}(\beta_i-\delta_i))(1-u_{\tilde{n}_i})\tag{15}$$

$$Q^{r_{\eta'}}(\tilde{n}_i)=\int_0^{1-v_{\tilde{n}_i}}\Big((\delta_i)+\frac{\zeta}{(1-v_{\tilde{n}_i})}(\beta_i-\delta_i)\Big)d\zeta=((\delta_i)+\frac{1}{2}(\beta_i-\delta_i))(1-v_{\tilde{n}_i})\tag{16}$$

S4. Calculate the similarity measure between two SVTriN-Numbers of $\mathfrak{R}$ with the following suggested relation

$$\begin{aligned}s(\tilde{n}_i,\tilde{n}_j) =& 2\{\alpha_i\alpha_j+\beta_i\beta_j+\delta_i\delta_j+w_{\tilde{n}_i}w_{\tilde{n}_j}+u_{\tilde{n}_i}u_{\tilde{n}_j}+v_{\tilde{n}_i}v_{\tilde{n}_j}+Q^{l_\mu}(\tilde{n}_i)Q^{l_\mu}(\tilde{n}_j)+Q^{l_{\pi'}}(\tilde{n}_i)Q^{l_{\pi'}}(\tilde{n}_j)\\ &+Q^{l_{\eta'}}(\tilde{n}_i)Q^{l_{\eta'}}(\tilde{n}_j)+Q^{r_\mu}(\tilde{n}_i)Q^{r_\mu}(\tilde{n}_j)+Q^{r_{\pi'}}(\tilde{n}_i)Q^{r_{\pi'}}(\tilde{n}_j)+Q^{r_{\eta'}}(\tilde{n}_i)Q^{r_{\eta'}}(\tilde{n}_j)\}/\{\alpha_i^2+\alpha_j^2+\beta_i^2\\ &+\beta_j^2+\delta_i^2+\delta_j^2+w_{\tilde{n}_i}^2+w_{\tilde{n}_j}^2+u_{\tilde{n}_i}^2+u_{\tilde{n}_j}^2+v_{\tilde{n}_i}^2+v_{\tilde{n}_j}^2+(Q^{l_\mu}(\tilde{n}_i))^2+(Q^{l_\mu}(\tilde{n}_j))^2\\ &+(Q^{l_{\pi'}}(\tilde{n}_i))^2+(Q^{l_{\pi'}}(\tilde{n}_j))^2+(Q^{l_{\eta'}}(\tilde{n}_i))^2+(Q^{l_{\eta'}}(\tilde{n}_j))^2+(Q^{r_\mu}(\tilde{n}_i))^2+(Q^{r_\mu}(\tilde{n}_j))^2\\ &+(Q^{r_{\pi'}}(\tilde{n}_i))^2+(Q^{r_{\pi'}}(\tilde{n}_j))^2+(Q^{r_{\eta'}}(\tilde{n}_i))^2+(Q^{r_{\eta'}}(\tilde{n}_j))^2\}\end{aligned} \tag{17}$$

5 Propositions

Theorem 5.1. *Let $\tilde{n}_1$ and $\tilde{n}_2$ are two SVTriN-Numbers. Demonstrate equation (17) satisfy the below properties:*

1) $0 \le s(\tilde{n}_1,\tilde{n}_2) \le 1$.

2) $s(\tilde{n}_1,\tilde{n}_2) = s(\tilde{n}_2,\tilde{n}_1)$.

3) $s(\tilde{n}_1,\tilde{n}_2) = 1 \Leftrightarrow \tilde{n}_1 = \tilde{n}_2$.

