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Clustering DMUs by projecting them onto the nearest MPSS

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Abstract

Cluster analysis in data envelopment analysis (DEA) is determining clusters for the units under evaluation regarding to their similarity. which measure of distances define their similarities. Over the years, researches have been carried out in the field of clustering of DMUs. In this paper, an algorithm for clustering units using projecting them on the frontier is presented. In fact, we gained for every decision-making unit (DMU), nearest most productive scale size (MPSS) as target, to find number of clusters 2 method applied. Silhouette index was used to measure similarity value for our clustering. Numerical examples are provided to illustrate the proposed method and its results.

Keywords: DEA, MPSS, Benchmarking (B.M), Clustering, Index Silhouette.

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1. Introduction

Data Envelopment Analysis (DEA) is a successful non-parametric mathematical method in assessing performance of DMUs, that use multiple inputs to produce multiple outputs by projecting DMUs on the piecewise linear efficient boundary, which it is formed by efficient DMUs and considered as an estimation of the production function to calculate the efficiency of each DMU. This method found uses in other fields except calculating efficiency, such as: ranking, grading, target setting, returns to scale, etc.

There are two basic models, models with constant returns to scale (CCR), were first presented in 1978 by Charnes et. al., and models with variable returns to scale (BCC), which it was proposed by Banker et al. 1984. Returns to scale (RTS) is an important concept in DEA that has attracted the attention of many researchers, since with this concept, they specified if it acceptable to increase or decrease or keeping it constant, the size of a DMU. At first Banker in 1984, proposed this concept [1,2].

Then Banker and Thrall in 1992, introduced a method to specification RTS [3]. In 1986, Banker and Maindiratta replaced the convexity principle with geometric convexity and thus the piece wise linear frontier used in DEA models, was replaced with the Log-Linear frontier [4]. Banker and Thrall in 1992 and Zhou and Shen in 1995, presented a method to estimate RTS [5]. Some researchers have investigated the returns to scale in DEA models under different conditions such as [6-21].

RTS is divided into three categories: 1- Increasing returns to scale (IRS), 2- Constant returns to scale (CRS), 3- Decreasing returns to scale (DRS); If a DMU has IRS, it is recommended to increase the size of DMU, if it has DRS, it is suggested to reduce the size of DMU, and if it has CRS, it is optimal size and suggested to keep it. In 1984, Banker first introduced the concept of MPSS, which is one of the most important and practical concepts in DEA and is related to RTS. Many scientists studied on MPSS issue in DEA [22-26].

In DEA method, by solving model, in addition to calculating the relative efficiency, introduced reference set, which is consist of some strong efficient DMUs on the frontier, thus inefficient DMUs can use the members of R.S to improvement their performance, so target setting is an important issue in DEA method.

In this process, first, it is necessary to determine what DMU's weaknesses is and improvement should take place in which areas. That is, during the bench-marking process, for each inefficient DMU, a DMU that is on the efficient border and is more similar to the DMU under evaluation in terms of performance, is determined as a target, although this target, may have a long distance from the DMU under evaluation; and it is difficult to achieve it.

To overcome this difficulty, the researchers have proposed some method to finding the nearest target [27-30]. Some also presented patterns using weight constraints to solve it [31-35].

Clustering is determining groups in a set of data, which are created based on how much DMUs are similar to each other. and Similarities, are achieved using their distance.

Here, we are going to use target setting in clustering DMUs. For this propose, initially benchmarking DMUs with the nearest MPSS. Then determine the maximum and minimum distances to the MPSS between all observed DMUs in PPS, and obtain radius ε using ratio of them or take average distances measure.

In next section, review literature, then in section 3, present algorithm of our

proposed method and illustrate algorithm in section 4, and in section 5, we have come to conclusions.

