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Ultra- Relativistic Solitons with Opposing Behaviors in Photon Gas Plasma

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Abstract: We have studied the formation of relativistic solitary waves due to nonlinear interaction of strong electromagnetic wave with the plasma wave. Here, our plasma is relativistic both in temperature and in streaming speed. A set of equations consisting of scalar and vector potentials together with a third order equation for the enthalpy in photon gas plasma is obtained analytically. Solutions with single-humped for the scalar potential and single and double-humped for the vector potential profiles are illustrated numerically. It is shown that the drifting velocity of moving solitons and plasma fluid velocity both play an important role in the formation of the solutions. The results show that the amplitude of the potentials increases for higher values of the plasma temperatures for the region that the flow velocity of the plasma is larger than the solitary wave velocity. For the region with larger amount of the soliton's velocity, the results show opposite behavior. It is also found that in the region where the plasma fluid velocity exceeds the soliton drifting velocity, all the solutions are excited at higher temperatures relative to the other area.

Keywords: Nonlinear phenomena, Photon Gas Plasma, Radiation, Relativistic Solitons

1. INTRODUCTION

The theoretical investigation of relativistic solitons in various environments is a relatively old problem which has been treated by many authors in the past. It has recently gained new attention by several authors [1–9]. The ion motion influence on relativistic soliton in electron-ion plasmas was first investigated numerically in Ref. [2]. In Ref. [10], the authors extended the previous works in the weak relativistic limit and studied the effect of relativistic trapped electrons in the propagation of an electro-sound wave in fully relativistic plasma with warm electrons and cold ions. It was observed that as the flow velocity of the plasma changes, the shape of the solitary wave shows two opposing behaviors depending on whether the solitary wave velocity is larger than the flow velocity

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or smaller. Recently, Lehmann and Spatschek [11] studied the existence of stationary electrostatic and electromagnetic waves in ultra-relativistic electron-ion plasmas. Using Poincare´ analysis, they investigated the influence of ion mobility on the phase-space structure. The results for electron-ion plasmas in the ultra-relativistic case were compared with the phenomena in electron-positron plasmas. It was shown that in this regime the electron-ion plasmas behave qualitatively like electron-positron plasma. Relativistic solitary waves in pure electron-positron plasma have been researched by many authors [12-17]. Lontano et al. in [13] derived the soliton-like distributions of electromagnetic radiation based on a relativistic kinetic model. Shukla et al. presented the interaction between intense laser pulses with electron-positron plasma by a nonlinear Schrödinger equation [14].

In [15], the authors investigated electromagnetic standing solitons with a zero drifting velocity for the case of ultra-relativistic hot electron-positron plasmas. They presented the variations of the standing solitons for different values of the plasma temperatures and plasma fluid velocities. The main purpose of current paper is to show the dependence of symmetric and anti-symmetric solution of moving electromagnetic solitary wave on the two velocities: the solitary wave velocity and the flow velocity of the plasma. To this end, a non-linear coupled differential equation for the scalar and vector potential together with a third order equation for the enthalpy in ultra-relativistic plasma is obtained analytically and solved numerically. In [16], the authors using the variational method studied the nonlinear interaction of an intense electromagnetic wave with hot relativistic electron-positron plasmas. They found an effective potential to describe the evolution of the system and showed the possibility of beam self-trapping in formation of stable two dimensional soliton structures.

The organization of the paper is as follows: in Sec. II, we derive the set of the relativistic hydrodynamic equations for propagation of an intense laser in two-component plasma consisting of hot electrons and positrons. Two coupled differential equations for the vector and scalar potential based on the plasma fluid velocity, solitary speed and the enthalpy are obtained. In order to close the set of the equations, we use the expressions for the enthalpy in ultra-relativistic regime and entropy for adiabatic plasma. Section III contains the numerical analysis for finding the acceptable solutions of the vector and scalar potential equations. Numerical solutions for the symmetric single hump scalar potential and symmetric single and anti-symmetric two humped vector potentials are presented. Sec. V. is devoted to concluding remarks.

2. THEORETICAL MODEL

We use the one dimensional relativistic hydrodynamic approximations to describe propagating electromagnetic solitary waves in plasmas with hot electrons and positrons.

