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The Investigation of Giant Magneto Resistance in an Inhomogeneous Ladder Lattice

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Abstract: The variation of the electrical resistivity of a material in the external magnetic field is known as magneto resistance. This phenomenon has been attracted both theoretical and experimental researchers in miniaturization of magneto meters in the recent years. In this paper, the magneto resistance of an inhomogeneous two dimensional conductor with ladder geometry is simulated by using a two dimensional resistor network model. Maxwell's equations have been solved for a point of lattice considered as disk and then, its magneto resistance was calculated using a network model. The results illustrate that the magneto resistance depends on the specific resistance ratios and their locations. Moreover, the results demonstrate when inhomogeneity is added properly, the magneto resistance will be increased, otherwise it will be reduced. The results also show that for special values of physical parameters especially the inhomogeneity, the magneto resistance is diverged at special magnetic field.

Keywords: Magnetic Sensor, Giant Magneto Resistance, Resistor Network Model.

1. INTRODUCTION

Giant magneto resistance usually occurs when a system contains ferromagnetic and paramagnetic metal layers exposed to a magnetic field [1]. A noticeable case of giant magneto resistance is the linear one. Although the theory of linear magneto resistance has not been developed completely, it is known that the disorder, in the form of impurity substitution is the cause of this phenomenon [2]. There are two theoretical possibilities for this phenomenon. The first one is classical magneto resistance that happens in a polycrystalline metal in a large field [3]. This kind of magneto resistance occurs when the Lorentz force is acting against the direction of electron motion thereby decreasing the conductivity of an

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electronic material [4]. This property has been considered to study the fundamental properties of electronic materials (such as the topology of the electron bands) and technological applications such as magnetic memory read-heads [5, 6]. Single electronic bands (in systems with a spatially homogenous carrier density) will have no magneto resistance, while the presence of two or more electronic bands with different carrier mobilities readily gives rise to the classical magneto resistance [4]. Classical magneto resistance has scientific applications in magnetic sensors [7-9]. When the carriers are of low density and small effective mass, in which all the carriers are placed in the lowest Landau, the quantum linear magneto resistance description is necessary [3]. Studying of magneto resistance leads to the observations of fascinating effects such as the integer and fractional quantum Hall effect [10,11]. The linear magneto resistance depends on the density fluctuations and the mobility of particles and is strongly enhanced by high mobility of the sample [2]. Linear magneto resistance is regularly observed in semi-metals, narrow band-gap semiconductors, multi-layer graphene and topological insulators [12-16]. Linear magneto resistance originates in an inhomogeneous conductor from distortions in the current paths induced by macroscopic spatial fluctuations in the carrier mobility [17]. In fact, this phenomenon arises from multiple scattering of the current-carrying electrons by low-mobility islands within the conducting layer [17]. Linear magneto resistance effect is observed in gold nanoparticle-decorated graphene and in bilayer mosaic graphene due to the two-dimensional resistor network [2, 9]. Linear magneto resistance due to inhomogeneity is attracting as it offers the potential of engineering materials for magnetic field sensor applications [2].

The magnetoresistance of the two dimensional networks have been subject of many researches in the past years [7-9, 18-22]. The 2D resistor networks have been considered in both classical [19, 20] and quantum [21, 22] approaches and confirmed by some experimental works [18, 19, 20].

In this paper, the magneto resistance of $N \times 2$ networks consists of two materials with columnar and zigzag configurations for different resistivity ratios are calculated, and the effect of the resistivity ratio on the magneto resistance has been investigated. The smallest network component is a two-dimensional homogeneous resistor, which is formed by connecting it to a larger network. The impedance matrix is obtained for one disk by using Maxwell's equations which leads to the entire network by the orbital relations.

2. THEORY

An inhomogeneous conductor is modeled by a random resistor network. The resistor network model is a classical method for solving magneto resistance that is sufficient for simulating a transverse magneto resistance. In this model, a resistor unit (Fig.1. [7]) is a homogeneous circular disk with four current terminals and four voltage differences between them with an external magnetic field applied perpendicular to the network.

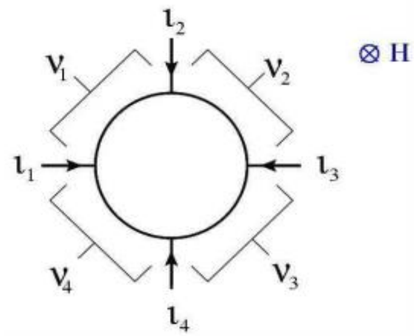

Fig.1 The unit of resistor network.