2. Literature review

2.1. DEA method

Suppose we have n decision-making units (DMU_j, j= 1… n), each of DMU consumes m non-negative inputs ($X_j = (x_{1j}, \ldots, x_{mj})$) and produces S non-negative outputs ($Y_j = (y_{1j}, \ldots, y_{sj})$). Charnes et al. (1978) proposed the CCR model to calculate the relative efficiency of DMUs.

$$Min\ \theta \qquad (1)$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, i = 1,...m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r = 1,..,s$$

$$\lambda_j \ge 0,\ j = 1,...,n,$$

In 1984, Banker et al. by developing CCR model, introduced the BCC model,

$$Min \quad \theta \qquad (2)$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, i = 1,...m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, r = 1,..,s$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \ge 0,\ j = 1,...,n,$$

The optimal value of these models is 1, and the difference between these two models is in the type of RTS of models, which lead to 1λ= 1, in the BCC model, which demonstrat the convexity axes.

The maximum optimal value of these models is equal to 1, and the difference between these two models is in the type of efficiency to their scale, which causes the limitation of

1λ =1, in the BCC model, which expresses the principle of convexity.

Definition of Returns to Scale (RTS)

In 1984, Banker defined the concept of Returns to scale as follows:

Returns to Scale (RTS) at a point (X, Y) on the efficient frontier of the production possibility set is represented with ρ, and is defined as follows:

$$\rho = \lim_{\beta \to 1} \frac{\alpha(\beta) - 1}{\beta - 1},$$

$$\alpha(\beta) = \{\alpha \mid (\beta X, \alpha Y) \in T\}, \beta > 0$$

which have $\rho: \begin{cases} > 1 \to IRTS \\ = 1 \to CRTS \\ < 1 \to DRTS \end{cases}$

The dual of the BCC model is as follows

$$\max \quad uy_o - u_0 \qquad (3)$$

$$s.t \quad vx_o = 1,$$

$$uy_j - vx_j - u_0 \le 0,$$

$$u \ge 0, v \ge 0$$

In this model, u_0 is corresponding to the constraint 1λ=1, and expresses the returns to scale. So, it can be identifying type of RTS with the sign of u_0. The next theorem proposed by Banker& Thrall in 1992, to specify situation of DMU's RTS.

Theorem: Let (X_o, Y_o) is a DMU located on the frontier of the PPS. The following conditions indicate the RTS of DMU.

1- For all optimal solutions $u_0^* < 0$, if and only if it has IRS.
2- For all optimal solutions $0 < u_0^*$ if and only if it has DRS.
3- In any optimal solution $u_0^* = 0$ if and only if it has CRS.

Then Banker, defined MPSS concept, which is dependent on RTS, as follows:

The (X_o, Y_o) is most production scale size (MPSS) if and only if for each $(\beta X_o, \alpha Y_o) \in T$, have $\frac{\alpha}{\beta} \leq 1$; Cooper et al.; in 1996 provided a model and theorem, which is define MPSS as follows:

$$Max \quad \beta / \alpha \qquad (4)$$

$$s.t \quad \sum_{j=1}^{n} \lambda_j x_j \leq \alpha x_o,$$

$$\sum_{j=1}^{n} \lambda_j y_j \geq \beta y_o,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0,\ j = 1,...,n,$$

$$\alpha \geq 0, \beta \geq 0$$

2.2. Clustering

Clustering is one of the data mining techniques which have application in various fields such as health, bioinformatics, image analysis, etc. Clustering algorithms are classified into two types of hard and soft clustering; hard clustering, i.e. each unit belongs to only one cluster, but in soft clustering, each unit can be assigned to some clusters with a degree. Generally, clustering is a process to grouping data set based on their similarities measure. This similarity depends on the distance between them, we can use Norms (Norm1, Euclidean, etc.) which can be used to calculate distance. One of the applications of clustering in DEA is ranking or grading data [34,36].

In this study, for clustering DMUs at first determining a benchmark which it is the nearest MPSS, then maximum distance is determined. next, the minimum distance or the average of distances to nearest MPSS is computed. And the radius ε, is consider as ratio of max distance to min distance, or the calculated average of distances.

