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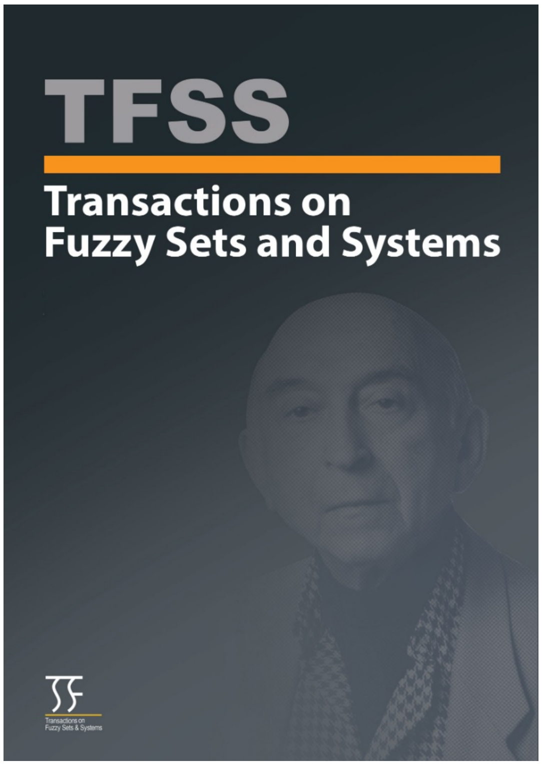

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Novel Generalisation of Some Fixed Point Results Using a New Type of Simulation Function

Mohd Hasan*

Abstract. I am utilizing a brand-new simulation function that has previously been developed by eminent mathematicians and that uses fuzzy metric-like spaces to establish new fixed point theorems. Here, this is demonstrated that the current conclusion is unquestionably a unified one that can generalize earlier current results. To further demonstrate the relevance of my findings, a few additional theorems and corollaries are demonstrated. Additionally, several excellent examples are provided to show how useful my findings are. I provide an application of my major finding in the conclusion.

AMS Subject Classification 2020: MSC 54H25; MSC 47H10
Keywords and Phrases: Metric space, Fuzzy metric space, Fuzzy metric-like space, α-admissible MA-simulation function.

1 Introduction

In 1951, Menger pioneered the idea of a metric which is statistical metric; see [1]. Kramosil and Michalek initiated the concept of a new metric called fuzzy metric in 1975([2]), building on the idea of a statistical metric. This idea is what is known as in a short form KM(Kramosil and Michalek)-fuzzy metric. In some ways, a KM(Kramosil and Michalek)- fuzzy metric is comparable to a metric based on statistics, but there are important distinctions in how they are explained and clarified. George and Veeramani [3], who are cited in [3, 4], inconsistently altered the fundamental idea of a KM(Kramosil and Michalek)- fuzzy metric; this improvement is known as a GV(George and Veeramani)-fuzzy metric. This improvement enables a number of realistic examples(some of them are very natural) of fuzzy metrics in unique fuzzy metrics established from measures. GV(George and Veeramani)-fuzzy metrics surface to be much more practical for looking at induced topological structures as well, in addition fuzzy metrics have sparked interest in between experts working in a variety of applied feilds of mathematics in addition to the main zest of many mathematicians based on theory phase of the principle of particularly fuzzy metrics, their topological and sequential components, their completeness, fixed points on maps, etc.

The Banach contraction principle guarantees the existence and uniqueness of a fixed point for a specific type of function. Fuzzy mathematics uses the Banach contraction principle to prove the existence and uniqueness of solutions to some fuzzy equations.

According to fuzzy mathematics, the Banach contraction principle is as follows:

There exists a constant $\alpha \in (0, 1)$ such that if (X, M) is a fuzzy metric space and $T : X \to X$ is a fuzzy contraction

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For every $x, y \in X$, $M(Tx, Ty) \leq \alpha M(x, y)$.

Then, in X, T has a fixed point which is unique.

A fuzzy contraction is a function that maps items to itself in a fuzzy metric space, therefore decreasing the fuzzy distances between those objects. The constant is known as the contraction constant. The Banach contraction principle states that a fuzzy contraction needs to have a clear fixed point in order to exist.

The map $\varsigma : [0,\infty) \times [0,\infty) \to \Re$ supposed to be a function which is simulation , it meets the given requirements:

(ς_1) ς(0, 0) = 0;

(ς_2) ς(r, w) <r - tw ∀t,r,w > 0;

(ς_3) if {r$_n$} and {w$_n$} re-orders(in (0, ∞) s.t.

$$0 < \lim_{n\to\infty} r_n = \lim_{n\to\infty} w_n,$$

if so

$$0 > \limsup_{n\to\infty} \varsigma(r_n, w_n).$$

The notion of simulation function was extended and modified, along with other concepts like b and θ-metric spaces, to produce the fixed point findings. By removing (ς_1), Argoubi [5] refined this idea in the same way, and Roldan et al. [6] simultaneously enhanced condition (ς_3) as follows:

(ς_3)‘ let two sequences {r$_n$} and {w$_n$} in (0, ∞) s.t.

$$\lim_{n\to\infty} w_n = \lim_{n\to\infty} r_n > 0, \ \ \& \ \ w_n > r_n, \ \ \forall \ \ n \in \mathbb{N}$$

if so

$$0 > \limsup_{n\to\infty} \varsigma(r_n, w_n).$$

By including α-admissible mappings, Karapinar [7] demonstrated a more broadly applicable version of the finding of Khojasteh [8].