The voltage differences between terminals are considered positive in the clockwise direction. Currents and voltages are related to each other via a 4×4 matrix (z). If the terminals are taken to be equally spaced and the angular width φ of the terminal is held fixed, then the impedance matrix will be as follows [7, 8, 19]:

$$z = \frac{\rho}{\pi t}\begin{pmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{pmatrix}, v_i = z_{ij} \iota_j \ , \ z_{ij} = v_i / I_j \tag{1}$$

where ρ and t are scalar resistivity and thickness of the disk respectively. The coefficients z_{ij} are obtained by solving the Laplace equation for the electric potential of a homogeneous disk, using the currents as boundary conditions [7, 8].

$$\begin{aligned} &a = -g(\varphi) + (\pi/4)B, \quad b = g(\varphi) + (\pi/4)B \\ &c = 0.35 - (\pi/4)B, \quad d = -0.35 - (\pi/4)B \end{aligned} \tag{2}$$

In this paper, the value of φ is considered to be 0.14 radians and $g(\varphi)$ is 4.0055 and B is magnetic field. The magnetic field is defined as a dimensionless variable $B = \mu H$, where μ is the mobility of the carriers and H is magnetic field intensity [7].

The impedance matrix (Z) is characterized by two parameters μ and $\rho/t\pi$. To construct an N × M resistor network, the disks are connected together using perfect conducting wires. Voltages and currents of network are connected via the impedance matrix. The impedance matrix (Z) can be obtained by grounding one terminal to provide a point of reference for the voltages (locating the ground or zero voltage) and also using the orbital relations. By classifying the voltages and currents in to 2N-1 longitudinal components and 2M Hall components, the impedance matrix is written as follows [8]:

$$Z = \begin{pmatrix} Z^{HH} & Z^{HL} \\ Z^{LH} & Z^{LL} \end{pmatrix} \tag{3}$$

To determine the magneto resistance of an N × M network, $I_i^H = 0$ and the right V^L is set to a constant potential *U*, and completely ground the left side of the longitudinal voltages [7]. The effective resistor a network is given by:

$$R_{NM}(H) = \frac{U}{\sum_i^N I_i^L} = \frac{U}{\sum_i^L (Z^{LL})_{ij}^{-1} V_j^L} \tag{4}$$

in which U is terminal voltage, I_i^L is the input currents along the ungrounded(right) edge.

The sum over input currents is performed along the right edge. If the ratio N / M is kept constant and take the limit where N → ∞, then the resistor network model output is the galvanomagnetic properties of the material [8]. The magneto resistance is defined as follows:

$$\frac{\Delta R}{R} = \frac{R(H) - R(0)}{R(0)} \tag{5}$$

3. RESULTS AND DISCUSSION

In this paper, N × 2 networks are investigated in which the left terminals are grounded and the right terminals are connected to the $U = -1\,V$ as shown in Fig. 2.

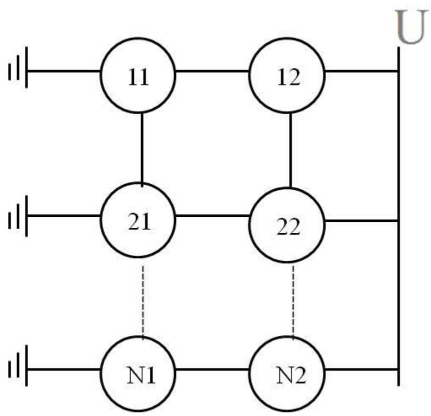

Fig.2: N × 2 resistor network.

A. Columnar Inhomogeneity

First, it was assumed that the resistivity of the first column is ρ_1 and it is ρ_2 for the second one ($r=\rho_1/\rho_2$, r is the resistivity ratio). Fig. 3 shows the computational results of Normalize Magneto resistance Variation (NMV) as a function of dimensionless magnetic field for N × 2 network with N= 9, 10, 19, 20, 49 and 50, and r=0.5 (Fig. 3(a)), r=1 (Fig. 3(b)), r=2 (Fig. 3(c)) and r=5 (Fig. 3(d)).

