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Studying Focusing Properties of Graded Index Photonic Crystals Made of Material with Different Refractive Index

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Abstract: In this paper we investigate focusing properties of graded index (GRIN) photonic crystal (PC) structures which are composed of different materials with different refractive indices. GRIN PC structure is constructed from air holes in dielectric background. The holes radii are varied in the normal direction to the propagation in such a way that a parabolic effective refractive index is produced. The focusing characteristic is studied relative to the refractive index variation of background material. While increasing refractive index of background material of the GRIN PC structure, the effective refractive index of the structure increases. With increasing effective refractive index, the focusing capability of the GRIN PC structure increase and outgoing wave at focal point will be more concentrated. The result shows that the designed GRIN PC structure work very well as a focusing lens. The finite-difference time-domain (FDTD) method was employed to compute field propagation through GRIN PC structure. Also, plane wave expansion (PWE) method has been carried out to extract the dispersion properties.

Key words: Effective refractive index, graded index photonic crystals, finite-difference time-domain, plane wave expansion.

1. INTRODUCTION

Photonic crystals (PCs) are periodic structures in two or three dimensions that effectively control electromagnetic wave propagation direction [1-2]. The unique property of the PCs is possessing photonic band gap (PBG) which enable them to block the input wave with any angle. The PC structures with large PBG are a

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good candidate for designing waveguides [3-4] and cavities [5]. Engineering photonic bands in the allowed part give birth to important optical phenomena such as negative refraction [6], super-prism [7-8], self-collimation [9-10]. The capability of designing the medium with anisotropic refractive index is an important concept for studying PCs. By changing the structural parameter of 2D PCs intentionally, one can obtain a GRIN PC structure in which the refractive index changes gradually. GRIN PCs are a valuable choice for GRIN medium design in which any desired type of refractive index distribution can be achieved. The GRIN PCs enhance the ability of controlling light propagation direction. The GRIN PC was studied for light bending purposes [11-12] for the first time. GRIN PCs have various applications such as mode-order convertor [13-14], coupler [15], wavelength de-multiplexer [16-17], and lenses [18]. One can design a GRIN PC structure by structural parametric modification such as varying lattice constant, the radii of rods (holes) and the refractive index of the material. Different GRIN PC structures were investigated in the literatures [19-23]. Various type of refractive index profile has been suggested to design a GRIN PC structure. Mengqian *et al.* put forward a proposal about studying GRIN PC with parabolic index profile in order to simulate beam aperture modifier and beam deflector [24]. Besides a GRIN PC with triangular refractive index profile has been suggested for beam compression of an electromagnetic wave and a beam aperture modifier has been designed [25]. A secant hyperbolic profile of effective refractive index in GRIN PC with varying lattice spacing was proposed in order to show focusing, diverging and collimation effect in both low and high frequency regimes [26]. Borislov *et al*. proposed an exact form of rod radii distribution and refractive index profile for designing lens-like medium [27].

In this paper, the various characteristic of focusing effect is investigated by the parabolic GRIN PC structure which is constructed from different material with different refractive index. The structure under study is composed of air holes in dielectric background. The Finite-Difference Time-Domain (FDTD) method has been carried out for simulating light propagation direction through the structures [28]. Furthermore, the plane wave expansion (PWE) method [29] has been used to calculate the band structure. The investigation has been done is valuable for designing GRIN PC lenses.

2. THEORY

The plane wave expansion (PWE) method has been implemented for calculating photonic band structure. The electromagnetic wave propagation can be determined by the Maxwell equation as follow

$$\vec{\nabla}\times\left[\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right]=\frac{\omega^2}{c^2}\vec{H}(\vec{r}) \tag{1}$$

In which c is light velocity in a vacuum, ω is the angular frequency of light and

$\varepsilon(\vec{r}) = \varepsilon(\vec{r}+\vec{R})$ is position dependent dielectric function that is periodic in terms of the lattice vector $\vec{R}$. The photonic band structure can be determined by solving (1) by PWE method [29]. The analytic analysis of 2D PC is much easier because of the fact that the (1) can be solved separately for two polarizations TM and TE. The vectorial form of main equation can be converted to two independent scalar problems. In the case of TM polarization the electric field is normal to the PC plane and in the case of TE the same is hold for magnetic field. Because of wave propagation through periodic structures the magnetic field can be expanded in terms of plane wave in wave vector space

$$\vec{H}(\vec{r}) = \sum_{\vec{G}} \sum_{\lambda=1}^{2} h_{\vec{G},\lambda} \hat{e}_{\lambda} e^{i(\vec{k}+\vec{G}).\vec{r}} \tag{2}$$

