

Investigation of energy transport velocity by leaky surface plasmon waves of graphene

Vahideh Mohadesi^{1,2,*}

¹ Department of Electrical Engineering, Sara. C., Islamic Azad University, Sarab, Iran

² Department of Physics, CT. C., Islamic Azad University, Tehran, Iran

ABSTRACT

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In graphene-based layered structures, the presence of inhomogeneous neighboring materials can significantly influence the propagation dynamics of surface plasmon waves. This interaction offers the potential to excite leaky surface plasmon waves in graphene. In this study, a configuration is investigated where a high refractive index medium is positioned near a graphene sheet, separated by a micrometer-scale gap. By systematically varying the gap thickness and tuning the Fermi level of graphene the resulting changes are analyzed in the dispersion characteristics and energy transport velocity of the leaky surface plasmon waves. Our findings shed light on the critical role of these parameters in optimizing plasmonic wave behavior, offering new avenues for advanced photonic and optoelectronic applications.

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INTRODUCTION

Surface Plasmon Polaritons (SPPs) are surface-bound electromagnetic waves that travel along the interface between dielectrics and materials with free electrons [1]. These waves can exist in two forms: bound SPPs, which are nonradiative and tightly confined near the interface [2-5], and leaky SPPs, where coupling with

free space allows for radiation [6]. The energy transport velocity, or group velocity, of plasmon waves is key to understanding how energy transfers and is confined in nanoscale photonic systems [7, 8]. Recently, graphene, a two-dimensional carbon material arranged in a honeycomb lattice, has gained significant attention for its ability to support SPPs [9-11].

*Corresponding Author Email: v.mohadesi@gmail.com



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Graphene supports transverse magnetic (TM)-polarized SPPs, particularly in the terahertz (THz) and infrared frequency ranges. Due to its exceptional optical and electrical properties, graphene has become a critical focus in modern plasmonic research [12-15].

In graphene, surface plasmon waves can appear as either bound or leaky, depending on the structural configuration [16-18]. Our last researches focused on the characteristics of leaky-type surface plasmons propagating along a graphene sheet in a layered, inhomogeneous structure [19-21]. We investigate their dispersion, propagation length, and radiation properties. However, the energy transport velocity associated with leaky-type surface plasmons in such configurations remains underexplored. This paper addresses this gap by analyzing the effects of leakage on the energy velocity of graphene surface plasmons under various tunable conditions. We derive and numerically solve the dispersion relation for both bound and leaky surface plasmons in a layered dielectric structure with a graphene sheet, with particular focus on the terahertz frequency range.

Basic Equations and Dispersion Relation

The system under investigation, depicted in Fig. 1, includes a monolayer graphene positioned on a substrate, separated from a high refractive index medium by an air gap of thickness d . The dielectric constants of the layers are real and denoted by ϵ_1 , ϵ_2 , and ϵ_3 as illustrated in figure 1. The structure is analyzed within a Cartesian coordinate system and can be simplified to two dimensions (x - z plane) due to its symmetry along the y -axis.

Our investigation focuses on the THz region, where only TM-polarized SPPs are supported in the graphene layer. Consequently, for the considered structure, the magnetic field is oriented along the y -direction, and the graphene can be treated as a zero-thickness layer characterized by its two-dimensional conductivity.

According to Kubo's formula in the THz region of frequency, the intra-band contribution is dominant

and the surface conductivity of graphene for considered time dependency can be determined by Eq.1. Where T represents temperature, ω is frequency, μ is the chemical potential of graphene (which is equal to fermi level of graphene), and τ is the transport relaxation time [22].

Assuming a time dependency of the form $e^{-i\omega t}$ and employing Maxwell's equations, the electromagnetic field components of the TM waves propagating along the $+x$ axis with z normal to the surfaces are given Eq.2. Where $k_{mz}^2 = q^2 - \epsilon_m k_0^2$; $m = 1,2,3$ and it has been assumed $\text{Re}(k_{mz}) > 0$.