Proof. 1) From Eq (17), we have $s(\tilde{n}_1,\tilde{n}_2) \ge 0$. Also,

$$\begin{aligned}&2\{\alpha_1\alpha_2+\beta_1\beta_2+\delta_1\delta_2+w_{\tilde{n}_1}w_{\tilde{n}_2}+u_{\tilde{n}_1}u_{\tilde{n}_2}+v_{\tilde{n}_1}v_{\tilde{n}_2}+Q^{l_\mu}(\tilde{n}_1)Q^{l_\mu}(\tilde{n}_2)+Q^{l_{\pi'}}(\tilde{n}_1)Q^{l_{\pi'}}(\tilde{n}_2)\\ &+Q^{l_{\eta'}}(\tilde{n}_1)Q^{l_{\eta'}}(\tilde{n}_2)+Q^{r_\mu}(\tilde{n}_1)Q^{r_\mu}(\tilde{n}_2)+Q^{r_{\pi'}}(\tilde{n}_1)Q^{r_{\pi'}}(\tilde{n}_2)+Q^{r_{\eta'}}(\tilde{n}_1)Q^{r_{\eta'}}(\tilde{n}_2)\}\\ &\le\{\alpha_1^2+\alpha_2^2+\beta_1^2+\beta_2^2+\delta_1^2+\delta_2^2+w_{\tilde{n}_1}^2+w_{\tilde{n}_2}^2+u_{\tilde{n}_1}^2+u_{\tilde{n}_2}^2+v_{\tilde{n}_1}^2+v_{\tilde{n}_2}^2+(Q^{l_\mu}(\tilde{n}_1))^2\\ &+(Q^{l_\mu}(\tilde{n}_2))^2+(Q^{l_{\pi'}}(\tilde{n}_1))^2+(Q^{l_{\pi'}}(\tilde{n}_2))^2+(Q^{l_{\eta'}}(\tilde{n}_1))^2+(Q^{l_{\eta'}}(\tilde{n}_2))^2\\ &+(Q^{r_\mu}(\tilde{n}_1))^2+(Q^{r_\mu}(\tilde{n}_2))^2+(Q^{r_{\pi'}}(\tilde{n}_1))^2+(Q^{r_{\pi'}}(\tilde{n}_2))^2+(Q^{r_{\eta'}}(\tilde{n}_1))^2+(Q^{r_{\eta'}}(\tilde{n}_2))^2\}\end{aligned}$$

So, $0 \le s(\tilde{n}_1,\tilde{n}_2) \le 1$.
2) Here,

$$\begin{aligned}s(\tilde{n}_1,\tilde{n}_2) =& 2\{\alpha_1\alpha_2+\beta_1\beta_2+\delta_1\delta_2+w_{\tilde{n}_1}w_{\tilde{n}_2}+u_{\tilde{n}_1}u_{\tilde{n}_2}+v_{\tilde{n}_1}v_{\tilde{n}_2}+Q^{l_\mu}(\tilde{n}_1)Q^{l_\mu}(\tilde{n}_2)+Q^{l_{\pi'}}(\tilde{n}_1)Q^{l_{\pi'}}(\tilde{n}_2)\\ &+Q^{l_{\eta'}}(\tilde{n}_1)Q^{l_{\eta'}}(\tilde{n}_2)+Q^{r_\mu}(\tilde{n}_1)Q^{r_\mu}(\tilde{n}_2)+Q^{r_{\pi'}}(\tilde{n}_1)Q^{r_{\pi'}}(\tilde{n}_2)+Q^{r_{\eta'}}(\tilde{n}_1)Q^{r_{\eta'}}(\tilde{n}_2)\}/\{\alpha_1^2+\alpha_2^2\\ &+\beta_1^2+\beta_2^2+\delta_1^2+\delta_2^2+w_{\tilde{n}_1}^2+w_{\tilde{n}_2}^2+u_{\tilde{n}_1}^2+u_{\tilde{n}_2}^2+v_{\tilde{n}_1}^2+v_{\tilde{n}_2}^2+(Q^{l_\mu}(\tilde{n}_1))^2+(Q^{l_\mu}(\tilde{n}_2))^2\\ &+(Q^{l_{\pi'}}(\tilde{n}_1))^2+(Q^{l_{\pi'}}(\tilde{n}_2))^2+(Q^{l_{\eta'}}(\tilde{n}_1))^2+(Q^{l_{\eta'}}(\tilde{n}_2))^2+(Q^{r_\mu}(\tilde{n}_1))^2+(Q^{r_\mu}(\tilde{n}_2))^2\\ &+(Q^{r_{\pi'}}(\tilde{n}_1))^2+(Q^{r_{\pi'}}(\tilde{n}_2))^2+(Q^{r_{\eta'}}(\tilde{n}_1))^2+(Q^{r_{\eta'}}(\tilde{n}_2))^2\}\\ &=2\{\alpha_2\alpha_1+\beta_2\beta_1+\delta_2\delta_1+w_{\tilde{n}_2}w_{\tilde{n}_1}+u_{\tilde{n}_2}u_{\tilde{n}_1}+v_{\tilde{n}_2}v_{\tilde{n}_1}+Q^{l_\mu}(\tilde{n}_2)Q^{l_\mu}(\tilde{n}_1)+Q^{l_{\pi'}}(\tilde{n}_2)Q^{l_{\pi'}}(\tilde{n}_1)\\ &+Q^{l_{\eta'}}(\tilde{n}_2)Q^{l_{\eta'}}(\tilde{n}_1)+Q^{r_\mu}(\tilde{n}_2)Q^{r_\mu}(\tilde{n}_1)+Q^{r_{\pi'}}(\tilde{n}_2)Q^{r_{\pi'}}(\tilde{n}_1)+Q^{r_{\eta'}}(\tilde{n}_2)Q^{r_{\eta'}}(\tilde{n}_1)\}/\{\alpha_2^2+\alpha_1^2\\ &+\beta_2^2+\beta_1^2+\delta_2^2+\delta_1^2+w_{\tilde{n}_2}^2+w_{\tilde{n}_1}^2+u_{\tilde{n}_2}^2+u_{\tilde{n}_1}^2+v_{\tilde{n}_2}^2+v_{\tilde{n}_1}^2+(Q^{l_\mu}(\tilde{n}_2))^2+(Q^{l_\mu}(\tilde{n}_1))^2\\ &+(Q^{l_{\pi'}}(\tilde{n}_2))^2+(Q^{l_{\pi'}}(\tilde{n}_1))^2+(Q^{l_{\eta'}}(\tilde{n}_2))^2+(Q^{l_{\eta'}}(\tilde{n}_1))^2+(Q^{r_\mu}(\tilde{n}_2))^2+(Q^{r_\mu}(\tilde{n}_1))^2\\ &+(Q^{r_{\pi'}}(\tilde{n}_2))^2+(Q^{r_{\pi'}}(\tilde{n}_1))^2+(Q^{r_{\eta'}}(\tilde{n}_2))^2+(Q^{r_{\eta'}}(\tilde{n}_1))^2\}=s(\tilde{n}_2,\tilde{n}_1)\end{aligned}$$