In which $\vec{k}$ is wave vector in the first Brilloun zone. $\hat{e}^{\lambda}_{\vec{G}}(\lambda = 1,2)$ are unit vectors that are perpendicular to each other and to the $\vec{k}+\vec{G}$. These three vectors $\{\hat{e}^{1}_{\vec{G}}, \hat{e}^{2}_{\vec{G}}, \vec{k}+\vec{G}\}$ constitute a right-hand system. Replacing (2) in (1) leads to the following equation

$$\sum_{\vec{G}} \left|\vec{k}+\vec{G}\right| \left|\vec{k}+\vec{G}'\right| \chi(\vec{G}-\vec{G}') \begin{pmatrix} \hat{e}_2.\hat{e}_2' & -\hat{e}_2.\hat{e}' \\ -\hat{e}_1.\hat{e}_2' & +\hat{e}_1.\hat{e}' \end{pmatrix} \begin{pmatrix} h_{\vec{G}',1} \\ h_{\vec{G}',2} \end{pmatrix} = \frac{\omega^2}{c^2} \begin{pmatrix} h_{\vec{G},1} \\ h_{\vec{G},2} \end{pmatrix} \tag{3}$$

Where $\chi(\vec{G})$ is Fourier transform of inverse $\varepsilon(\vec{r})$ that is a key parameter in calculating photonic band structure which is a function of primitive lattice vector. In a 2D PC, $\vec{k}+\vec{G}$ is placed in x-y plane for all $\vec{G}$, so $\hat{e}_2.\hat{e}_1{}' = \hat{e}_1.\hat{e}_2{}' = 0$. For incident light normal to the segment axis, the matrix form of (3) converts to the two scalar problems for two polarizations. For TM polarization and for all $\vec{G}$ the eigenvalue problem will be as follow

$$\sum_{\vec{G}'} \left|\vec{k}+\vec{G}\right| \left|\vec{k}+\vec{G}'\right| \chi(\vec{G}-\vec{G}') h_{\vec{G}',2} = \frac{\omega^2}{c^2} h_{\vec{G},2} \tag{4}$$

This equation is known as “Master equation” for 2D PC. All the variables depend on the reciprocal lattice vector. The important characteristic of equation (4) is the absence of the coordinate dependence. That reveals the importance of the PWE method.

3. GEOMETRY

The PC structure under study is composed of air holes in dielectric background with circular elements in a square array. There are three methods for creating a GRIN PC by changing the structural parameters of normal PC such as, varying filling factor of unit cell, altering the lattice spacing and using different materials with different refractive index. Here, the first method has been used to create a GRIN PC with parabolic profile of refractive index. To obtain a gradient of refractive index according to a parabolic function, the air holes radii are altered in the direction transverse to the propagation. Figure 1 shows a schematic representation of the designed GRIN PC structure. The parabolic GRIN PC structure is made of 9 columns in x-direction and 21 layers in y-direction which makes y-direction in the range of [-10a, 10a] in which *a* is lattice constant. To curve and bend light propagation direction, the GRIN PC structure is designed and we investigated beam propagation through GRIN structure by studying iso-frequency curves in wave vector space. Manipulation and controlling light propagation direction is based on gradual change of structural parameter of PCs that gradually modify the refractive index of the structure which results in changing dispersion properties.

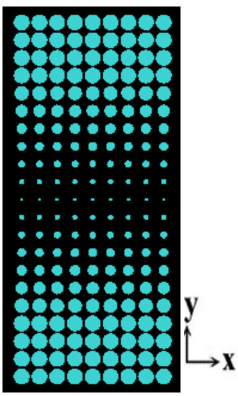

Fig. 1. The schematic representation of GRIN PC structure with parabolic refractive index