In this paper, I prove a new type fixed point theorem in fuzzy metric-like spaces using the recently created MA-simulation function, a novel simulation function proposed by Perveen and Imdad [9] (see also [10]). Furthermore, I show that our results can be used more widely to synthesize several current conclusions from the literature and develop a few new findings as corollaries. I also offer a solid illustration to back up our conclusion. As an application of my theorems, I finally give the Fredholm nonlinear integral equation, which has an existential solution.

2 Preliminaries

Definition 2.1. [11] A t-norm which is continouos of a mapping(binary operation) $\star : (-\infty, 1] \cap [0,\infty) \times (-\infty, 1] \cap [0,\infty) \to (-\infty, 1] \cap [0,\infty)$ if the subsequent circumstances holds:

(I) $\star$ is continuous evrywhere;

(II) $\star$ is associative & commutative;

(III) for all $r \in [0, 1]$, $\quad r \star 1 = a$;

(IV) $\forall \quad r, s, t, u \in [0,1]$ $r \star s \leq t \star u$ whenever $r \leq t$ and $s \leq u$.

For further details on continuous t-norms and their classical instances, consider the t-norms of maximum, product, and minimum, which are represented by the symbols $T_l(r,s), = \max(r+s-1,0)$, $T_p(r,s) = rs$ & $T_m(r,s) = \min(r,s)$, respectively.

The definition below was provided in 1994 by George and Veeramani ([3]), who also made major changes to Kramosil and Michalek's definition ([2]).

Definition 2.2. [3] Given that X is arbitrary and M be a fuzzy set, and $\star$ be a t-norm which is a continuous on this triplet, it is called to be a FMS that meets the criteria listed below, $s, t > 0$ & $\forall \quad x, z, y \in X$:

(I) $M(x, y, t)$ greator than zero;

(II) $M(x, y, t)$ equals to 1, for all $t > 0$ if and only if x and y are same;

(III) $M(x, y, t)$ is commutative:

(IV) $M(x, y, t)$ is holds traingular inequality i.e. $M(x, y, t) \star M(y, z, s) \leq M(x, z, t+s)$;

(V) $M(x, y, t)$ is continouos defined as $M(x, y, .) : (0, \infty) \to [0,1]$.

If x is not equal to y, $M(x, y, t)$ is greater than 0 and less than 1, as shown by (1) and (2) (cf. [12]), for all $t > 0$. It is clear that $M(x, y, .)$ is an increasing function for any $x, y \in X$. See the following works for further details: Citations for George and Veeramani [3], Gregori et al. [12], Roldan et al. [13] and Sapena [14].

Remark 2.3. **Remark 2.3.**[15] $M(x, y, .)$ be a non-decreasing function on $\forall \ x, y \in X$ & $\Re \cap (0, \infty)$.

Definition 2.4. [16] Let $\star$ is a continuous t-norm on the triplet $(X, \mathbb{F}, \star)$, here $\mathbb{F}$ is a fuzzy set and the set X be an arbitrary set. This triplet is referred to as a fuzzy metric-like space if it meets the conditions listed below $t, s > 0$ & $\forall \quad x, y, z \in X$.

(I) $\mathbb{F}(x, y, t)$ is greator than 0;

(II) If $\mathbb{F}(x, y, t)$ is equals to 1, then $x = y$, $\forall \ t > 0$,;

(III) $\mathbb{F}(x, y, t)$ is commutative;

(IV) $\mathbb{F}(x, y, t) \star F(y, z, s) \leq \mathbb{F}(x, z, t+s)$;

(V) $\mathbb{F}$ is continuous where $\mathbb{F}(x, y, .) : \Re \cap (0, \infty) \to [0,1]$.

In this case, $\mathbb{F}$ (fitted with $\star$) is described as a fuzzy metric-like on X.

Remark 2.5. This fuzzy metric-like space has an additional constraint, which is that $\mathbb{F}(x, x, t)$ may be smaller than 1 for all $t > 0$ for all (or may be some) $x \in X$. Shukla et al. [16] to make this argument. Additionally, for all $t > 0$ and for all $x \in X$, any fuzzy metric space is the same as a fuzzy metric-like space when $F(x, x, t) = 1$.

The fact that the value of $\mathbb{F}(x, x, t)$ may be less than 1 indicates that the definition above is usable when the degree of proximity between y and x is not the same, whereas this is not the case for the George and Veeramani [3] definition.

Example 2.6. Let this $(X, \mathbb{F}, \star_l)$ is a fuzzy metric-like space, with $X = \Re \cap [0,1]$, then, the $\mathbb{F}$ be a fuzzy set is defined like this;

$$\mathbb{F}(x,y,t) = \begin{cases} 1 & \text{if} \quad \text{x and y are same and equal to 0;} \\ \frac{x+y}{2} & \text{if} \quad \text{else} \end{cases},$$

$\forall \ \ t > 0$.

The following propositions can be used to identify different examples of triplet $(X, \mathbb{F}, \star)$ (fuzzy metric-like spaces).

Proposition 2.7. *[16] Let metric-like space be (X, σ) (see Harandi [17]. The fuzzy set $\mathbb{F}$ is provided by, and $(X, \mathbb{F}, \star_p)$ is a fuzzy metric-like space*

$$\mathbb{F}(x,y,t) = \frac{kt^n}{kt^n + m\sigma(x,y)}$$

$\forall \ x, y \in X, \ t > 0, \ m > 0$ *and* $n \geq 1$ *where* $k \in \Re$.

Remark 2.8. [16] Given that $k = n = m = 1$ in standard metric-like space induces a fuzzy metric-like space, this fuzzy metric-like space is known as standard fuzzy metric-like space. This fuzzy metric-like space is where

$$F_\sigma(x,y,t) = \frac{t}{t + \sigma(x,y)}$$

$\forall \ \ t > 0, \ \ x, y \in X$.