3) If $\tilde{n}_1 = \tilde{n}_2$, then $\alpha_1 = \alpha_2, \beta_1 = \beta_2, \delta_1 = \delta_2, w_{\tilde{n}_1} = w_{\tilde{n}_2}, u_{\tilde{n}_1} = u_{\tilde{n}_2}, v_{\tilde{n}_1} = v_{\tilde{n}_2}$ So, $s(\tilde{n}_1,\tilde{n}_2) = 1$.

Conversely, if $s(\tilde{n}_1,\tilde{n}_2)=1$, we have

$$2\{\alpha_1\alpha_2+\beta_1\beta_2+\delta_1\delta_2+w_{\tilde{n}_1}w_{\tilde{n}_2}+u_{\tilde{n}_1}u_{\tilde{n}_2}+v_{\tilde{n}_1}v_{\tilde{n}_2}+Q^{l_\mu}(\tilde{n}_1)Q^{l_\mu}(\tilde{n}_2)+Q^{l_{\pi'}}(\tilde{n}_1)Q^{l_{\pi'}}(\tilde{n}_2)$$
$$+Q^{l_{\eta'}}(\tilde{n}_1)Q^{l_{\eta'}}(\tilde{n}_2)+Q^{r_\mu}(\tilde{n}_1)Q^{r_\mu}(\tilde{n}_2)+Q^{r_{\pi'}}(\tilde{n}_1)Q^{r_{\pi'}}(\tilde{n}_2)+Q^{r_{\eta'}}(\tilde{n}_1)Q^{r_{\eta'}}(\tilde{n}_2)\}=$$
$$\{\alpha_1^2+\alpha_2^2+\beta_1^2+\beta_2^2+\delta_1^2+\delta_2^2+w_{\tilde{n}_1}^2+w_{\tilde{n}_2}^2+u_{\tilde{n}_1}^2+u_{\tilde{n}_2}^2+v_{\tilde{n}_1}^2+v_{\tilde{n}_2}^2+(Q^{l_\mu}(\tilde{n}_1))^2+(Q^{l_\mu}(\tilde{n}_2))^2$$
$$+(Q^{l_{\pi'}}(\tilde{n}_1))^2+(Q^{l_{\pi'}}(\tilde{n}_2))^2+(Q^{l_{\eta'}}(\tilde{n}_1))^2+(Q^{l_{\eta'}}(\tilde{n}_2))^2+(Q^{r_\mu}(\tilde{n}_1))^2+(Q^{r_\mu}(\tilde{n}_2))^2+(Q^{r_{\pi'}}(\tilde{n}_1))^2+(Q^{r_{\pi'}}(\tilde{n}_2))^2$$
$$+(Q^{r_{\eta'}}(\tilde{n}_1))^2+(Q^{r_{\eta'}}(\tilde{n}_2))^2\}$$