Wave propagation direction is defined by the group velocity which is perpendicular to the iso-frequency curves. We need to modify the direction of group velocity travelling through GRIN PC to obtain light bending. Within a GRIN PC structure the group velocity is position dependent. Using this unique property of GRIN PC, continuous light bending will be obtained. The GRIN PC structures are not strictly periodic though when the gradient of refractive index is small enough, we can extract the optical properties from the unmodified PC structure. Arising from gradual change in filling factor of unit cells, group velocity is position dependent so light propagation direction slowly change going through the GRIN structure. The iso-frequency curves of normal PCs made of air holes with the radius equal 0.15a, 0.25a, 0.35a and 0.45a are represented in Figure 2(a)-(d). As can be seen from Figure 2, for different value of *r/a* and nearly for all frequencies located at the first band of band structures, the iso-frequency curves are almost circular. Therefore, we can approximate the PC structure as an isotropic medium.

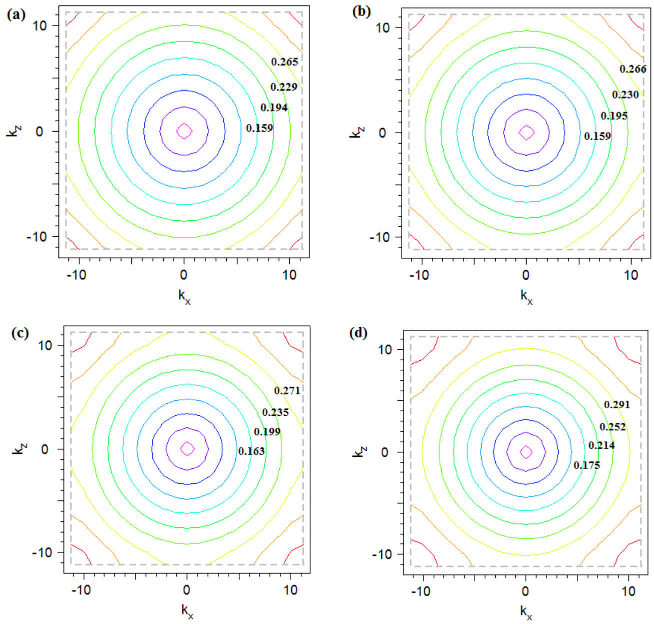

Fig. 2. Iso-frequency curves for PCs made of air holes in dielectric background in which the hole radius equal (a) 0.15a, (b) 0.25a, (c) 0.35a and (d) 0.45a

At first, the PWE method has been employed to calculate dispersion diagram along Γ-X direction for the first band of PCs with different air hole radii. The results are represented in Fig 3(a). Increasing radius of the holes while refractive index and lattice spacing is kept constant, results in shifting the related bands to the higher frequencies. For a certain column, holes radius increase from 0.05a to 0.45a. The value of air hole radius decreases from the edges to the center. The gradient of refractive index only is present in the transverse y-direction. To change refractive index of air holes, we need using different material so we keep the refractive index of background material equals 2.

As a second step, we calculate the group index from the slop information of each band of dispersion diagram. The group index can be calculated from the below relation

$$N_g = c / \nabla_k \omega(k) \tag{5}$$

In which c is light velocity in vacuum, ω is angular frequency and k is wave vector. Hereafter, the group index is referred as effective refractive index (n_{eff}). The result is shown in Figure 3(b) that represents effective refractive index as a function of normalized frequency for different value of air hole radii. For the longer wavelength or lower frequency the effective index curves are closely spaced. So, we have smooth variation in effective refractive index. Therefore, by getting closer to the edges (cut region); we can see a nonlinear behavior of dispersion in effective index curves. Each curve enters the cut region at different frequency and strong dispersion can be seen in these regions. As a last step, we are going to design a GRIN PC structure with a certain profile of refractive index in a fixed frequency. This frequency is selected from the region that we have smooth variation in effective index. The GRIN PC structure is designed at normalized frequency equals a/λ=0.18. Now we can calculate effective index as a function of air hole radius when frequency is fixed. Figure 3(c) shows effective refractive index as a function of air hole radius variation. To obtain a GRIN PC structure with any desired profile of refractive index distribution in the range of 1.25 to 1.98 we need to obtain intermediate point of refractive index which provides a smooth variation in effective index. So interpolation method has been applied to obtain intermediate point by fitting the effective refractive index profile of square cells with different radius of air holes which varies from 0.10a to 0.45a. Then the desired GRIN PC structure is constructed by gradually changing the air hole radius having those intermediate n_{eff} values in such a way that concerning index distribution become manifest.