Proposition 2.9. *[16] Let's say that the fuzzy set F is defined as $\mathbb{F}(x,y,t) = e^{-\frac{\sigma(x,y)}{t^n}}$, where $n \geq 1$ (here, (X, σ) is metric-like sapce) is true for any $x, y \in X \ , t > 0$. Then $(X, \mathbb{F}, \star_p)$ is a space that resembles a fuzzy metric.*

Example 2.10. Let $\mathbb{F}$ be a fuzzy set in $X^2 \times \Re \cap (0, \infty)$ by $\mathbb{F}(x,y,t) = \frac{1}{e^{\max\{x,y\}/t}}$ and X be a natural numbers. Here we take prduct t-norm(i.e.. $a \star b = ab$) and $\forall \ \ x, y \in X, t > 0$. Therefore, according to Proposition 2.9, the triplet $(X, \mathbb{F}, \star)$ is not a fuzzy metric space but rather a fuzzy metric-like space since $\sigma(x,y) = \max(x,y)$, for any $x, y \in X$, is a fuzzy metric-like on X as $\mathbb{F}(x,x,t) = \frac{1}{e^{x/t}} \neq 1, \quad \forall \ \ x > 0$ and $t > 0$.

Example 2.11. ([16]) Let $\mathbb{F}$ be a fuzzy set in $X^2 \times (0, \infty)$ by

$$\mathbb{F}(x,y,t) = \begin{cases} \frac{x}{y^3} & \text{if } x \leq y; \\ \frac{y}{x^3} & \text{if } y \leq x \end{cases},$$

for all $x, y \in X, t > 0$. and $X = \Re \cap [0,1]$. Define t-norm by product norm($a \star b = ab$). Then triplet $(X, \mathbb{F}, \star)$ is a fuzzy metric-like space.

Even if we use the minimum $t-$norm $\star_m(a \star b = \min\{a, b\}$ instead of the product $t-$norm $a \star b = ab$ (see [16]), the Propositions 2.7 and 2.9 are still valid.

Proposition 2.12. *If $K > 0$ exists and $\sigma(x,y) \leq K$ for all u, v in X, then (X, σ) is the bounded metric-like space and the fuzzy set $\mathbb{F}$ is defined by $\mathbb{F}(u,v,t) = 1 - \frac{\sigma(u,v)}{K+t}$, where $t > 0$ for all u, v in X. A fuzzy metric-like space is thus represented by the triplet $(X, \mathbb{F}, \star_l)$.*

Proof. The characteristics (i)-(iii) and (v) (defined in Definition 2.4) are clear and simple to prove. For (iv)(Definition 2.4), let $t > 0$ and $u, v, w \in X$, then since $\sigma(u, w) \leq \sigma(u, v) + \sigma(v, w)$, we have

$$1 - \frac{\sigma(u, w)}{K + t} \geq 1 - \frac{\sigma(u, v) + \sigma(v, w)}{K + t}.$$

From the above inequality it follows that

$$\max\left\{1 - \frac{\sigma(u, v) + \sigma(v, w)}{K + t}, 0\right\} \leq 1 - \frac{\sigma(u, v)}{K + t}.$$

This demonstrates that (iv) was met.

□

I will now determine Cauchy sequences, completeness, and convergence in fuzzy metric-like spaces.

Definition 2.13. [16] Let $\{u_n\}$ be a sequence in any X and the triplet $(X, \mathbb{F}, \star)$ be a fuzzy metric-like space. Then

(a) A u is referred to be the limit of a u_n sequence, and a u_n sequence is referred to as convergent to $u \in X$ if for all $t > 0$,

$$\lim_{n\to\infty} \mathbb{F}(u_n, u, t) = \mathbb{F}(u, u, t)$$

(b) The limit $\lim_{n\to\infty} \mathbb{F}(u_{n+p}, u_n, t)$ exists if $\forall\ t > 0$ and each $p > 1$. The sequence u_n is then referred to as Cauchy.

(c) if every Cauchy sequence u_n in any X converges to a particular u point in X. The triplet $(X, \mathbb{F}, \star)$ is therefore said to be complete if and only if

$\lim_{n\to\infty} \mathbb{F}(u_n, u, t) = \mathbb{F}(u, u, t) = \lim_{n\to\infty} \mathbb{F}(u_{n+p}, u_n, t)$, for each $p \geq 1$ and $\forall\ t > 0$.

Lemma 2.14. *[15] The mappings in the fuzzy metric-like space $(X, \mathbb{F}, \star)$ are continuous on $X \times X \times (0, \infty)$.*

In the debate that follows, the following may be necessary.

Definition 2.15. [18] Let triplet $(X, \mathbb{F}, \star)$ is a fuzzy metric-like space. A mapping $h : X \to X$ is said to be α-admissible if $\exists$ a function $\alpha : X \times X \times \Re \cap (0, \infty) \to \Re \cap [0, \infty)$ such that for all $t > 0$

$$u, v \in X, \alpha(u, v, t) \geq 1 \text{ implies } \alpha(hu, hv, t) \geq 1.$$

Definition 2.16. [19] Let the space $(X, \mathbb{F}, \star)$ represent a fuzzy metric-like. If $\forall\ t > 0$, a triangular α-admissile mapping $h : X \to X$ is said to exist.

$$u, v, w \in X, \alpha(u, v, t) \geq 1 \quad \text{and} \quad \alpha(v, w, t) \geq \implies \alpha(u, w, t) \geq 1.$$

Lemma 2.17. *[19] Assume that the triplet $(X, \mathbb{F}, \star)$ is a fuzzy metric-like space and that the mapping $h : X \to X$ is α-admissible. Assume there is a point u_0 in X where $\alpha(u_0, hu_0, t)$ is true. Define a sequence$u_0 \subseteq X$ by $u_n = fu_{n-1}$, $\forall\ n \in \mathbb{N}$. Then comes*

$$\alpha(u_n, u_m, t) \geq 1, \quad n < m, \quad \textit{for all} \quad m, n \in \mathbb{N},.$$

3 Results

A novel simulation function, the MA-simulation function, is introduced by Khojasteh et al. [8], Parveen and Imdad [9]. Using this function, I have created a new sort of contraction called the α-admissible $\Gamma_M A$-contraction, which will be used to deduce several new findings while also establishing a new result that unifies numerous results from the literature already in existence.