3. Proposed clustering method

Let we have n DMUs $(DMU_j, j = 1,...,n)$, which consume inputs $X_j = (x_{1j},...,x_{mj})$ to produce outputs $Y_j = (y_{1j},...,y_{sj})$. The main goal of this study is introducing a method for clustering DMUs, which units with similar performance (their efficiency scores) are placed in a cluster, and the concept of MPSS is used to target setting, i.e. each unit is benchmarked with the nearest MPSS.

3.1. Benchmarking by using MPSS

For benchmarking, estimating performance of each DMU by model 1, which project them on the MPSS border and compute their distance of their target by Norm1.

3.2. Determination of radius ε

To determine the radius ε**,** first calculate d^{max}, which is the maximum distance to MPSS, and d^{min}, the lowest distance to MPSS or the average distance to MPSS (d^{avr}). We can set value of the ratio d^{max} to d^{min}, or d^{avr} as ε. Now, by segmentation, d^{max} intervals are obtained as follows:

$$d^{\max}, d^{\max} - \varepsilon, d^{\max} - 2\varepsilon, ...$$

$$d^i = d^{\max} - i\varepsilon$$

$$\left[d^{i-1}, d^i\right]$$

3.3. Clustering

Now, by comparing the distance of each DMU to its corresponding target, with the

segmentation carried out, proper group for DMUs will be determined.

The general algorithm is as follows:

1- Determine the MPSS target of each DMU using $\left(\frac{\theta^* x_p - s^{-*}}{1\lambda^*}, \frac{y_p + s^{+*}}{1\lambda^*} \right)$

2- Calculate the distance of each unit to its corresponding target,

3- finding $d^{\max}$, $d^{\min}$

4- Calculate ε , $\varepsilon = \frac{d^{\max}}{d^{\min}}$ or ε=d^{avr},

5- finding d^i , $d^i = d^{\max} - i\varepsilon, d^i \geq 0$.

6- Specifying the segments by comparing the distance between each unit to its nearest MPSS target and determining proper cluster for every DMU.

7- After clustering DMUs with the mentioned method, calculating the Silhouette coefficient for each cluster (so, we define differences in their efficiency value as their unsimilarity). If we get $b_j \geq a_j$, for DMU_j, it means, DMU_j belongs to the appropriate cluster; otherwise, obtaining the distance of that DMU to the elements of other clusters, and according to obtained distances assigned this DMU to the new cluster, and again calculate Silhouette coefficient to assessing clusters situation. And continued it until reach a suitable clustering.

3.4. Silhouette Coefficient

The basic idea is how much assigning units to each cluster is appropriate, and that, members of a cluster should be more similar to each other than members of other clusters.

b_o, a_o and S_o, are defined as follows:

a_o: The average distance of "DMU_o" to the other members of its cluster.

b_o: is the Minimum of average distances of "DMU_o" to other clusters.

$$a_o = \frac{1}{|C_o|} \sum_{p \in C_o} dis(o, p)$$

$$b_o = \underset{C_k \neq C_o}{Min} \left(\frac{1}{|C_k|} \sum_{p \in C_k} dis(o, p) \right)$$

$$S_o = \begin{cases} 0 & a_o = 0, e.g. |C_k| = 1 \\ \frac{b_o - a_o}{Max\{b_o, a_o\}} & else \end{cases}$$

The value range of silhouette index (S_o) is from, -1 to, 1.

Assuming $a_o \neq 0$:

$-b_o \gg a_o \rightarrow s_o \approx 1$ => Unit O has proper allocation to its cluster

$-b_o \approx a_o \rightarrow s_o \approx 0$ => Unit O is between clusters A and B.

$-b_o \approx a_o \rightarrow s_o \approx -1$ => Almost, unit O is closer to members of cluster B

For each cluster, the silhouette index is defined as follows

$$Silh(C_k) = \frac{1}{|C_k|} \sum_{o \in C_k} S_o$$

And in general, the silhouette index for clustering, is defined as follows.

$$Silh(C) = \frac{1}{|D|} \sum_{o \in D} S_o$$

The silhouette coefficient S_c, is the average silhouette of all DMUs.