As shown in Fig. 3(a), it is indicated that by increasing the number of rows, the magneto resistance increases, and as the number of odd rows increases, the magneto resistance in the higher magnetic field becomes saturated. For r=1 (Fig. 3(b)), increasing the number of rows in the network, is caused the magneto resistance is increased. Fig. 3(c) and (d) illustrates that the magneto resistance has increased and the differences in magneto resistance between odd-N and even-N networks diminish as resistivity ratio increases with increasing the number of even and odd rows. The general trend of magneto resistance for even-N networks, is non-saturating but for odd-N networks is saturating. This behavior is similar to one-dimensional networks N × 1 [7, 8, 23]. In networks consisting of low numbers of atoms, there are a lot of differences in the magneto resistance between even and odd atoms, however these differences diminish as the number of atoms increases [8] simultaneously, and the magneto resistance of the network increase. By comparing these figures, it is concluded that by increasing the resistivity ratio, the magneto resistance is increased. There is asymmetry in this system, so that by increasing resistivity of the grounded column, the magneto resistance is raised.

Fig. 4 shows the magneto resistance variations as a function of the resistivity ratio at B=300, for the odd-N networks (Fig. 4(a)) and even- N networks (Fig. 4(b)).

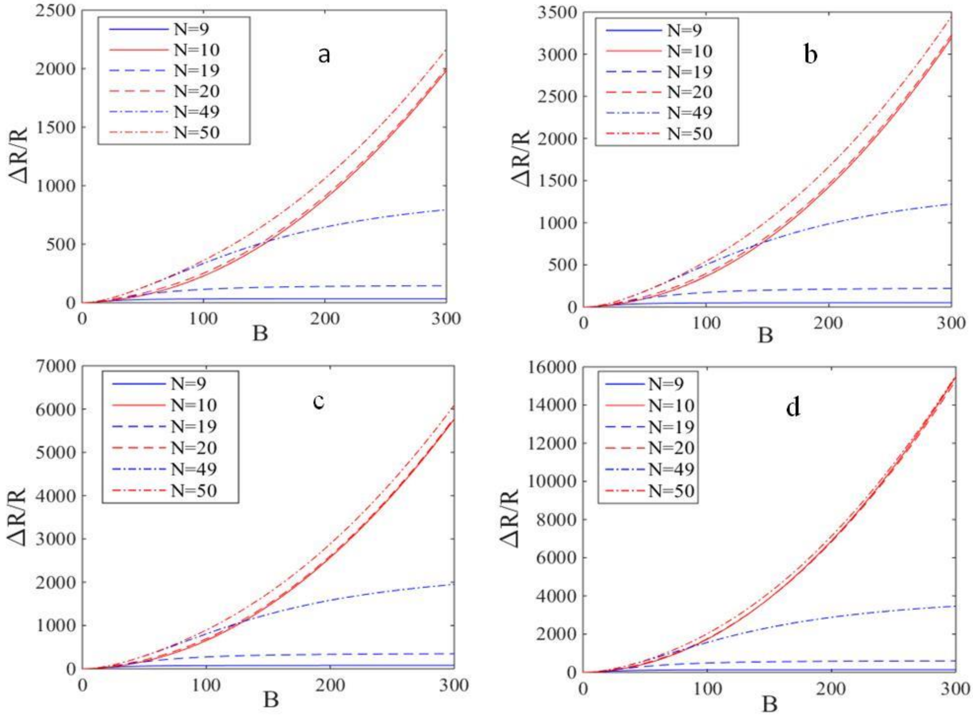

Fig.3: Magneto resistance of the N × 2 network as a function of the magnetic field for the resistivity ratio (a) 0.5 (b) 1 (c) 2 (d) 5.

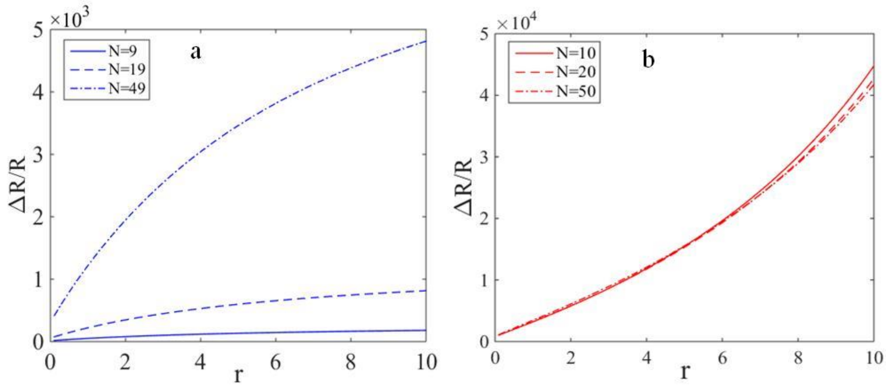

Fig.4: The magneto resistance variations as a function of the resistivity ratio r at B=300 (a) odd-N networks (b) even-N networks.