Then,

$$\begin{aligned}&\{(\alpha_1-\alpha_2)^2+(\beta_1-\beta_2)^2+(\delta_1-\delta_2)^2+(w_{\tilde{n}_1}-w_{\tilde{n}_2})^2+(u_{\tilde{n}_1}-u_{\tilde{n}_2})^2+(v_{\tilde{n}_1}-v_{\tilde{n}_2})^2\\&+(Q^{l_\mu}(\tilde{n}_1)-Q^{l_\mu}(\tilde{n}_2))^2+(Q^{l_{\pi'}}(\tilde{n}_1)-Q^{l_{\pi'}}(\tilde{n}_2))^2+(Q^{l_{\eta'}}(\tilde{n}_1)\\&-Q^{l_{\eta'}}(\tilde{n}_2))^2+(Q^{r_\mu}(\tilde{n}_1)-Q^{r_\mu}(\tilde{n}_2))^2+(Q^{r_{\pi'}}(\tilde{n}_1)-Q^{r_{\pi'}}(\tilde{n}_2))^2+(Q^{r_{\eta'}}(\tilde{n}_1)-Q^{r_\mu}(\tilde{n}_2))^2\}=0\end{aligned}$$

Hence,

$$\begin{aligned}&\alpha_1=\alpha_2,\beta_1=\beta_2,\delta_1=\delta_2,w_{\tilde{n}_1}=w_{\tilde{n}_2},u_{\tilde{n}_1}=u_{\tilde{n}_2},v_{\tilde{n}_1}=v_{\tilde{n}_2}\\&Q^{l_\mu}(\tilde{n}_1)=Q^{l_\mu}(\tilde{n}_2),Q^{l_{\pi'}}(\tilde{n}_1)=Q^{l_{\pi'}}(\tilde{n}_2),Q^{l_{\eta'}}(\tilde{n}_1)=Q^{l_{\eta'}}(\tilde{n}_2),\\&Q^{r_\mu}(\tilde{n}_1)=Q^{r_\mu}(\tilde{n}_2),Q^{r_{\pi'}}(\tilde{n}_1)=Q^{r_{\pi'}}(\tilde{n}_2),Q^{r_{\eta'}}(\tilde{n}_1)=Q^{r_\mu}(\tilde{n}_2)\end{aligned}$$

So, $\tilde{n}_1=\tilde{n}_2$. □

Property 1: If $\tilde{n}_1=\langle(\alpha,\alpha,\alpha);1,0,0\rangle$ and $\tilde{n}_2=\langle(\beta,\beta,\beta);1,0,0\rangle$, then

$$s(\tilde{n}_1,\tilde{n}_2)=\frac{2\{9\alpha\beta+1\}}{9\alpha^2+9\beta^2+2}.$$

Property 2: If $\tilde{n}_1=\langle(\alpha,\alpha,\alpha);1,0,0\rangle$ and $\tilde{n}_2=\langle(0,0,0);1,0,0\rangle$, then

$$s(\tilde{n}_1,\tilde{n}_2)=\frac{2\{1\}}{9\alpha^2+2}.$$

Property 3: If $\tilde{n}_1=\langle(1,1,1);1,0,0\rangle$ and $\tilde{n}_2=\langle(0,0,0);1,0,0\rangle$, then

$$s(\tilde{n}_1,\tilde{n}_2)=\frac{2\{1\}}{11}.$$

Property 4: If $\tilde{n}_1=\langle(\alpha,\alpha,\alpha);1,0,0\rangle$, then

$$s(\tilde{n}_1,\tilde{n}_1)=\frac{2\{9\alpha^2+1\}}{18\alpha^2+2}.$$

6 SVTriN-TOPSIS Approach based on Novel Similarity Measure

The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a prominent method in multi-criteria decision-making (MCDM), commonly used to rank alternatives based on their proximity to an ideal solution. However, when dealing with neutrosophic environmentswhich are characterized by indeterminate and inconsistent informationthe traditional TOPSIS approach faces significant challenges. The method often struggles with uncertainty and contradictions in the data, potentially leading to less reliable or inaccurate outcomes. To address these challenges, we propose an innovative similarity measure that, when integrated into the TOPSIS framework, enables more effective handling of neutrosophic data. Our similarity measure calculates the closeness between Single-Valued Triangular Neutrosophic Numbers (SVTriNs) by thoroughly considering the components of truth-membership, indeterminacy-membership, and falsity-membership. By encompassing all aspects of neutrosophic information, this approach allows for a more precise and comprehensive comparison of alternatives.

Incorporating this similarity measure into the TOPSIS method modifies the traditional distance calculations to better align with neutrosophic data. This integration results in more reliable and meaningful rankings of alternatives by effectively managing the uncertainties and contradictions inherent in the data. Consequently, the decision-making process is enhanced, and the outcomes are more closely aligned with the complex and uncertain realities of real-world scenarios. The following steps express the detail of applying the suggested similarity measure in the SVTriN-TOPSIS method to deal with an MCDM problem with $\mathscr{o}$ candidates and $\mathscr{g}$ criteria.