$0.7 < S_c \leq 1$: Strong structure

$0.5 < S_c \leq 0.7$: Medium structure

$0.25 < S_c \le .5$: Poor structure

$0.25 \le S_c$: Improper structure

4. Numerical Example

Here we are going to illustrate the proposed method with a simple example in which units use an input to produce an output and then examine the suggested method with a practical example

Example 1- There are 20 DMUs that produce an output using an input. The inputs and outputs and efficiency of the units are shown in Table 1.

In this example, units 5, 6 and 9 are MPSS units.

Therefore, by calculating the efficiency score of all DMUs and finding the nearest MPSS to each DMU, and finding the closest MPSS to every DMU, and determine its target $\left(\frac{\theta^* x_p - s^{-*}}{1\lambda^*}, \frac{y_p + s^{+*}}{1\lambda^*} \right)$, $d^{\max}$ and compute the value of $d^{\min}$ then obtain the value of ε, and also, calculate (d^i)s. using the mentioned method and clustering of DMUs is done.

The primary clusters are:

$C_1 = \{5,6,9\}$, $C_2 = \{14\}$,

$C_3 = \{2,3,4,7,8,10,11,12,18\}$,

$C_4 = \{\ \}$, $C_5 = \{1,13\}$,

$C_6 = \{15,16,17,19,20\}$.

To check if a cluster is appropriate ،we calculate the a_j, b_j, S_j values. The results are as follows $DMU_1, DMU_2, DMU_3, DMU_4, DMU_8$, are closer to cluster 2, $DMU_7, DMU_{10}, DMU_{11}, DMU_{12}$, are nearest to cluster 6, and DMU_{15} is between clusters 5 and 6. And $S_C = 0.0043$. So, we do cluster again by proposed method. In this example, after 4 repetitions, gain a suitable clustering of DMUs which condition $b_j \ge a_j$, is satisfies for all DMU.

Table No. 1- Values of inputs, outputs and efficiency of DMUs

	i1	r1	Efficiency
DMU1	22	30	0.73
DMU2	4	5	0.75
DMU3	5	7.5	0.85
DMU4	6	6	0.58
DMU5	8	15	1
DMU6	12	22.5	1
DMU7	15	5	0.2
DMU8	3	4.5	0.8
DMU9	16	30	1
DMU10	20	11	0.3
DMU11	16	9	0.31
DMU12	13	13	0.92
DMU13	32	28	0.65
DMU14	10	12	0.65
DMU15	37	30	0.43
DMU16	42	26	0.33
DMU17	45	25	0.3
DMU18	15	23	0.82
DMU19	46	20	0.23
DMU20	48	10	0.12

The resulted clusters are as follows

$C_1 = \{5,6,9\}$, $C_2 = \{1,2,3,8,14,18\}$,

$C_3 = C_4 = \{\ \}$, $C_5 = \{4,13,15\}$,

$C_6 = \{7,10,11,12,16,17,19,20\}$.

In fact, we have 4 cluster. And it is also obtained: $S_C = 0.675$, which represents the proper clustering structure. The first cluster contains only MPSS units, and each cluster also contains DMU with similar performance.

The obtained clusters are shown in Figure 1. In this figure, the area is displayed in orange color.