For the odd-N networks, by increasing the resistivity ratio, the magneto resistance of the network has been significantly enlarged. Also, in this network it can be seen that by increasing N, magneto resistance is augmented. Moreover,

shown in Fig. 4(b), the magneto resistance variations are non-saturated, and in the high resistivity ratio, the magneto resistance will reduce as N increases.
In Fig. 5 the magnetoresistance variations as a function of N for even (Fig. 5(a)) and odd (Fig. 5(b)) network in B=300 and different resistivity ratios is depicted.

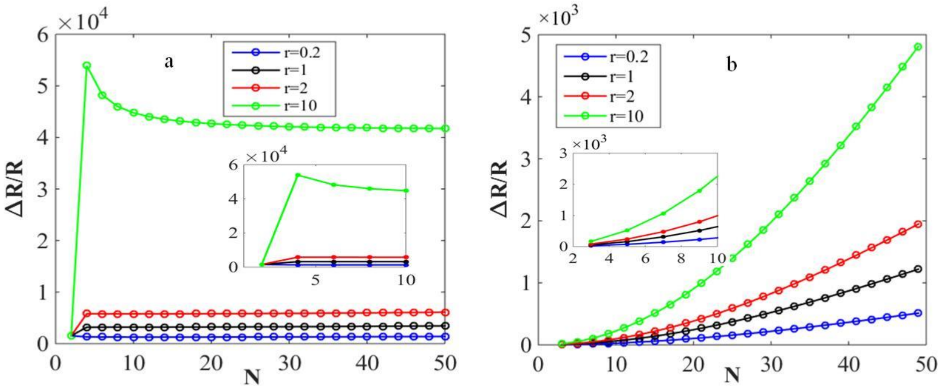

Fig.5: The magneto resistance variations in B=300 and different resistivity ratio as a function of the number of network rows (a) the number of even rows (b) the number of odd rows.

In Fig. 5(a) it is obvious that, for a resistivity ratio less than 1, the magneto resistance is maximum for two disks and decreases slowly with increasing the number of disks. For the resistivity ratios equal or larger than 1, the magneto resistance is minimum for two disks and is maximized in the number of four disks, and then decreases slightly so that it approaches a constant value. As shown in Fig. 5(b), by increasing the number of disks for and resistivity ratio for odd network the magneto resistance increases.

As one of important results, Fig. 6 exhibits the bottom voltage of the disks (Fig. 6(a)) and, the vertical current (Fig. 6(b)) between the adjacent disks n and n + 1 indices, for N × 2 network with N=49, 50 and different resistivity ratio, in B= 300 and U=-1V. As shown in Fig. 6(a), the voltage variations of the first and second columns are oscillating and their amplitudes are increasing as the near-bottom atoms (larger n) are approached. In homogeneous system (*r*=1) the voltage fluctuation amplitude of the first column is greater than the second one. But, for *r*= 0.5, the voltage fluctuation amplitude of the second column is greater than the first one. For *r*=2 the voltage fluctuation amplitude of the first column is significantly greater than the second column. Fig. 6(b) reveals the fluctuating behavior of vertical current in both columns. Furthermore, the current fluctuation amplitude is maximum, while the system is homogeneous.