Definition 3.1. [9] If a mapping $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ satisfies the following criteria, it is said to be an MA-simulation function:

(γ_1) $\gamma(r, w) < \frac{1}{r} - \frac{1}{w}, \quad \forall \quad r, w \in (0, 1)$;

(γ_2) if $\{r_n\}$ and $\{w_n\}$ are given sequences lies in $(0, 1]$ such that $\lim_{n\to\infty} r_n = \lim_{n\to\infty} w_n = l \in (0, 1)$ and $r_n < w_n, \quad \forall \quad n \in \Re$ then

$$\limsup_{n\to\infty} \gamma(r_n, w_n) < 0.$$

The set of all MA-simulation functions represented by the notation Γ_{MA}.

I provide several instances of MA-simulation function in the lines that follow.

Example 3.2. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear valuet as

$$\gamma(r, w) = c\Big(\frac{1}{r} - 1\Big) - \Big(\frac{1}{w} - 1\Big),$$

$\forall \quad r, w \in (0, 1]$ and $c \in (0, 1)$.

Example 3.3. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear valuet as

$$\gamma(r, w) = \psi\Big(\frac{1}{r} - 1\Big) - \Big(\frac{1}{w} - 1\Big),$$

$\forall \quad r, w \in (0, 1]$ where ψ is self mapping at the interval $[0, \infty)$ and $\forall \quad r > 0, \psi(r)r$ are right continuous functions.

Example 3.4. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear value as

$$\gamma(r, w) = \Big(\frac{1}{r} - 1\Big) - \psi\Big(\frac{1}{r} - 1\Big) - \Big(\frac{1}{w} - 1\Big),$$

$\forall \quad r, w \in (0, 1]$ where ψ is a self-mapped variable at the range $[0, \infty)$ and$r > 0,$, $\psi(r) > 0$, and $\psi(0) = 0$.

Example 3.5. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear value as

$$\gamma(r, w) = w - \psi(r), \quad \forall \quad r, w \in (0, 1]$$

where $\psi(r) > r$,for all r in $(0, 1)$ and $\psi : (0, 1] \to (0, 1]$ are left-continuous and non-decreasing, respectively.

Example 3.6. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear value as

$$\gamma(r, w) = \Big(\frac{1}{r} - 1\Big)\psi\Big(\frac{1}{w} - 1\Big) - \Big(\frac{1}{r} - 1\Big),$$

$\forall \quad r, w \in (0, 1]$ where $\psi : \Re \cap [0, \infty) \to \Re \cap (0, 1)$ is a function that is specified so that $\forall \;\; R > 0, \quad \lim_{r\to R^+} \psi(r) < 1$.

Example 3.7. Suppose $\gamma : (-\infty, 1] \cap (0, \infty) \times (-\infty, 1] \cap (0, \infty) \to \Re$ having a clear value as

$$\gamma(r, w) = \Big(\frac{1}{r} - 1\Big) - \int_0^{\frac{1}{w}-1} \psi(w)dw,$$

$\forall \quad r, w \in (0, 1]$ and $\forall \quad s > 0$ where ψ is a self-mapped variable at the range,$[0, \infty)$ and $\int_0^s \psi(w)dw > s$, respectively.

Now, I am able present the notion for fuzzy metric-like space called it α-admissible Γ_{MA}-contraction.

Definition 3.8. The triplet $(X, \mathbb{M}, \star)$ is a fuzzy metric-like space, and a self mapping h on set X is said to be a α-admissible Γ_{MA}-contraction defind on this triplet. If a $\gamma \in \Gamma_{MA}$ exists and is such that for any $t > 0$, it fulfills the following

$$x, y \in X, \quad \alpha(x, y, t) \geq 1 \Rightarrow \gamma\Big(M(x, y, t), M(hx, hy, t)\Big) \geq 0, \tag{3.1.1}$$

I am prepared to offer our primary finding right here.

Theorem 3.9. *If h is a self-mapping on X a α-admissible Γ_{MA}-contraction in respect of γ, then $(X, \mathbb{M}, \star)$ is a fcomplete fuzzy metric-like space. Assume the following circumstances are true:*

(i) $\exists\ x_0 \in X$ *like that* $\alpha(x_0, hx_0, t) \geq 1$*;*

(ii) h *to be triangular* α*-admissible;*

(iii) h *to be continuous*
or
if $\forall \quad n \in N, \quad t > 0$ *and* $\{x_n\} \to x$*, such that* $\alpha(x_n, x_{n+1}, t) \geq 1$*, where* $\{x_n\}$ *is a sequence in* X *for some* $x \in X$*,* $\exists$ *a subsequence* $\{x_{n_k}\} \in \{x_n\}$ *such that* $\alpha(x_{n_k}, x, t) \geq 1, \quad$ *for all* $\quad k$ *is natural number and* t *greater than* 0*.*

Next, h maintain a fixed point.