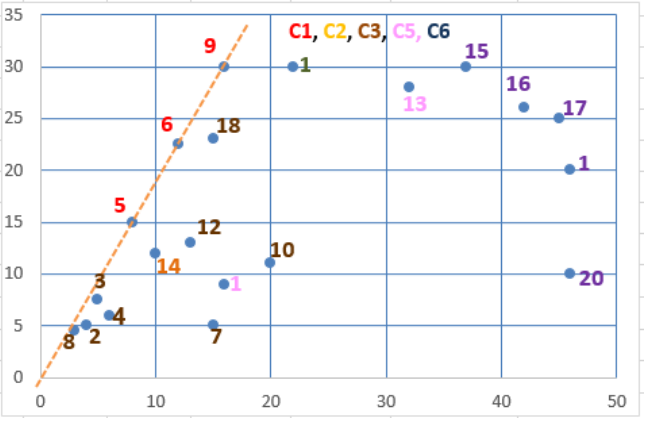

Figure 1. The PPS and the initial clusters obtained with this method

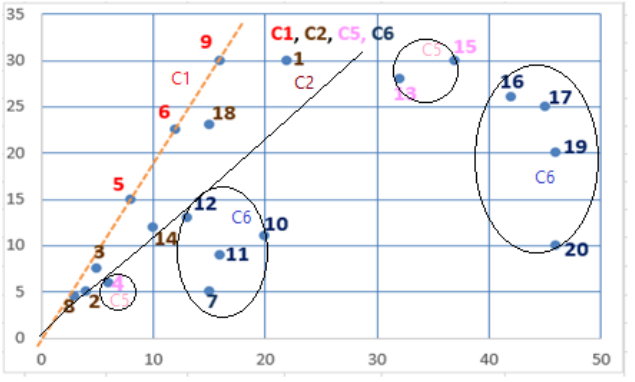

Figure 2. the PPS and the final clusters obtained with this method

Example 2- In this example, we will cluster the bank branches which data is given in Table 2.

For the data set, we first get the efficiency score of the DMUs by model (1). Then by applying obtained targets for inefficient DMUs, the distance of nearest MPSS is calculated, and gaining the value of $d^{\max}, d^{\min}$, then calculating the radius ε, as ratio of $d^{\max}, d^{\min}$, and d^i. So, the initial clusters are determined, as follows:

$$C_1 = \left\{ \begin{matrix} 2*5,7,10,12,14,15,17,27,28, \\ 29,31,32,35,36,37 \end{matrix} \right\},$$

$$C_2 = \{1,8,9,19,24,25\},$$

$$C_3 = \{6,21,22,26,30,34\},$$

$$C_4 = \{16,20,33\}, \qquad C_5 = \{11\},$$

$$C_6 = \{13\}, \qquad C_7 = \{18,23\}.$$

By calculating a_j, b_j and s_j index, and 5 iterations of this method, reaching to the final clusters which are:

$$C_1 = \left\{ \begin{matrix} 2,3,4,5,7,8,10,12,14,15,17,22, \\ 27,28,29,31,32,34,35,36,37 \end{matrix} \right\},$$

$$C_5 = \{11,16,24,30,33\}, \; C_6 = \{6,13,26\},$$

$$C_7 = \{1,9,18*21,23,25\}.$$

The value of a_j, b_j, s_j, calculated which shows the strong structure of clustering.

It is worth mentioning, there is a possibility of occurrence of empty clusters in this method and all MPSS units are in the first cluster.

S_{c1}: 0.942044912
S_{c5}: 0.473856222
S_{c6}: 0.694501196
S_{c7}: 0.582400017,
S_c : 0.780944

5. Conclusion

In this paper, clustering of DMUs with nearest MPSS unit was used for clustering. Norm1, was used for calculate the distance of each DMU to its nearest MPSS target. And calculate ε, and dividing the furthest distance to the nearest MPSS by using radius ε (the number of which was obtained using the ratio of the largest distance to nearest MPSS to the lowest distance to the furthest MPSS or average of all distances).

In general, this method was applicable for clustering units. And it is easy to analyse the performance of DMUs in a cluster. Using this cluster, it can be used to gradually benchmarking the units by first remodelling each DMU in its own group, then in another group that has a higher average efficiency than its original group, and this continues until reach to an MPSS unit, so, deciding on the size of a DMU is easier for the manager and more practical to implement. It can also be used to ranking or grading DMUs.