According to the ray theory, in the GRIN media, the light rays bend toward higher refractive index [30]. As can be deduce from Fig 3(b), the effective refractive index of structure along the optical axis is larger than both sides, then the incident wave converges toward the central region.

The effective refractive index of the GRIN PC structure obey from parabolic function according to the following relation

$$N^2(y) = N_0^2(1-\alpha^2 y^2) \tag{6}$$

Where N_0 is refractive index along the optical axis (x-direction) which equals 1.970 and α is gradient coefficient. The gradient coefficient α for the GRIN PC structure is 0.074. The required radius values for refractive index distribution according to the equation (6) are extracted from the fitting curve in Figure 3(c). The effective refractive index and radius distribution for parabolic GRIN PC structure is plotted in Figure 3(d).

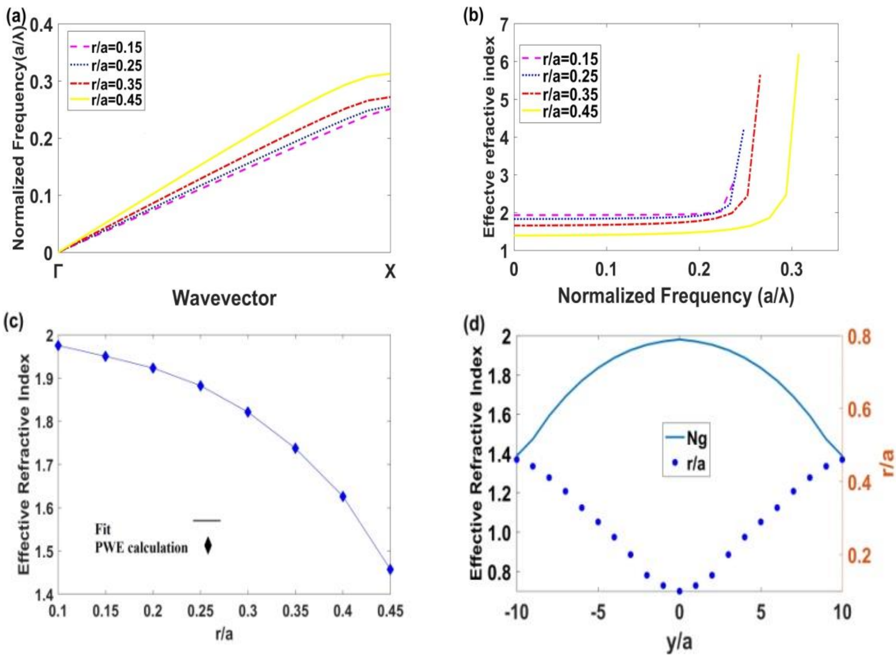

Fig 3. (a) Dispersion diagram along Γ-X direction for the first band, (b) The effective refractive index as a function of normalized frequency (a/λ), (c) The effective refractive index as a function of air hole radius at fixed frequency 0.18. (d) Effective refractive index and r/a for GRIN PC structure

The focusing effect can be studied from the iso-frequency curves. In a homogenous medium the iso-frequency curves are circular but in PCs they are strongly anisotropic and sensitive to frequency. The propagation of light through GRIN PC structures can be understood by analyzing the iso-frequency curves related to the PCs that successively will be passed by the incoming beam. The Gaussian beam propagates layer by layer through the PCs and the group velocity can be determined by the local dispersion curves. Moreover, in a homogenous medium the revolution of iso-frequency curves can be determined by the change of effective refractive index.

4. RESULTS OF SIMULATION

For the GRIN PC structure in which the holes radii vary from 0.10a to 0.45a and the refractive index of the background material equal 1.4, 1.8 and 2.1, the iso-frequency curves are circular in a fixed frequency equals 0.23 (ωa/2πc). In homogenous regime the PC structure can be replaced by a homogenous medium with an effective index that obeys from the Snell-descart law [31]. Therefore, the focusing effect can be explained according to the effective refractive index. Due to the isotropic iso-frequency curves related to the first band, the effective medium theory holds [32]. So, the 2D GRIN PC with flexible hole's radius can be described using effective medium theory [33]. In the case of TM polarization in which electric field is parallel to the hole axis, the average permittivity of the holes and background can be calculated as follow [34-35]:

$$\varepsilon_{eff} = f\varepsilon_r + (1-f)\varepsilon_b \qquad (7)$$

Where the permittivity of the air holes is ε_r, ε_b is the background permittivity and f is filling factor of the holes which equals $\pi r^2 / a^2$. r and a represents hole's radius and lattice constant. To change the effective refractive index of the base PC, the air hole radius is changed that leads to the change of the filling factor of the unit cells. The variation of effective refractive index leads to curve the flow of light and guide light to the higher refractive index part. Curving the light path and deflecting light rays to the higher index part results to observe the focusing effect.

Here, we investigated the focusing effect of the GRIN PC structures with different refractive index of background material. Using equation (7) the effective refractive index (N_{eff}) variation in terms of the variation of refractive index of background material has been calculated. The GRIN PC structures are made of different dielectric material with refractive index equal 1.4, 1.8 and 2.1. Figure 4 represents the effective refractive index as a function of air holes radius in a fixed frequency.

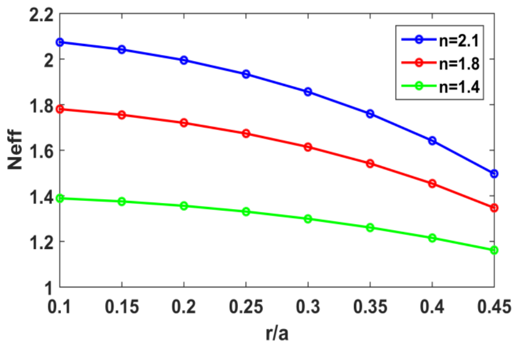

Fig. 4.The effective refractive index as a function of air hole radius for different value of refractive index of the background material

As can be seen, while increasing the refractive index of background material, the effective index of the structure increases. For the GRIN PC structure made of air holes in dielectric background with refractive index equal 1.4, the effective index varies from 1.18 to 1.4. For the structure with refractive index equal 1.8 and 2.1 the effective index interval vary from 1.35 to 1.79 and from 1.50 to 2.09.

5. DISCUSSION

In this paper, we investigate the focusing effect of the GRIN PC structures made of air holes in dielectric background with different refractive index. At first, the transmission spectrum of parabolic GRIN PC for different value of background material refractive index is calculated. We limit our study to the case of TM polarization in which the electric field is parallel to the segment axis. Afterward, the finite-difference time-domain (FDTD) analysis has been performed in order to measure the transmission spectra over a wide bandwidth. A continoues source is placed at the left side of the GRIN PC and a power monitor is located at the output side of the structure in order to measure the transmission spectrum. The result is shown in Figure 5.

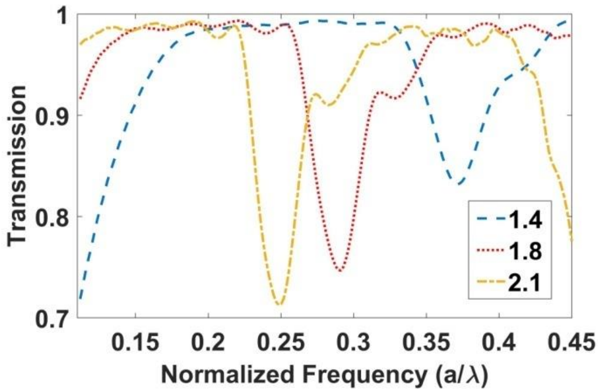

Fig. 5. The transmission spectrum of parabolic GRIN PC structures with different refractive index of background material calculated by the FDTD method

As it is obvious, there are reflectance gap as well as transmission bands. That is because of the facts that periodical characteristics along the propagation direction are kept unchanged and the gradient index only presents along the transverse direction to propagation. As it is obvious, by increasing the refractive index of background material, the reflectance gap gets narrower and moves toward the higher wavelength. The frequency intervals of a/λ={0.14 to 0.24 and 0.33 to 0.45} are presenting high transmission window of TM polarizations. As a second step, we are going to investigate the focusing characteristics of GRIN PC in relative to the refractive index of background material. A wide Gaussian source is placed at the left side of the parabolic GRIN PC structure. The whole structure is surrounded by a perfectly matched layer (PML) boundry condition. The FDTD method is applied to simulate electromagnetic wave propagation through the structure. Gaussian beam with a frequency of 0.23 (ωa/2πc) is radiated to the structure which refractive index of background material equal 1.4, 1.8 and 2.1. The electric field propagation through the GRIN PC structures is shown in Figure 6(a)-(c).