Proof. Assume that $x_0 \in X$ is a random point. Explain the Picard sequence. $\{x_n = h^n x_0\}$. Suppose $\exists$ some $m_0 \in \mathbb{N}$ such that $h^{m_0}(x_0) = h^{m_0+1}x_0$, i.e., $x_{m_0} = x_{m_0+1}$, then x_{m_0} is a fixed point of h. Now, suppose that $h_{n-1}x_0 \neq h_n x_0$, $\forall \quad n \in \mathbb{N}$. Using Lemma 2.2, we then have

$$\alpha(x_n, x_m, t) \geq 1, \quad \text{for all} \quad \text{m,n be are natural numbers}, \quad n < m, \tag{3.1.2.}$$

In light of (3.1.2) and (3.1.1), for $y = x_n$ and $x = x_{n-1}$ I obtain

$$0 \leq \gamma\Big(\mathbb{M}(x_{n-1}, x_n, t), \mathbb{M}(hx_{n-1}, hx_n, t)\Big) = \gamma\Big(\mathbb{M}(x_{n-1}, x_n, t), \mathbb{M}(x_n, x_{n+1}, t)\Big)$$

$$< \frac{1}{\mathbb{M}(x_{n-1}, x_n, t)} - \frac{1}{\mathbb{M}(x_n, x_{n+1}, t)},$$

which implies

$$\mathbb{M}(x_{n-1}, x_n, t) < \mathbb{M}(x_n, x_{n+1}, t)$$

Therefore, $\{\mathbb{M}(x_n, x_{n+1}, t)\}$ is non-decreasing(an increasing) sequence of $\Re_+$ in $\Re \cap (0, 1]$. Let $\lim_n \to \infty \mathbb{M}(x_n, x_n + 1, t) = r(t)$. I claim that $r(t) = 1$, for every $t > 0$. On the other hand, suppose that for some $t_0 > 0$, $r(t_0) < 1$. Then, as $\{r_n = \mathbb{M}(x_{n-1}, x_n, t_0)\} \to r(t_0)$ and $\{w_n = \mathbb{M}(x_n, x_{n+1}, t_0)\} \to s(t_0)$ so using (γ_2), I obtain

$$0 \leq \limsup_{n\to\infty} \gamma\Big(\mathbb{M}(x_{n-1}, x_n, t_0), \mathbb{M}(x_n, x_{n+1}, t_0)\Big) < 0.$$

a contradiction, thus, we obtain $(\forall t > 0)$ from the expression $r(t) = 1, \forall t > 0$.

$$\lim_{n\to\infty} \mathbb{M}(x_n, x_{n+1}, t) = 1 \qquad (3.1.3.)$$

The next step is to demonstrate that x_n is a Cauchy sequence. Let's say it's not true, then $\exists\ 0 < \epsilon_0 < 1,\quad t_0 > 0$ and 2 sub-sequences $\{\{x_{n_k}\}, \{x_{m_k}\}\}$ of $\{x_n\}$ such that $m(k) > n(k) \geq k$ and

$$\mathbb{M}(x_{n(k)}, x_{m(k)}, t_0) \leq 1 - \epsilon_0.$$

From the Remark 2.3, we have

$$\mathbb{M}\Big(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}\Big) \leq 1 - \epsilon_0 \qquad (3.1.4.)$$

Let's now assume that $m(k)$ is the smallest integer that can be used to represent $n(k)$ and yet fulfill (3.1.4). Then comes

$$\mathbb{M}\Big(x_{n(k)}, x_{m(k)-1}, \frac{t_0}{2}\Big) \leq 1 - \epsilon_0. \qquad (3.1.5)$$

Now, using condition ((iv of Definition 2.5), (3.1.4) and (3.1.5), we obtain

$$1 - \epsilon_0 \geq \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0)$$

$$\geq \mathbb{M}\Big(x_{n(k)}, x_{m(k)-1}, \frac{t_0}{2}\Big) \star \mathbb{M}\Big(x_{m(k)-1}, x_{m(k)}, \frac{t_0}{2}\Big)$$

$$> (1 - \epsilon_0) \star \mathbb{M}\Big(x_{m(k)-1}, x_{m(k)}, \frac{t_0}{2}\Big)$$

Applying the t-norm and allowing $k \to \infty$, it produces

$$1 - \epsilon_0 \geq \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0) \geq 1 - \epsilon_0$$

and hence

$$\lim_{n\to\infty} \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0) = 1 - \epsilon_0. \qquad (3.1.6)$$

Also, again by (3.1.1) and (γ_2), for $x = x_{n_k-1},\quad y = x_{m_k-1}$ and $t = t_0$, we get

$$0 \leq \gamma\Big(\mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0), \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0)\Big)$$

$$< \frac{1}{\mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0)} - \frac{1}{\mathbb{M}(x_{n(k)}, x_{m(k)}, t_0)},$$

so that

$$\mathbb{M}(x_{n(k)}, x_{m(k)}, t_0) > \mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0)$$

$$\geq \mathbb{M}\Big(x_{n(k)-1}, x_{n(k)}, \frac{t_0}{2}\Big) \star \mathbb{M}\Big(x_{n(k)}, x_{m(k)-1}, \frac{t_0}{2}\Big)$$

$$> \mathbb{M}\Big(x_{n(k)-1}, x_{n(k)}, \frac{t_0}{2}\Big) \star (1 - \epsilon_0)$$

which on letting $k \to \infty$ and using t-norm yields

$$1 - \epsilon_0 > \lim_{k\to\infty} \mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0) \geq 1 - \epsilon_0.$$

Hence, we have

$$\lim_{k\to\infty} \mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0) = 1 - \epsilon_0. \tag{3.1.7}$$

As a result, according to (3.1.2), we obtain $\alpha(x_{n(k)-1}, x_{m(k)-1}, t_0) \geq 1$, assuming $\{r_k = \mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0)\}$ and $\{w_k = \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0)\}$ and applying (γ_2), we obtain

$$0 \leq \lim\sup_{k\to\infty} \gamma\Big(\mathbb{M}(x_{n(k)-1}, x_{m(k)-1}, t_0), \mathbb{M}(x_{n(k)}, x_{m(k)}, t_0)\Big) < 0,$$

a contradiction. Thus, $(X, \mathbb{M}, \star)$ has a Cauchy sequence (x_n). Now, due to X's completeness, $\{x_n\} \to x$ exists within X. If h is continuous, then we have $\{hx_n\} \to hx$, which implies that $hx = x$ by the uniqueness of the limit.