In this structure, bound SPPs correspond to non-radiative waves strongly confined to the graphene sheet, identifiable by a field peak on the graphene ($z=0$) and exponential decay in the perpendicular direction. Conversely, leaky SPPs radiate power into space, with their amplitude growing exponentially away from the graphene sheet. Thus, since $\text{Re}(k_{mz}) > 0$, the upper/lower sign of k_{1z} in Eq. 1 corresponds to bound/leaky SPPs [19].

Using Maxwell's equations electric fields can be calculated by magnetic field (Eq.3).

The dispersion equation for bound or leaky SPPs in this configuration is determined by the sign (positive or negative) in the Eq.5. For a leaky wave field launched at $x=0$ and present for $x>0$, it can be interpreted as the radiation field of a finite aperture along the x direction [20]. The coupling of surface waves to far-field radiation occurs by satisfying the momentum-matching condition within a specific angle (θ). The validity criterion for determining the leaky region is the point of intersection of the dispersion curve with the light line of ϵ_1 . Therefore, the dispersion relation with the lower sign is only valid for frequencies lower than the crossing point frequency [19].

The energy transport velocity of bound/leaky surface plasmons of graphene represents the group velocity of the surface waves. This velocity can be

obtained using Eq. 5 and is defined as Eq.6.

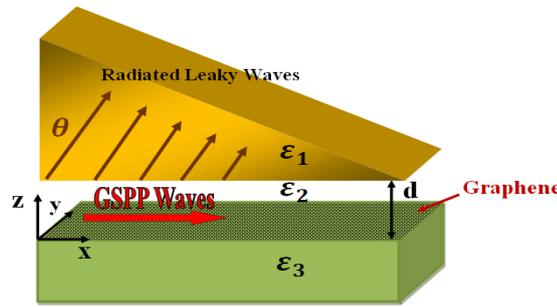


Fig.1: Schematic of a layered structure including a graphene sheet on the substrate and a high refractive index media located in a distance of d from it

$$\sigma_{intra}(\omega, \mu, \tau, T) = \frac{ie^2 \mu}{\pi \hbar^2 (\omega + i/\tau)} \times \left(1 + 2 \frac{k_B T}{\mu} \ln \left(e^{-\frac{\mu}{k_B T}} + 1 \right) \right) \quad (1)$$

$$H_y^1(x, z) = \exp(iqx) \{A_1 e^{\pm k_{1z}(z+d)}\}, \quad z < -d$$

$$H_y^2(x, z) = \exp(iqx) \{A_2 e^{k_{2z}(z+d)} + B_2 e^{-k_{2z}(z+d)}\}, \quad -d < z < 0 \quad (2)$$

$$H_y^3(x, z) = \exp(iqx) \{B_3 e^{-k_{3z}z}\} \quad z > 0$$

$$E_x^m(x, z) = -\frac{i}{\omega \epsilon_0 \epsilon_m} \frac{\partial H_y^m(x, z)}{\partial z} \quad (3)$$

$$E_z^m(x, z) = \frac{i}{\omega \epsilon_0 \epsilon_m} \frac{\partial H_y^m(x, z)}{\partial x}$$

Applying boundary conditions at $z=0$ and $z=-d$ (Eq.4):

$$z = -d, \quad E_x^1(x, -d) = E_x^2(x, -d), \quad H_y^1(x, -d) = H_y^2(x, -d) \quad (4)$$

$$z = 0, \quad E_x^2(x, 0) = E_x^3(x, 0), \quad H_y^2(x, 0) - H_y^3(x, 0) = \sigma E_x^2(x, 0)$$