S1.

a) Define the MCDM problem.

b) Identify the set of candidates; $O = o_1, o_2, \ldots, o_{\mathscr{o}}$.

c) Identify the set of criteria; $G = g_1, g_2, \ldots, g_{\mathscr{g}}$.

d) Identify the weight vector of criteria; $\xi = (\varsigma_1, \varsigma_2, \ldots, \varsigma_g).(\sum_{j=1}^{g} \varsigma_j = 1, \varsigma_j \in [0,1])$

S2. Organize SVTriN-decision matrix based on the evaluations of criteria from decision-maker opinion

$$D = [\tilde{n}_{ij}]_{\mathscr{o}*\mathscr{g}}, \text{where} \tilde{n}_{ij} = \langle (\alpha_{ij}, \beta_{ij}, \delta_{ij}); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \rangle.$$

S3. Calculate the normalized SVTriN-decision matrix

$$\tilde{D} = [\tilde{\tilde{n}}_{ij}]_{\mathscr{o}*\mathscr{g}},$$

where

$$\tilde{\tilde{n}}_{ij} = \left\langle \left(\tilde{\alpha}_{ij}, \tilde{\beta}_{ij}, \tilde{\delta}_{ij} \right); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \right\rangle = \left\langle \left(\frac{\alpha_{ij}}{\sum_{i=1}^{o} (\alpha_{ij})^2}, \frac{\beta_{ij}}{\sum_{i=1}^{o} (\beta_{ij})^2}, \frac{\delta_{ij}}{\sum_{i=1}^{o} (\delta_{ij})^2} \right); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \right\rangle$$

for the benefit criteria (BC), and

$$\tilde{\tilde{n}}_{ij} = \left\langle \left(\tilde{\alpha}_{ij}, \tilde{\beta}_{ij}, \tilde{\delta}_{ij} \right); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \right\rangle = \left\langle \left(\frac{\alpha_{ij}}{\sum_{i=1}^{o} (1/\alpha_{ij})^2}, \frac{\beta_{ij}}{\sum_{i=1}^{o} (1/\beta_{ij})^2}, \frac{\delta_{ij}}{\sum_{i=1}^{o} (1/\delta_{ij})^2} \right); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \right\rangle$$

for the cost criteria (CC).

S4. From the Definition 2.6. Eq (11), obtain the weighted normalized SVTriN-decision matrix.

$$\tilde{D}^{\varsigma} = [\tilde{\tilde{n}}_{ij}^{\varsigma}]_{\mathscr{o}*\mathscr{g}},$$

where

$$\tilde{\tilde{n}}_{ij}^{\varsigma} = \langle (\tilde{\alpha}_{ij}^{\varsigma}, \tilde{\beta}_{ij}^{\varsigma}, \tilde{\delta}_{ij}^{\varsigma}); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \rangle = \langle (\varsigma\tilde{\alpha}_{ij}, \varsigma\tilde{\beta}_{ij}, \varsigma\tilde{\delta}_{ij}); w_{\tilde{n}_{ij}}, u_{\tilde{n}_{ij}}, v_{\tilde{n}_{ij}} \rangle.$$

S5. Determine the SVTriN-positive ideal solution and SVTriN-negative ideal solution from $\tilde{D}^{\varsigma}$. The following equations are applied in this step.

$$ID^P = [(\tilde{\tilde{n}}_j^{\varsigma})^+]_{1*\mathscr{g}}, \text{ where } (\tilde{\tilde{n}}_j^{\varsigma})^+ = \langle (\max_i \tilde{\alpha}_{ij}^{\varsigma}, \max_i \tilde{\beta}_{ij}^{\varsigma}, \max_i \tilde{\delta}_{ij}^{\varsigma}); \max_i w_{\tilde{n}_{ij}}, \min_i u_{\tilde{n}_{ij}}, \min_i v_{\tilde{n}_{ij}} \rangle.$$

$$ID^N = [(\tilde{\tilde{n}}_j^{\varsigma})^-]_{1*\mathscr{g}} \text{ where } (\tilde{\tilde{n}}_j^{\varsigma})^- = \langle (\min_i \tilde{\alpha}_{ij}^{\varsigma}, \min_i \tilde{\beta}_{ij}^{\varsigma}, \min_i \tilde{\delta}_{ij}^{\varsigma}); \min_i w_{\tilde{n}_{ij}}, \max_i u_{\tilde{n}_{ij}}, \max_i v_{\tilde{n}_{ij}} \rangle.$$

S6. Compute the similarity measure between each candidate and the SVTriN-ideal solutions according to equation (17).