□

We now give the example below, which illustrates how Theorem 3.1 can be used.

Example 3.10. Let $X = [0, 1]$. Define $* : [0, 1] \times [0, 1] \to [0, 1]$be a t-norm as $p * q = \min\{p, q\}$. Define fuzzy metric-like space $\mathbb{M}$ by

$$\mathbb{M}(x, y, t) = \frac{t}{\sigma(x, y) + t},$$

where $\sigma(x, y) = x^2 + y^2$ is metric-like space. This is $(X, \mathbb{M}, .)$ a complete fuzzy metric-like space. A mapping with the definitions of $h : X \to X$ and $\alpha : X \times X \times \Re_+ \to [0, \infty)$ is as follows:

$$\alpha(x, y, t) = \begin{cases} 1 & \text{if } x, y \in [0, \frac{1}{2}]; \\ 0 & \text{if } otherwise \end{cases},$$

and

$$hx = \begin{cases} \frac{ax}{1+x} & \text{if } x \in [0, \frac{1}{2}]; \\ x & \text{if } otherwise \end{cases},$$

in where $a \in (0, 1)$. Then, there is ($\forall \quad x, y \in X$ and $t > 0$)

$$\frac{1}{\mathbb{M}(x, y, t)} - 1 = \frac{t + \sigma(x, y)}{t} - 1 = \frac{\sigma(x, y)}{t} = \frac{x^2 + y^2}{t}$$

Also, for $x, y \in X$ such that $\alpha(z, y, t) \geq 1$, we have

$$\frac{1}{\mathbb{M}(hx, hy, t)} - 1 = \frac{t + \sigma(hx, hy)}{t} - 1 = \frac{\sigma(hx, hy)}{t}$$

$$= \frac{(hx)^2 + (hy)^2}{t} = \frac{(\frac{ax}{1+x})^2 + (\frac{ay}{1+y})^2}{t}$$

$$= \frac{\frac{a^2x^2}{(1+x)^2} + \frac{a^2y^2}{(1+y)^2}}{t}.$$

Then, using the formula $\gamma(t, s) = k(\frac{1}{t} - 1) - (frac1s - 1)$, we can obtain (for $x, y \in X$) for any $k \in [a, 1]$ and $a \in (0, 1)$.

$$\alpha(x,y,t) \geq 1 \Rightarrow \xi\Big(\mathbb{M}(x,y,t), \mathbb{M}(hx,hy,t)\Big)$$
$$= k\Big(\frac{x^2+y^2}{t}\Big) - \Big(\frac{\frac{a^2x^2}{(1+x)^2} + \frac{a^2y^2}{(1+y)^2}}{t}\Big)$$
$$= \frac{x^2}{t}\Big(k - \frac{a^2}{(1+x)^2}\Big) + \frac{y^2}{t}\Big(k - \frac{a^2}{(1+y)^2}\Big) \geq 0$$

$\forall \;\; t > 0$. Thus, Theorem 3.1's prerequisites are all met, and the theorem's conclusionthat h has a unique fixed point, namely $x = 0$. However, the Gregori and Sapena [12] result cannot be applied. In fact, there is no k in $(0, 1)$ such that (1.1) is met for any $x, y \in (\frac{1}{2}, 1]$.

Next theorem shows the uniqueness of fixed point.

Theorem 3.11. *Theorem 3.9's premise is met. along with one extra following observation is fulfilled:*

(iv) for each $x, y \in Fix(h)$, $\exists \;\; w \in X$ like that $1 \leq \alpha(y, w, t)$, and $\alpha(x, w, t) \geq 1$ for all $t > 0$,

then $h(x) = x$ is unique.

Proof. Theorem above follows the existence portion. In order to determine if a fixed point is unique, let's suppose that x and $x^\star$ are two separate fixed points of h. Then, according to condition (iv), there is a point $w \in X$ where $\forall \;\; t > 0$, $\alpha(x^* \star w, t) \geq 1$ and $\alpha(x, w, t) \geq 1$.

Create a sequence $w_n \subseteq X$ by setting $w_n + 1 = Tw_n$ and $w_0 = w$ and , for every $n \in \mathbb{N} \cup \{0\}$. Triangular α-admissibility provides us with

$$\alpha(x^*, w_n, t) \geq 1 \quad \text{and} \quad \alpha(x, w_n, t) \geq 1, \quad \forall \quad t > 0 \quad \text{and} \quad n \in \mathbb{N} \cup \{0\} \tag{3.1.8}$$

Using 3.1.8 and 3.1.1 (for $x = x$ and $y = w_n$), we can now deduce

$$M(x, w_{n+1}, t) > M(x, w_n, t), \quad \forall \quad t > 0 \quad \text{and} \quad n \in \mathbb{N} \cup \{0\} \tag{3.1.9}$$

which demonstrates that the sequence $\{M(x, w_n, t)\}$ is an increasing series of positive real numbers in the range $\lim\limits_{n\to\infty} M(x, w_n, t) = L(t)$. Our contention is that $\forall \;\; t > 0$ gives $L(t) = 1$, . On the other hand, suppose that certain $t_0 > 0$ exist and that $L(t_0)1$. As a result, for $\{t_n = M(x, w_n, t_0)\}$ and $\{s_n = M(x, w_n + 1, t_0)\}$, we obtain (γ_2) by using 3.1.1.

$$0 \leq \lim_{n\to\infty} \gamma\Big(M(x, w_{n+1}, t_0), M(x, w_n, t_0)\Big) < 0,$$

a contradiction. As a result, $L(t) = 1$ and for all$t > 0$. As a result, $\lim\limits_{n\to\infty} w_n = x$ from $\lim\limits_{n\to\infty} M(x, w_n, t) = 1$, for all $t > 0$. The same pattern allows us to demonstrate that $\lim\limits_{n\to\infty} w_n = x*$. We get to $x = x*$ through the uniqueness of the limit.