$$e^{k_{2z}d} \left[\left(\frac{\epsilon_2}{k_{2z}} \pm \frac{\epsilon_1}{k_{1z}} \right) \left(\frac{\epsilon_3}{k_{3z}} + \frac{\epsilon_2}{k_{2z}} + i \frac{\sigma}{\omega \epsilon_0} \right) \right] + e^{-k_{2z}d} \left[\left(\frac{\epsilon_2}{k_{2z}} \mp \frac{\epsilon_1}{k_{1z}} \right) \left(\frac{\epsilon_3}{k_{3z}} - \frac{\epsilon_2}{k_{2z}} + i \frac{\sigma}{\omega \epsilon_0} \right) \right] = 0 \quad (5)$$

$$v_G(\omega, k) = \frac{d\omega}{dk} \quad (6)$$

RESULTS AND DISCUSSIONS

In this section, the numerical results are presented from the previous section concerning the

dispersion relation and energy transport velocity of bound/leaky surface plasmon waves in graphene within the considered structure. We examine two tuning parameters: the air gap thickness (d), which regulates

the coupling between graphene surface waves and far-field radiation in the vicinity, and the chemical potential (μ) of the graphene sheet, which governs its conductivity.

The dispersion curves for the surface plasmon waves in graphene are depicted in Fig. 2. The curves related to $d \rightarrow \infty$ correspond to bound surface plasmon waves, which are nonradiative waves trapped near the graphene sheet. In addition to these curves, Fig.2 shows dispersion curves of leaky surface plasmons for high frequencies.

The effect of the gap thickness and the chemical potential of graphene were also examined on the energy transport velocity of the graphene surface plasmon waves. In Fig.3 we see the energy velocity (v_E) versus the frequency calculated by Eq.6 for graphene surface plasmon waves related to the dispersion equation shown in Fig.2.

As it is observed in Fig. 2 the energy velocity decreases by increasing the frequency. We can see in panel (a) that in low-frequency region, the energy velocity of leaky surface plasmons decreases by

$\mu=2$ eV and different values of d in panel (a) and different values of μ in panel (b), when $\epsilon_1=16$, $\epsilon_2=1$, $\epsilon_3=4$ and $\tau=10$ ps. The light line of ϵ_1 is also shown with the dotted black line. It can be seen that with decreasing the thickness of the air gap (d), dispersion curves deviate from that for bound modes. Furthermore, increasing the chemical potential of graphene (μ) affects the dispersion curves and extends the leaky region to

decreasing of the thickness of the gap. Panel (b) shows that the value of the chemical potential of graphene affects the energy velocity of surface plasmon waves. At a certain frequency, the energy velocity increases when the chemical potential of graphene increases. For low frequencies, the energy velocity of leaky surface plasmons is down-shifted and this effect is more obvious in low levels of μ . As a result of the above discussion, it is clear that the dispersion relation and the energy transport velocity of graphene surface plasmon waves can be controlled in multi-layered structures.