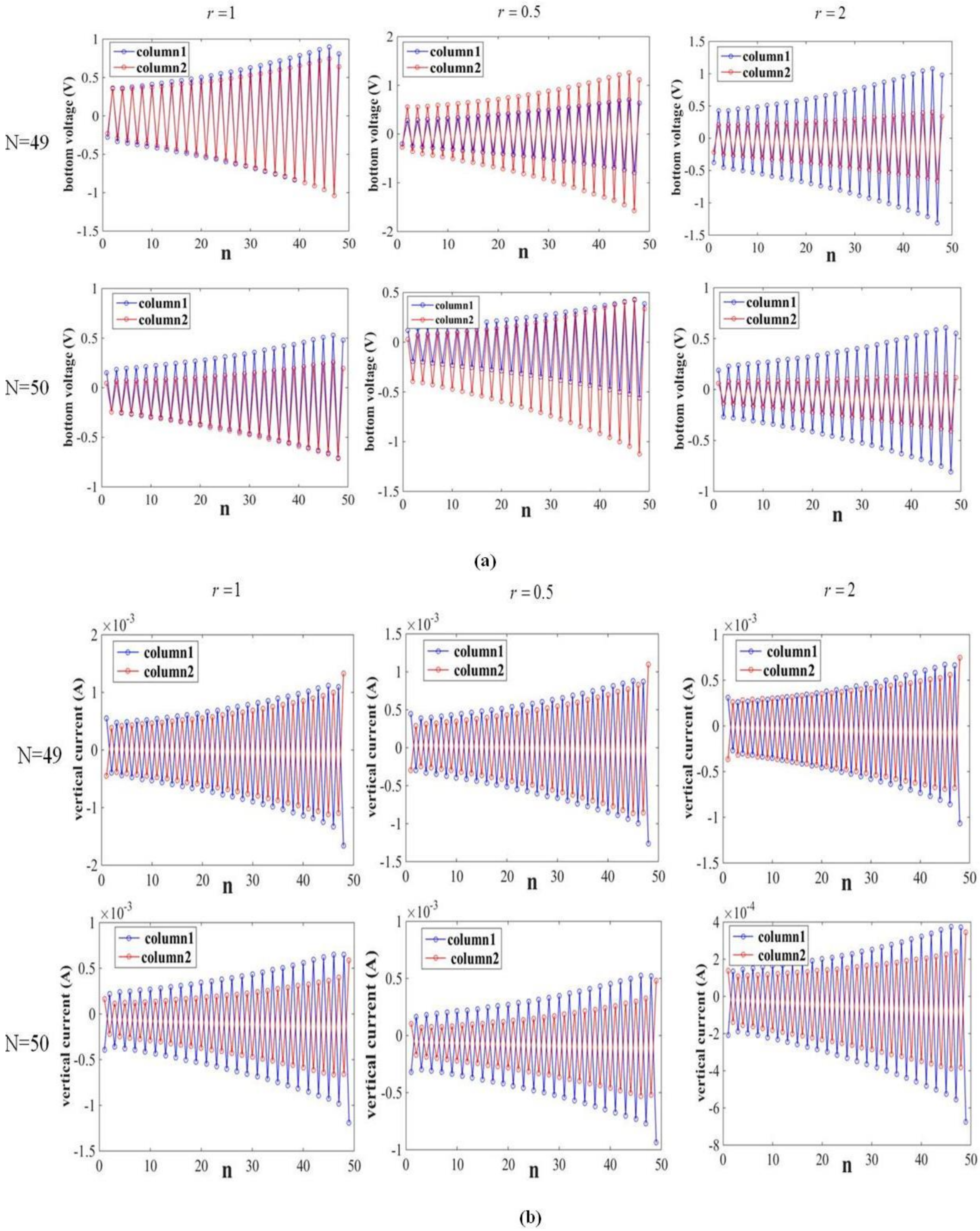

Fig.6: (a) The bottom voltage of disks (b) Vertical current between disks with resistivity ratio r=1, 0.5 and 2 for N × 2 networks with N = 49, 50 and for B= 300.

B. Zigzag Inhomogeneity

For zigzag inhomogeneity that is proposed, it is assumed that the resistivity of the disks with even sum is ρ_1 and odd sum is ρ_2 ($r=\rho_1/\rho_2$). Fig. 7 shows the normalize magneto resistance variations as a function of magnetic field for a network with N= 49 (Fig. 7(a)) and N=50 (Fig. 7(b)). Fig. 7(a) depicts that, for the resistivity ratio of 0.5, 1 and 5, by increasing magnetic field, the magneto resistance is raised, and for higher fields, the magneto resistance becomes saturated. For resistivity ratio of 0.2, the NMV is negative and its magnitude is increased by increasing the magnetic field. Fig. 7(b) indicates for the resistivity ratio 1 and less than 1, the NMV are non-saturated. By increasing the resistivity ratio, the NMV is grown, so when the system is homogeneous, the minimum NMV is observed. It is noticed, for a resistivity ratio of 5, the NMV is diverged in the field about 110. The divergent of NMV due to extreme variation of resistivity versus magnetic field could be applied as high precision magnetic sensors.