□

Next example below, which illustrates how Theorem 3.11 where fixed point is unique.

Example 3.12. Let $X = [0, 1]$. Define $* : [0, 1] \times [0, 1] \to [0, 1]$be a t-norm as $p * q = \min\{p, q\}$. A mapping with the definitions of $h : X \to X$ and $\alpha : X \times X \times \Re_+ \to [0, \infty)$ is as follows:

$$hx = \begin{cases} \frac{ax}{1+x} & \text{if } x \in [0, \frac{1}{2}]; \\ x & \text{if } \textit{otherwise} \end{cases},$$

in where $a \in (0,1)$. and

$$\alpha(x,y,t) = \begin{cases} 1 & \text{if } x,y \in [0,\frac{1}{2}]; \\ 0 & \text{if } otherwise \end{cases},$$

Define fuzzy metric-like space $\mathbb{M}$ by

$$\mathbb{M}(x,y,t) = \frac{t}{\sigma(x,y)+t},$$

where $\sigma(x,y) = x^2 + y^2$ is metric-like space. This is $(X,\mathbb{M},\star)$ a complete fuzzy metric-like space. One can easily varify this exapmle on lines of Example 3.10, in this example fixed point is unique.

4 Consequences

I now derive a few corollaries for fuzzy metric-like spaces as a result of Theorem 3.1, starting with the one that follows.

Corollary 4.1. *([20] type) Assume that $(X,,M,\star)$ is a complete fuzzy metric-like space and h is a satisfied self mapping on X.*

$$x,y \in X \quad \alpha(x,y,t) \geq 1 \Rightarrow \frac{1}{\mathbb{M}(hx,hy,t)} - 1 \leq k\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big),$$

Both $k \in (0,1)$ and $\forall\ \ t > 0$. After that, h has a unique fixed point.

Proof. One can proof this corollary from Theorem 3.9 and Example 3.2. □

Corollary 4.1 may be reduced to the following result by assuming that $\alpha(x,y,t) = 1$, for any $x,y \in X$ and $t > 0$ by Gregori and Sapena [12].

Corollary 4.2. *Let triplet $(X,\mathbb{M},\star)$ be a complete fuzzy metric-like space, and let $h : X \to X$ be a satisfied .*

$$k\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big) \geq \frac{1}{\mathbb{M}(hx,hy,t)} - 1,$$

$\forall\ k \in (0,1)$ and $t > 0$, $\ x,y \in X$,. After that, $hx = x$ i.e. h has a fixed point which is unique.

The Boyd and Wong [21] type result for fuzzy metric-like spaces will be presented in the following corollary.

Corollary 4.3. *If h is a satisfied self mapping on X, then triplet $(X,\mathbb{M},\star)$ is a complete fuzzy metric-like space. Then*

$$\alpha(x,y,t) \geq 1 \Rightarrow \frac{1}{\mathbb{M}(hx,hy,t)} - 1 \leq \psi\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big),$$

$\forall\ \ x,y \in X$ and $t > 0$, where $\psi : \Re \cap [0,\infty) \to \Re \cap [0,\infty)$ is a given function like that $\psi(r) < r$, $\psi(0) = 0$ and $\forall\ \ r > 0$. After that, $hx = x$ i.e. h has a fixed point which is unique.

Proof. The conclusion arises from Theorem 3.9 and Example 3.3. □

The fixed point result from Abbas et al. [22] is shown below.

Corollary 4.4. *Suppose triplet $(X, \mathbb{M}, \star)$ is a complete fuzzy metric-like space and $h : X \to X$ is satisfying*

$$\alpha(x,y,t) \geq 1 \Rightarrow \frac{1}{\mathbb{M}(hx,hy,t)} - 1 \leq \Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big) - \psi\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big),$$

$\forall$ $t > 0$ and $x, y \in X$, where $\psi : \Re \cap [0,\infty) \to \Re \cap [0,\infty)$ is a function such that $\psi(0) = 0$, and $\psi(r) > 0$ for all $r > 0$. After that, h has a fixed point which is unique.

Proof. The conclusion follows in light of Theorem 3.9 and Example 3.4. □

The results that follow are known in some natural settings but appear novel in the fuzzy context.

Corollary 4.5. *Suppose triplet $(X, \mathbb{M}, \star)$ is a complete fuzzy metric-like space and $h : X \to X$ is satisfying*

$$\alpha(x,y,t) \geq 1 \Rightarrow \mathbb{M}(hx,hy,t) \geq \psi(\mathbb{M}(x,y,t)),$$

$\forall$ $t > 0$ and $x, y \in X$, where $\psi : \Re \cap (0,1] \to \Re \cap (0,1]$ is a left-continuousfunction and nondecreasing such that $\forall$ $r \in \Re \cap (0,1]$, $\psi(r) > r$,. After that, h has a fixed point which is unique.