Table 2. Input and output values of branches

	I1	I2	I3	O1	O2	O3	O4	O5
DMU01	57.16	7700.462917	73102.753504	407492.441691	338858.069972	4433.589964	1865.081618	10121.357004
DMU02	33.11	6582.232881	90134.057029	540962.565034	395389.445006	4113.085379	5316.468446	44724.315709
DMU03	14.91	3715.848251	43838.306149	208044.742548	365317.585492	379.200138	2626.670580	7968.102717
DMU04	24.68	5805.036011	6255.436770	223047.786315	261668.505899	549.186505	3223.602333	22372.515014
DMU05	31.45	13298.332581	338.134831	418272.109599	338696.987458	2630.105133	1271.257096	5526.220059
DMU06	24.11	10879.748209	62671.126831	170402.587568	648686.646318	2894.197123	345.215197	4448.625255
DMU07	57.47	8207.589534	660.023992	474685.853671	252014.284855	16761.736021	1031.476939	5833.900282
DMU08	32.31	9990.341554	36920.585084	635851.928121	388329.968337	4873.553145	2876.273049	37008.080276
DMU09	81.18	14429.599187	440511.595561	1317289.233251	407215.695896	13724.803958	4741.183069	15241.207718
DMU10	56.47	19709.435214	366979.273605	1282334.086247	1183968.719239	28895.391222	13853.020949	108131.361301
DMU11	30.37	21721.578702	149038.874689	730390.714126	464214.422949	31889.276043	202.986136	4385.749726
DMU12	51.58	5590.799793	176674.330004	577873.763711	269727.079288	6293.415973	4178.903208	51224.128271
DMU13	58.4	57767.217488	1078327.713515	2823470.371372	1175604.116610	74871.695475	5288.325390	71888.213335
DMU14	29.71	2925.417361	69207.309533	166234.795066	418832.384938	210.068820	509.255781	4285.784257
DMU15	63.77	63744.797380	103881.394771	4511464.971057	1273629.024945	208534.733035	2780.450807	19939.924498
DMU16	54.87	29533.938975	595394.442389	324065.857453	630627.154709	1937.989519	239.142925	1800.810471
DMU17	88.97	114743.434408	864837.382612	7198238.035934	5328076.541808	16706.234229	154574.276846	79204.484916
DMU18	44.35	54283.998889	773839.420084	2498760.289139	1299757.641021	55369.107239	4165.383922	24912.873942
DMU19	52.94	21644.167552	122089.919076	1246567.205885	1094402.253966	2024.085800	2753.427275	20972.358936
DMU20	46.99	66342.159905	400218.926805	519914.361239	1388684.155952	9323.281997	2552.605575	37278.559615
DMU21	34.54	28067.867398	103782.163541	605692.835677	1007795.491399	3441.854552	1483.351227	4052.865593
DMU22	42.26	9947.145354	458470.772280	324300.180556	956523.931150	5442.078512	1774.702342	17707.370260
DMU23	48.68	58145.106414	209650.304125	739077.152958	1447795.978582	2171.797688	955.483463	19070.683058
DMU24	55.73	19856.358108	118918.395381	185578.010926	466380.632813	10685.264563	4047.671147	29901.946513
DMU25	43.79	23191.220929	100388.941829	2181915.772330	553530.351963	7083.379475	5010.638944	17265.593388
DMU26	21.44	23481.720667	43491.390957	118388.026350	704443.567804	950.601578	1170.301759	9193.105059
DMU27	17.68	12161.248003	19869.589235	1644624.899689	382158.334356	1.753425	2622.049198	39647.099558
DMU28	42.78	15143.739120	345431.339017	1343001.142300	1076072.484771	50758.250299	1542.325695	1798.237340
DMU29	33.24	106810.917611	20639.718181	2874887.372349	1662510.242494	541.505628	86.063470	437.420000
DMU30	23.35	3296.967069	47329.929928	82713.558809	81738.662033	696.437541	168.213750	1569.553742
DMU31	39.56	10094.268507	70740.166481	1747971.711645	409751.890368	21468.687150	1679.894635	11734.531671
DMU32	33.55	45925.290055	57310.466000	4547716.524577	743058.138165	3.851404	162.047528	8170.138845
DMU33	42.25	12343.401251	213315.040451	335945.390879	265298.443632	9518.569517	3476.357041	30048.696462
DMU34	38.94	4539.574801	76872.365617	640355.623852	177959.844215	10140.870618	953.300702	8033.512822
DMU35	37.08	34081.801322	83015.776255	863923.771998	1482666.381908	988.372823	1887.869477	1890.272744
DMU36	35.68	16649.315128	112804.565649	947940.465701	1337766.393686	19402.890832	2559.354899	4323.092928
DMU37	18.83	19010.314739	7343.368031	441683.547083	430574.726613	10615.620539	720.914682	19529.275807