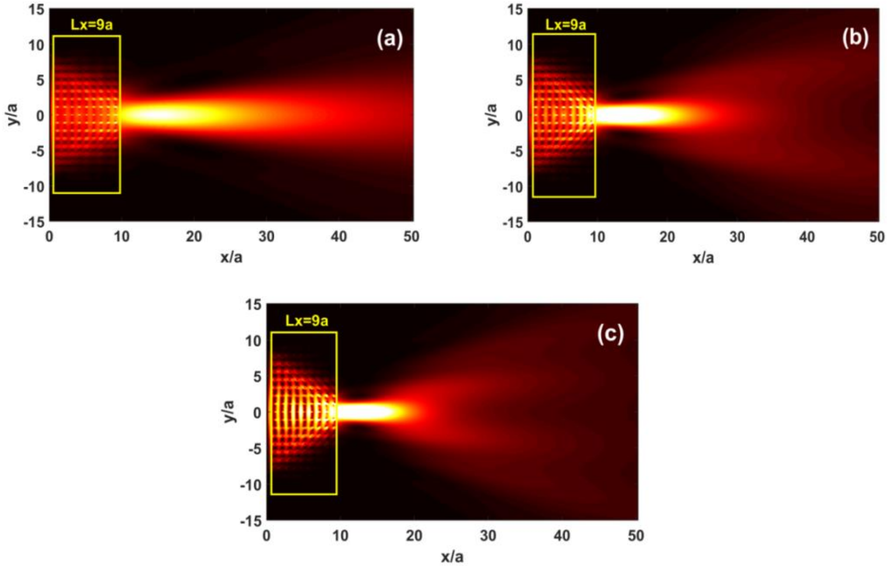

Fig. 6.The electromagnetic field propagation through GRIN PC structures with different refractive index of background material equal (a) n=1.4, (b) n=1.8 and (c) n=2.1

As can be seen, by increasing refractive index of background material the focusing effect becomes more apparent. The outgoing wave becomes more confined and more focused in the focal point. As mentioned before, by increasing the refractive index of material, the effective index enhances and also the focusing power increases. The cross sectional electric field profile at focal point of three structures is plotted in Figure 7. As it is obvious, while enhancing the refractive index of background material, wave becomes more confined. The width decreases and the confinement intensity increases.

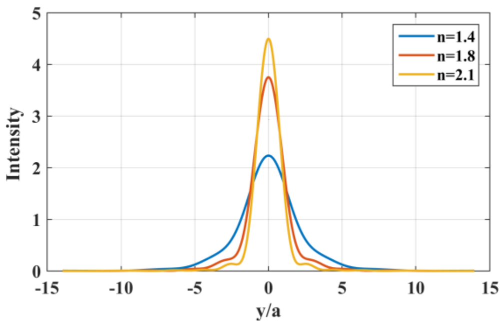

Fig. 7. The normalized intensity at focal point position of GRIN PC structures with different refractive index of background material

The full-width at half-maximum (FWHM) of the outgoing wave at the output of the GRIN PC structures with refractive index equal {1.4, 1.8, 2.1} in terms of lattice constant *a* equal {2.89a, 3.60a, 5.32a} and in terms of wavelength λ are {0.66λ, 0.82λ, 1.22λ}. As can be seen from Figure 7, while increasing the refractive index of background material, the concentration field intensity increases too.

6. CONCLUSION

Here, we studied the focusing characteristics of a parabolic GRIN PC structure made of air holes in dielectric substrate. The investigations of electromagnetic wave behavior through GRIN PC structures with different refractive index of background material show that while increasing refractive index of background, the focusing effect become more apparent. Due to the isotropic and circular iso-frequency curves the effective medium theory holds. Furthermore, we studied how the refractive index variation affect the transmission spectrum. The effective refractive index calculation shows that while increasing the refractive index of background material the effective index of GRIN PC structure increase. As a result, the outgoing wave become more confined and normalized intensity at focal point position increase.

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