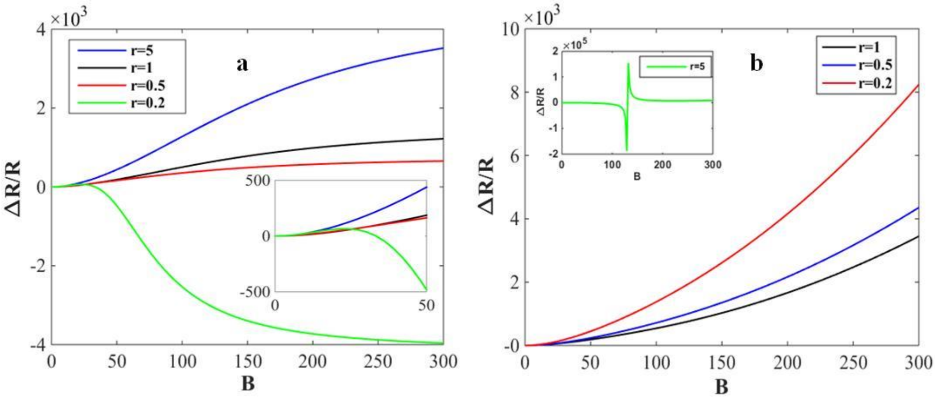

Fig. 7: The magneto resistance variation for N × 2 network (a) N=49 and (b) N=50 (the sum even arrays has a resistivity r=1 and the sum odd arrays has different resistivity r= 0.2, 1, 2 and 5).

The NMV as a function of the different resistivity ratio for B=300, for N=10, 20 and 50 are shown in Fig. 8. This figure shows that by increasing inhomogeneity, the NMV increases. It is also seen that for each number of rows, there is a certain resistivity ratio at which point the magneto resistance diverges. This resistivity ratio is increased by growing the network sizes. This figure also indicates that for smaller networks, the peaks are sharper.

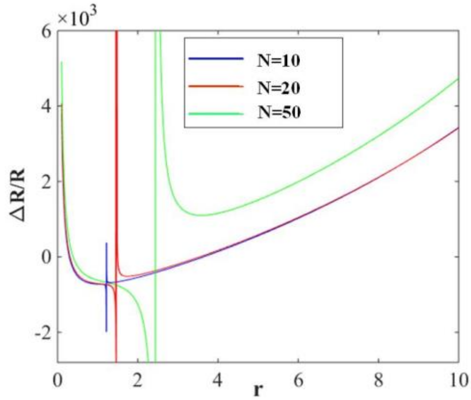

Fig.8: The magneto resistance variation as a function of the different resistivity ratio to B=300, for the network with even N

The bottom-point voltages of the disks and the vertical current between the adjacent disks indices n and n + 1, for a network with N=50 with different resistivity ratios is shown in Fig. 9. It is worth mentioning that for the resistivity ratios of 1, 0.2, 0.5, the curves are plotted for magnetic field 300 and for the resistivity ratio of 5 that has NMV peak, the curves are plotted for magnetic field 130. In Fig. 9a, the oscillation of voltage variations of the first and second columns, for the all resistivity ratios were illustrated. It also shows that, when we get closer to the bottom of the network (larger n), the voltage oscillation amplitudes are increased (except for ratio of 0.2). Also it is observed that, as the magneto resistance increases, the voltage amplitude of the second column increases so that for *r*=5, the voltage amplitude of the second column is considerably higher than the voltage amplitude of the first column.

Fig 9(b) shows that by increasing the resistivity ratio, the amplitude of the current variations of the second column with respect to the first column is improved, but for *r*=5, there is a significant difference in the amplitude of the current variations of two-column.