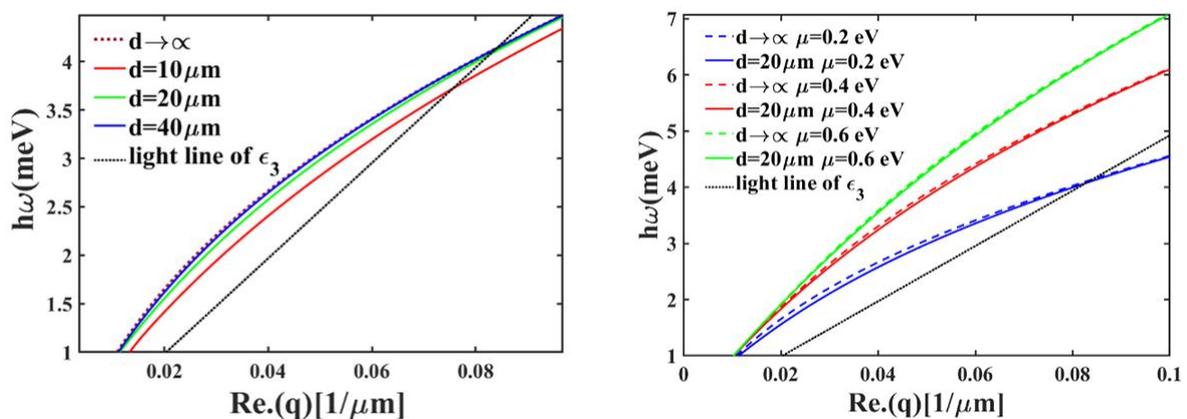


Fig. 2: Dispersion curves of the bound ($d \rightarrow \infty$) and leaky surface plasmon waves of graphene in the considered configuration for (a) $\mu=0.2$ eV and different values of d and (b) different values of μ , when $\epsilon_1 = 16$, $\epsilon_2 = 1$, $\epsilon_3 = 4$ and $\tau = 10$ ps.

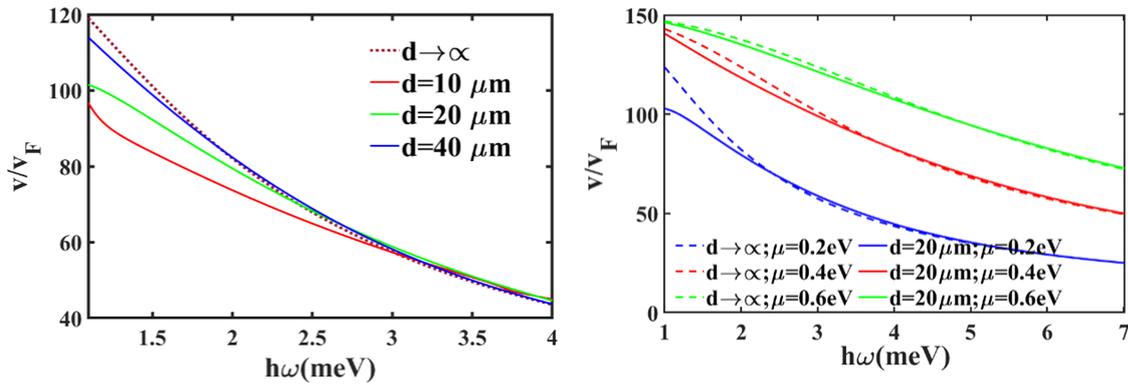


Fig. 3: Energy velocity versus the frequency of the bound ($d \rightarrow \infty$) and leaky surface plasmon waves of graphene in the considered configuration for (a) $\mu = 0.2$ eV and different values of d and (b) different values of μ , when $\epsilon_1 = 16$, $\epsilon_2 = 1$, $\epsilon_3 = 4$ and $\tau = 10$ ps.

CONCLUSION

In conclusion, our investigation focused on a multilayered structure including graphene, which supports the propagation of leaky-type surface plasmon waves. We conducted a thorough analysis of the dispersion relation and energy velocity of leaky waves within this structure. Through numerical simulations, we examined the impact of varying the thickness of the air gap between graphene and a high refractive index medium, as well as the chemical potential of graphene, on both bound and leaky surface plasmons. Numerical results demonstrate that as the material with a high refractive index approaches the graphene sheet, the dispersion curve of leaky surface plasmon waves in graphene is influenced. With closer proximity, the difference between the dispersion curve of leaky surface plasmon waves and that of bound waves increases, bringing it closer to the light line of the surrounding medium with a high refractive index. Furthermore, variations in the chemical potential of graphene from 0.2 eV to 0.6 eV strongly affect the dispersion curves of leaky waves and extend the leaky region to higher frequencies. Furthermore, as the air gap narrows, the velocity of surface plasmon waves in this frequency range decreases, with a more significant difference

observed at lower frequencies. Increasing the chemical potential of graphene enhances the energy transfer velocity by leaky surface plasmon waves. These findings could be valuable in the design of tunable sensors and other optoelectronic devices.

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