The similarity measure to the SVTriN-positive ideal solution is

$$S_i^+ = \sum_{j=1}^{g} s(\tilde{\tilde{n}}_{ij}^{\varsigma}, (\tilde{\tilde{n}}_j^{\varsigma})^+) \qquad i = 1, 2, \ldots, o.$$

And, the similarity measure to the SVTriN-negative ideal solution is

$$S_i^- = \sum_{j=1}^{g} s(\tilde{\tilde{n}}_{ij}^{\varsigma}, (\tilde{\tilde{n}}_j^{\varsigma})^-) \qquad i = 1, 2, \ldots, o.$$

S7. Obtain the relative closeness of each candidate to the SVTriN-ideal solutions:

$$RC_i = \frac{S_i^+}{(S_i^- + S_i^+)}, \qquad i = 1, 2, \ldots, o.$$

S8. Rank alternatives based on RC_i values.

7 Numerical Example and Comparative Analysis

Every year, the lives of thousands of people around the world are affected by natural disasters such as floods, earthquakes, fires, severe storms, and droughts. In order to prepare preventive measures, governments can assess the risk of accidents in high-risk areas using the decision problem. As an example, here, to assess the risk of flooding for four coastal cities in India, we want to use the four criteria provided by Yang [26], named Disaster-inducing factors ($\mathcal{g}_1$), Hazard-formative environment ($\mathcal{g}_2$), Characters of hazard affected body ($\mathcal{g}_3$), and Social disaster bearing capacity ($\mathcal{g}_4$) [7]. (see Fig. 1)

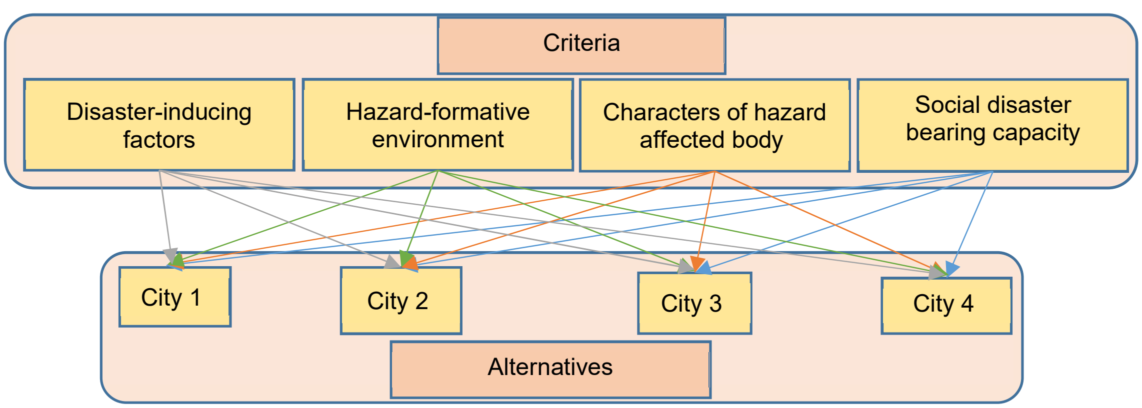

Figure 1: Hierarchical system between criteria and alternatives for flood disaster.

Here, cities have been evaluated in criteria with the SVTriN-numbers (according to the information in Table 1). The first and second criteria are the cost type, and the third and fourth criteria are the profit type. In addition, the weights of the criteria are considered equally.

Table 1: SVTriN-decision matrix.

Alts	Criteria $\mathscr{g}_1$	$\mathscr{g}_2$	$\mathscr{g}_3$	$\mathscr{g}_4$
o_1	$\left\langle \begin{array}{c}(4.7, 5.4, 8.5); \\ 0.4, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(5.7, 6.8, 8.7); \\ 0.6, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(5.3, 6.6, 9.8); \\ 0.3, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(4.4, 5.8, 7.3); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$
o_2	$\left\langle \begin{array}{c}(6.2, 7.5, 8.3); \\ 0.4, 0.2, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(7.2, 7.6, 8.2); \\ 0.5, 0.3, 0.4\end{array} \right\rangle$	$\left\langle \begin{array}{c}(6.2, 8.8, 9.0); \\ 0.6, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(6.3, 7.4, 8.8); \\ 0.7, 0.5, 0.6\end{array} \right\rangle$
o_3	$\left\langle \begin{array}{c}(5.5, 6.3, 7.3); \\ 0.8, 0.2, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(4.7, 6.8, 8.5); \\ 0.7, 0.3, 0.6\end{array} \right\rangle$	$\left\langle \begin{array}{c}(7.1, 8.6, 8.9); \\ 0.5, 0.3, 0.7\end{array} \right\rangle$	$\left\langle \begin{array}{c}(6.6, 8.7, 10); \\ 0.6, 0.3, 0.2\end{array} \right\rangle$
o_4	$\left\langle \begin{array}{c}(5.5, 6.3, 7.3); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(4.9, 6.3, 8.1); \\ 0.3, 0.4, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(7.5, 8.4, 8.3); \\ 0.7, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(6.9, 8.3, 10); \\ 0.8, 0.3, 0.4\end{array} \right\rangle$