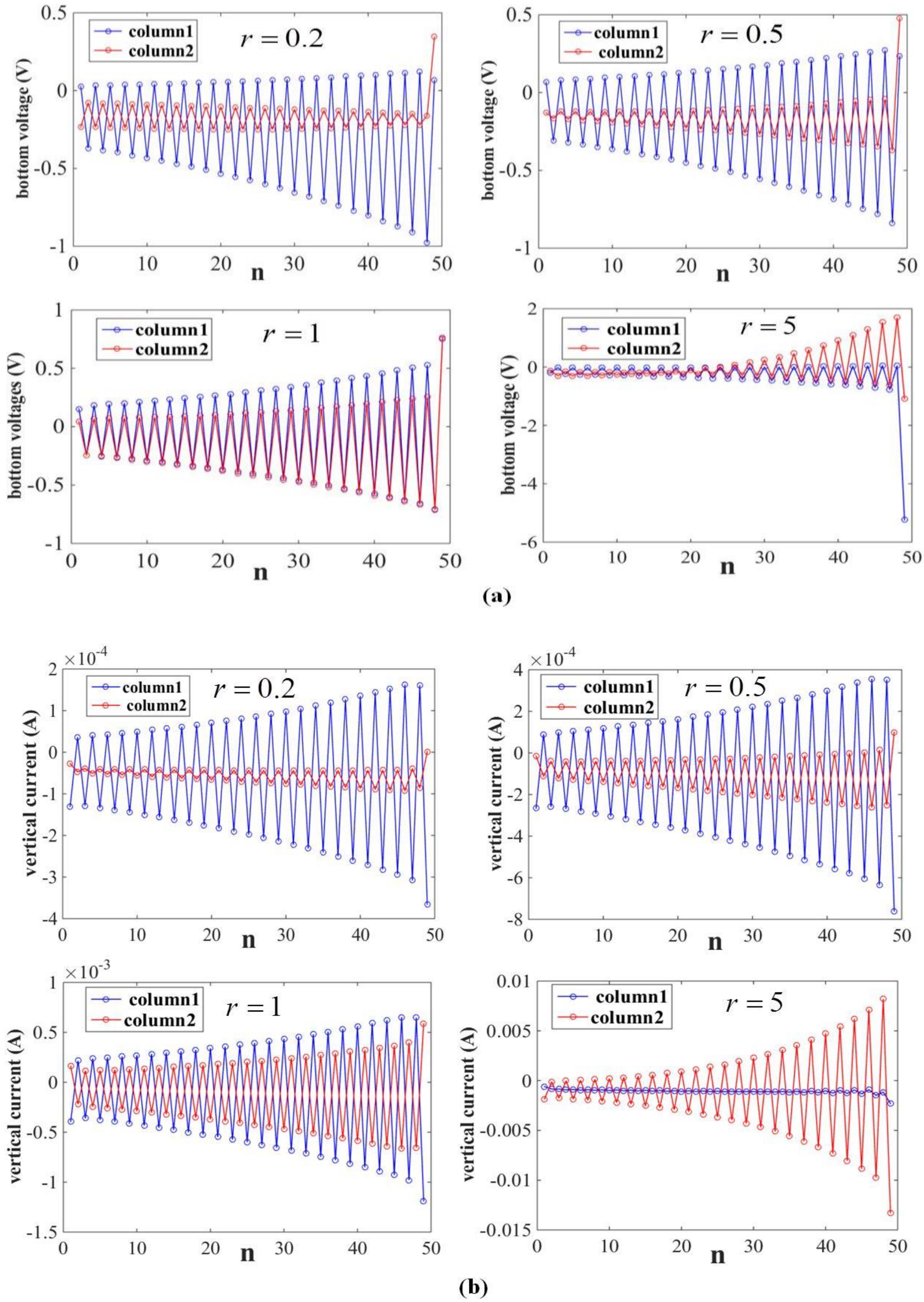

Fig.9: (a) the bottom voltage of the disks and (b) the vertical currents between the disks for a network with N=50, B=300 (for a resistivity ratio of 5, the magnetic field is 130).

4. CONCLUSION

In this paper, the magneto resistance of an inhomogeneous conductor can be calculated numerically using a resistor network model consist of four-terminals. As it is seemed, in a 2 × N network for even- N networks, the magneto resistance is non-saturated, whereas for odd- N networks, the magneto resistance will be saturated. In columnar inhomogeneity the higher value of resistivity of the material that is attached to the constant voltage, the greater the magnetic resistance. For a network with odd-N, by increasing resistivity ratios, the magneto resistance is saturated in lower fields. In zigzag inhomogeneity the higher the resistivity ratio of the disks, the greater the NMV. In this case, the system with most inhomogeneity has the minimum NMV. It is also seen that the magneto resistance is divergent for networks with different sizes at specific resistivity ratio and magnetic field. Generally, the increase or decrease of NMV value as a function of the dimensionless magnetic field, as well as the voltages variations and the vertical currents between the disks, depends on the resistivity ratio and the arrangement of the two materials.

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