Proof. Theorem 3.9 and Example 3.5 lead to the proof. □

Corollary 4.6. *Suppose triplet $(X, \mathbb{M}, \star)$ is a complete fuzzy metric-like space and $h : X \to X$ is satisfying*

$$x, y \in X \quad \alpha(x,y,t) \geq 1 \Rightarrow \frac{1}{\mathbb{M}(hx,hy,t)} - 1 \leq \Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big).\psi\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big),$$

$\forall$ $t > 0$ and $x, y \in X$, where $\psi : \Re \cap [0,\infty) \to \Re \cap [0,\infty)$ is a given function such that $\lim_{r \to s+} \psi(r) > 0$, $\forall$ $r > 0$. After that, h has a fixed point which is unique.

Proof. The conclusion is inferred from Example 3.6 and Theorem 3.9. □

5 An Application

Many authors have recently used various sufficient conditions to determine the existence and uniqueness of integral equation solutions in various contexts. Here, I focus on a Fredholm nonlinear integral equation and use our established finding for fuzzy metric-like spaces to identify the problem's one and only solution. I see that by using Theorem 3.1, this Fredholm non-linear integral equation has a unique solution under particular circumstances, and that if these circumstances are not met, I am unable to use our findings to obtain the unique solution.

To illustrate this, I take into account the following:

$$x(t) = \int_a^b K(t,s)h(x(w))dw + g(r), \tag{5.1}$$

$\forall$ $t \in \Omega = [a,b](a, b \in \Re)$, $\quad g, h \in C(\Omega, \Re) \quad K \in C(\Omega \times \Omega, \Re)$.

Let Phi represent the collection of all mappings from $\phi : \Re \cap [0,\infty) \to \Re \cap [0,\infty)$ that meet the criteria listed below:

(ϕ_1) $\forall$ $t \in [o, \infty)$, $\qquad \phi(t) \leq t$;

(ϕ_2) ϕ is non-decreasing.

I can now state our theorem as follows in this section:

Theorem 5.1. *The following requirements must be met for the integral equation (5.1) with the variables $K \in C(\Omega \times \Omega, \Re)$ and $g \in C(\Omega, \Re)$ to be valid:*

(i) $\exists$ a +ve number $\phi \in \Phi$ and λ such that the following is true for any $x, y \in C(\Omega, \mathbb{R})$:

$$h(x) - h(y) \leq \lambda \phi(x - y) \qquad (5.2);$$

(ii) $\lambda \sup_{t \in \Omega} \int_a^b |K(r, w)| dr \leq \frac{1}{2}$.

Then, $C(\Omega, \Re)$ is the ounique solution to equation (5.1).

Proof. Be aware that $X = C(\Omega, \Re)$ is a complete metric space in terms of its sup-metric.

$$\sigma(x, y) = \sup_{t \in \Omega}(|x(t)| + |y(t)| + a).$$

Additionally, the space $(X, \mathbb{M}, \star)$

$$\forall \quad t > 0 \quad \text{and} \quad x, y \in X \quad \mathbb{M}(x, y, t) = \frac{t}{t + \sigma(x, y)},$$

be a complete fuzzy metric-like space with product t-norm.

Now we define a mapping $h : X \to X$as:

$$Sx(r) = \int_a^b K(r, w) h(x(w)) dw + g(r) \qquad (5.3)$$

$\forall \quad r \in \Omega$. Using (5.2) and (5.3), we have

$$hx(r) - hy(r) = \int_a^b K(r, w)[h(x(w)) - h(y(w))] dw$$

$$\leq \lambda \int_a^b K(r, w) \phi(x(w) - y(w)) dw \qquad (5.4).$$

Using(ϕ_1), we have

$$\phi(x(w) - y(w) \leq \phi(\sup(|x(w)| + |y(w)| + a)) = \phi(\sigma(x, y)). \qquad (5.5).$$

Applying (5.5) in (5.4), we obtain

$$\phi(x(w) - y(w) \leq \lambda \int_a^b K(r, w) \phi(\sigma(x, y)) dw.$$

Taking supremum over $r \in \Omega$, using conditions (II) and (ϕ_2), we get

$$\sigma(hx, hy) \leq \lambda \phi(\sigma(x, y)) \int_a^b |K(r, w)| dw$$

$$\leq \frac{1}{2} \phi(\sigma(x, y)) \leq \frac{1}{2}(\sigma(x, y)). \qquad (5.6).$$

Now, we have

$$\frac{1}{\mathbb{M}(hx,hy,t)} - 1 = \frac{\sigma(hx,hy)}{t}$$

$$\leq \frac{\sigma(x,y)}{2t} = \frac{1}{2}\Big(\frac{1}{\mathbb{M}(x,y,t)} - 1\Big)$$

By using the formula $\gamma(r,w) = \frac{1}{2}, \Big(\frac{1}{r} - 1\Big) - \Big(\frac{1}{w} - 1\Big)$ and $\alpha(x,y,t) = 1$ for all $t > 0$ and $x, y \in X$ satisfies all the criteria of Theorem 3.1 for every $r, w \in (0,1])$. Theorems 3.1 and 3.2's results lead to the conclusion that $C(\Omega, \Re)$ is the only solution to equation (5.1). $\square$

6 Conclusion

In this paper, motivated by the work of Khojasteh et al. [8], Perveen and Imdad [9] and Karapinar citeKarapinar, we propose the idea of a new contraction called the α-admissible Γ_{MA}-contraction and use it to prove fixed point results, ensuring the existence and uniqueness of fixed points. We also introduce a new simulation function. Additionally, we show through a few corollaries that our main finding is broad enough to encompass a number of findings from the body of literature already in existence. Finally, we demonstrate the utility of our primary result by showing an application.

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