From the S3. of the presented method, the weighted normalized SVTriN-decision matrix are obtained according to Table 2.

Table 2: Normalized SVTriN-decision matrix.

Alts	Criteria $\mathscr{g}_1$	$\mathscr{g}_2$	$\mathscr{g}_3$	$\mathscr{g}_4$
o_1	$\left\langle \begin{array}{c}(0.31, 0.58, 0.81); \\ 0.4, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.31, 0.51, 0.73); \\ 0.6, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.29, 0.40, 0.74); \\ 0.3, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.24, 0.38, 0.60); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$
o_2	$\left\langle \begin{array}{c}(0.32, 0.42, 0.62); \\ 0.4, 0.2, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.33, 0.45, 0.58); \\ 0.5, 0.3, 0.4\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.34, 0.54, 0.68); \\ 0.6, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.53, 0.48, 0.72); \\ 0.7, 0.5, 0.6\end{array} \right\rangle$
o_3	$\left\langle \begin{array}{c}(0.37, 0.50, 0.70); \\ 0.8, 0.2, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.32, 0.51, 0.89); \\ 0.7, 0.3, 0.6\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.39, 0.53, 0.68); \\ 0.5, 0.3, 0.7\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.36, 0.57, 0.82); \\ 0.6, 0.3, 0.2\end{array} \right\rangle$
o_4	$\left\langle \begin{array}{c}(0.37, 0.50, 0.70); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.33, 0.55, 0.85); \\ 0.3, 0.4, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.42, 0.52, 0.63); \\ 0.7, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.38, 0.54, 0.82); \\ 0.8, 0.3, 0.4\end{array} \right\rangle$

Furthermore, from the S4. the weighted normalized SVTriN-decision matrix is obtained according to Table 3.

Table 3: Weighted normalized SVTriN-decision matrix.

Alts	Criteria $\mathscr{g}_1$	$\mathscr{g}_2$	$\mathscr{g}_3$	$\mathscr{g}_4$
o_1	$\left\langle \begin{array}{c}(0.08, 0.15, 0.20); \\ 0.4, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.08, 0.13, 0.18); \\ 0.6, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.07, 0.10, 0.19); \\ 0.3, 0.6, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.06, 0.10, 0.15); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$
o_2	$\left\langle \begin{array}{c}(0.08, 0.11, 0.16); \\ 0.4, 0.2, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.08, 0.11, 0.15); \\ 0.5, 0.3, 0.4\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.09, 0.14, 0.17); \\ 0.6, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.13, 0.12, 0.18); \\ 0.7, 0.5, 0.6\end{array} \right\rangle$
o_3	$\left\langle \begin{array}{c}(0.09, 0.13, 0.18); \\ 0.8, 0.2, 0.2\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.08, 0.13, 0.22); \\ 0.7, 0.3, 0.6\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.10, 0.13, 0.17); \\ 0.5, 0.3, 0.7\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.10, 0.14, 0.21); \\ 0.6, 0.3, 0.2\end{array} \right\rangle$
o_4	$\left\langle \begin{array}{c}(0.09, 0.13, 0.18); \\ 0.7, 0.3, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.08, 0.14, 0.21); \\ 0.3, 0.4, 0.3\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.11, 0.13, 0.16); \\ 0.7, 0.4, 0.5\end{array} \right\rangle$	$\left\langle \begin{array}{c}(0.10, 0.14, 0.21); \\ 0.8, 0.3, 0.4\end{array} \right\rangle$