References

[1] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the efficiency of decision-making units, Eur. J. Oper. Res. 2 (1978) 429{444.

[2] R. D. Banker, (1984). Estimating most productive scale size using data envelopment analysis. European Journal of Operational Research, 17, 35–44.

[3] R. D. Banker, & R. M. Thrall, (1992). Estimation of returns to scale using data envelopment analysis. European Journal of Operational Research, 62, 74–84.

[4] R. D. Banker, & A. Maindiratta, (1986). Piecewise log linear estimation of efficient production surfaces. Management Science, 32, 126–135.

[5] J. Zhu, Z.H. Shen, a discussion of testing DMU's returns to scale in DEA, Eur. J. Oper. Res., 1995; 81, 590–596.

[6] M. Omidi, M. Rostamy-Malkhalifeh, A. Payan, Estimation of Overall Returns to Scale (RTS) of a Frontier Unit Using the Left and Right RTS. Comput Econ, 2019; 53, 633–655.

[7] B. Golany, G. Yu, Estimating returns to scale in DEA. European Journal of Operational Research, 1997; 103: 28–37.

[8] M. Zarepisheh, M. Soleimani-damaneh, L. Pourkarimi, Determination of returns to scale by CCR formulation without chasing down alternative optimal solutions. Applied Mathematics Letters, 2006; 19(9),964–967.

[9] B. K. Sahoo, M. Khoveyni, R. Eslami, p. Chaudhury, Returns to scale and most productive scale size in DEA with negative data. European Journal of Operational Research, 2016; 255(2),545-558.

[10] L. M. Seiford, J. Zhu, An investigation of returns to scale under Data Envelopment Analysis. Omega, 1999; 27, 1–11.

[11] G. R. Jahanshahloo, M. Soleimani-damaneh, M. Rostamy-malkhalifeh, an enhanced procedure for estimating returns-to-scale in DEA. Appl Math Comput, 2005; 171: 1226–1238.

[12] M. Soleimani-damaneh, G. R. Jahanshahloo, S. Mehrabian, M. Hasannasab, Scale elasticity and returns to scale in the presence of alternative solutions. J Comput Appl Math, 2009; 233: 127–136.

[13] M. Soleimani-damaneh, G. R. Jahanshahloo, S. Mehrabian, M. Hasannasab, Returns to scale and scale elasticity in the presence of weight restrictions and alternative solutions. Knowl-Based Syst, 2010; 23: 86–93.

[14] M. Soleimani-damaneh, A. Mostafaee, a comment on ''Returns to scale and scale elasticity in data envelopment analysis''. Eur J of Oper Res., 2008; 184:1179–1181.

[15] M. Soleimani-damaneh, A. Mostafaee, Stability of the classification of returns to scale in FDH models. Eur J of Oper Res., 2009; 196: 1223-1228.

[16] T. Sueyoshi, M. Goto, Measurement of returns to scale and damages to scale for DEA-based operational and environmental assessment: How to manage desirable (good) and undesirable (bad) outputs? Eur J of Oper Res., 2011; 211: 76–89.

[17] T. Sueyoshi, K. Sekitani, Returns to scale in dynamic DEA. Eur J of Oper Res., 2005; 161: 536–544.

[18] T. Sueyoshi, K. Sekitani, Measurement of returns to scale using a non-radial DEA model: A range-adjusted measure approach. Eur J of Oper Res., 2007a; 176: 1918–1946.

[19] T. Sueyoshi, K. Sekitani, the measurement of returns to scale under a simultaneous occurrence of multiple solutions in a reference set and a supporting hyperplane. Eur J of Oper Res., 2007b; 181: 549–570.

[20] M. Zarepisheh, M. Soleimani-damaneh, A dual simplex-based method for determination of the right and left returns to scale in DEA. Eur J of Oper Res., 2009; 194: 585–591.

[21] A. Hatami-Marbini, Z. G. Beigi, J. L. Hougaard, K. Gholami, Measurement of Returns to-Scale using Interval Data Envelopment Analysis Models, Computers & Industrial Engineering (2017),

[22] G. R. Jahanshahloo, and M. Khodabakhshi, using input–output orientation model for determining most productive scale size in DEA. Applied Mathematics and Computation.2003, 146(2–3), 849–855

[23] R. Eslami, M. Khodabakhshi, G.R. Jahanshahloo, F. Hosseinzadeh lotfi, M. Khoveyni, estimating most productive scale size with imprecise-chance constrained input–output orientation model in data envelopment analysis, Computers & Industrial Engineering 63(1) 2012,254-261.

[24] C. Y. Lee, most productive scale size versus demand fulfillment: A solution to the capacity dilemma. European Journal of Operational Research, 2016; 248(3): p. 954-962.

[25] Y. M. Wang, and Y.-X. Lan, Estimating most productive scale size with double frontiers data envelopment analysis. Economic Modelling, 2013; 33: p. 182-186.

[26] M. Khodabakhshi," Estimating Most Productive Scale Size in Stochastic Data Envelopment Analysis ", Economic Modelling, 26, 2009, 968-973.

[27] Aparicio. J., Ruiz. JL., Sirvent. I. Closest targets and minimum distance to the Pareto-efficient frontier in DEA. Journal of productivity Analysis 28(3):209-218, 2007.

[28] L. Karamali, A. Memariani, G.R. Jahanshahloo, M. Rostamy-Malkhalifeh, Benchmarking by an integrated data envelopment analysis-artificial neural network algorithm. J. Basic. Appl. Sci. RES, 2013; 3,6,892-898.

[29] J. L. Ruize, I. Sirvent, common benchmarking and ranking of units with DEA. Omega,2016; 65,1-9.

[30] H. Didehkhani, F. Hosseinzade Lotfi, S. Sadi-Nezhad, Practical benchmarking in DEA using artificial DMUs. Journal of Industrial Engineering International, 2019; 15. 293-301.

[31] F. Hosseinzadeh Lotfi, A. Hatami-Marbini, PJ. Agrell, N. Aghayi, K. Gholami, allocating fixed resources and setting targets using a common weights DEA approach. Computer and industrial engineering, 2013; 64,2. 631-640.

[32] S. Razipour-GhalehJough, F. Hosseinzadeh Lotfi, G. R. Jahanshahloo, M. Rostamy-malkhalifeh, H. Sharafi, finding closest target for bank branches in the presence of weight restrictions using data envelopment analysis. Annals of Operations Research, 2020; 288, 755–787.

[33] J. Wu, L. Liang, M. Song, Performance based clustering for benchmarking of container ports: an application of DEA and cluster analysis technique. International journal of computational intelligence system, 2010; 3(6), 709-722.

[34] X. Dai, T. Kuosmanen, Best-practice bench marking using clustering methods: Application to energy regulation. Omega, 2014; 42(1): 179-188.

[35] R. W. Po, Y.Y. Guh, M.S. Yang, a new clustering approach using data envelopment analysis. European Journal of operational research, 2009; 199, 276-284.

[36] J. Han, M. Kamber, j. Pei., (2011), Data mining concepts and Techniques, morgan kaufmann, 3rd edition, 744 pages.