From the information in Table 3, ID^P and ID^N are compute as follows

$$ID^P = \left[\left\langle \begin{array}{c} (0.09, 0.15, 0.20); \\ 0.8, 0.2, 0.2 \end{array} \right\rangle \left\langle \begin{array}{c} (0.08, 0.14, 0.22); \\ 0.7, 0.3, 0.3 \end{array} \right\rangle \left\langle \begin{array}{c} (0.11, 0.14, 0.19); \\ 0.7, 0.3, 0.2 \end{array} \right\rangle \left\langle \begin{array}{c} (0.13, 0.14, 0.21); \\ 0.8, 0.3, 0.2 \end{array} \right\rangle \right],$$

$$ID^N = \left[\left\langle \begin{array}{c} (0.08, 0.11, 0.16); \\ 0.4, 0.6, 0.3 \end{array} \right\rangle \left\langle \begin{array}{c} (0.08, 0.11, 0.15); \\ 0.3, 0.4, 0.6 \end{array} \right\rangle \left\langle \begin{array}{c} (0.08, 0.10, 0.16); \\ 0.3, 0.6, 0.7 \end{array} \right\rangle \left\langle \begin{array}{c} (0.06, 0.10, 0.15); \\ 0.6, 0.5, 0.6 \end{array} \right\rangle \right]$$

The results from S6 to S8 are sumerized in Table 4.

Table 4: Rankig Altratives.

Alts	S_i^+	S_i^-	RC_i	Rankig Alts
o_1	3.5431	3.5852	0.4971	4
o_2	3.6214	3.6594	0.4974	3
o_3	3.7371	3.4519	0.5198	1
o_4	3.7531	3.5792	0.5119	2

From Table 4, it can be observed that the desirable alterative is o_3.

Although in section 3, by presenting several counterexamples, the limitations of some existing tools for solving the MCDM problem with the SVTriN-numbers were pointed out. In this section, a summary of the results related to the ranking of options based on the proposed method compared to other methods is given in Table 5.

Table 5: Results of multi-criteria decision-making based on different tools.

Solution Method	Ordering	Best Choice
Deli & Subass approach [4]	$o_3 \succ o_4 \succ o_1 \succ o_2$	
Deli & Subass approach ($\theta = 0.5$) [5]	$o_3 \succ o_4 \succ o_1 \succ o_2$	
Yes approach [28]	$o_3 \succ o_4 \succ o_1 \succ o_2$	
Liang et al.s approach [12]	$o_3 \succ o_4 \succ o_1 \succ o_2$	o_3
Biswas et al.s approach [1]	$o_3 \succ o_4 \succ o_1 \succ o_2$	
Garai et al.s approach [7]	$o_3 \succ o_2 \succ o_4 \succ o_1$	
Proposed approach	$o_3 \succ o_4 \succ o_2 \succ o_1$	

As can be seen, although the methods presented in Table 5 have differences in the details of ranking the options, in all methods, the optimal choice is option 3.

8 Conclusion

Due to the lack of resources related to the similarity measure for continuous SVTriN numbers, our primary focus in this study has been providing a new similarity measure between numbers. Therefore, the innovations of this research are as follows:

- Introducing a new and logical similarity measure for SVTriN numbers
- Examining the characteristics of the proposed similarity measure
- Practical application of the proposed similarity measure in the TOPSIS method
- Solving the SVTriN-MCDM decision-making problem using the proposed hybrid method and comparing the results with some existing methods

This study successfully bridges the gap in existing MCDM approaches by integrating the proposed similarity measure with the TOPSIS method, thereby significantly enhancing the decision-making process in neutrosophic environments. By effectively handling the truth, indeterminacy, and falsity components inherent in SVTriNs, the integrated approach provides more accurate and consistent rankings of alternatives. This addresses the limitations of existing methods that often struggle with indeterminate information and may produce contradictory results.

It should be noted that applying the proposed similarity measure on SVTriN numbers is not limited to solving the MCDM problem. The enhanced SVTriN-TOPSIS method not only improves the reliability of decision-making outcomes but also offers practical applicability in various fields where uncertainty and indeterminacy are prevalent. The illustrative example demonstrates the practicality and effectiveness of our approach, showcasing its potential for widespread adoption.

The proposed SVTriN measure can be used in clustering methods, disease diagnosis, pattern recognition, and etc., with neutrosophic data. Future research could explore the application of this integrated method in more complex decision-making scenarios and compare its performance with other advanced MCDM methods. Additionally, extensions to other types of neutrosophic sets and integration with other decision-making frameworks could further enhance its utility.

In addition, proving some features related to the proposed similarity measure and its relationship with the concept of distance measure and entropy measure are practical issues for future research. The newly introduced similarity measure in this study paves the way for pioneering research possibilities in domains such as clustering methods, pattern recognition, and beyond. Moreover, future studies could investigate the connections between the proposed similarity measure and other metrics, including distance and entropy.

Conflicts of Interest: All co-authors have seen and agree with the manuscript's contents, and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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