The model equations are the continuity equation, the momentum balance equation, Poisson's equation and the perpendicular component of the electromagnetic waves in the Lorentz gauge,

$$\frac{d}{d\xi}\left(\gamma_\alpha n_\alpha \left(v_{\alpha x} - V\right)\right) = 0 \tag{1}$$

$$\frac{d}{d\xi}\left(\gamma_\alpha w_\alpha + e\ \sigma_\alpha \varphi - V P_{\alpha x}\right) = 0, \tag{2}$$

$$\frac{d^2\phi}{d\xi^2} = -\sum_\alpha \sigma_\alpha \gamma_\alpha n_\alpha, \tag{3}$$

$$\frac{d^2 a}{d\xi^2} + \omega^2 a = -\sum_\alpha \sigma_\alpha \gamma_\alpha n_\alpha v_{\alpha\perp}, \tag{4}$$

where the length, time, velocity, momentum, enthalpy, pressure, vector and scalar potential and number density are normalized by c/ω_p, ω_p^{-1}, c, mc, mc^2, $mc^2 n_0$, mc^2/e and n_0 respectively; Here, $\omega_p = \left(4\pi\, n_0\, e^2/m_e\right)^{\frac{1}{2}}$ is the electron plasma frequency, m is the electron and positron rest mass, c the speed of light in vacuum, e the magnitude of the electron charge and n_0 is the unperturbed particle density.

In Eq. (2), P_α is related to the kinetic momentum $p_\alpha = \gamma_\alpha w_\alpha v_\alpha$ through $\mathbf{P}_\alpha = \mathbf{p}_\alpha + \sigma_\alpha \mathbf{A}$ and called canonical momentum and $\gamma_\alpha = \left(1 + p_\alpha^2\right)^{1/2}$ is the relativistic factor. Also, α represents the two species of the plasma, a is the vector potential, ϕ the scalar potential, ω the electromagnetic wave frequency, V the group velocity of the soliton and v_α the plasma species velocity. Using equations (1-2) and the definition of the kinetic momentum, we have

$$p_{\alpha x} = \gamma_V^2\left(\pm w_\alpha R_\alpha - V\,\Psi_\alpha\right) \tag{5}$$

$$\gamma_\alpha = \gamma_V^2\left(\pm V\, R_\alpha - \frac{\Psi_\alpha}{w_\alpha}\right) \tag{6}$$

$$n_\alpha = \pm\gamma_0 \frac{\left(\beta_{x_0} - V\right)}{R_\alpha} \tag{7}$$

where

$$\Psi_\alpha = \gamma_V^{-2}\sigma_\alpha\phi + \gamma_0 w_0\left(\beta_{x_0}V - 1\right),\ \beta_{x_0} = \frac{v_{x_0}}{c},\ R_\alpha = \frac{1}{w_\alpha}\sqrt{\Psi_\alpha^2 - \gamma_V^{-2}\left(w_\alpha^2 + a^2\right)} \quad (8)$$

Eqs. (5-7) are the expressions for the x component of the momentum, relativistic factor and number densities of the species as function of potentials and enthalpy. The subscript zero in equations (5-7) shows the quantities at infinity ($\xi \to \pm\infty,$), where the vector and scalar potential vanishes. The plus and minus signs are associated to $\beta_{x_0} > V$ and $\beta_{x_0} < V$ cases, respectively. For the boundary condition that the plasma is at rest at infinity $\beta_{x_0} = 0$, these areas are equivalent to $V < 0$ and $V > 0$ respectively [18]. From the physical point of view, the case $\beta_{x_0} < V$ $\left(\beta_{x_0} > V\right)$ is associated to the situation that the plasma fluid and the soliton are moving in the same (opposite) direction. Eqs. (1-8) finally can be expressed as

$$a_{\xi\xi} + \omega^2 a = \gamma_0\left(\beta_{x_0} - V\right)\sum_\alpha \frac{a}{\sqrt{\Psi_\alpha^2 - \gamma_V^{-2}\left(w_\alpha^2 + a^2\right)}} \quad (9)$$

$$\phi_{\xi\xi} = \gamma_0\gamma_V^2\left(\beta_{x_0} - V\right)\sum_\alpha \frac{\sigma_\alpha\Psi_\alpha}{\sqrt{\Psi_\alpha^2 - \gamma_V^{-2}\left(w_\alpha^2 + a^2\right)}} \quad (10)$$

where $\gamma_V = \left(1 - V^2\right)^{-1/2}$ is a relativistic factor associated with the velocity of the soliton. In order to find our closed system of equations, we will find the enthalpy as a function of the potentials. To this end, we restrict the problem to an adiabatic ultra-relativistic plasma, $w_\alpha = w_0 n_\alpha^{1/3}$ [15] in which $w_0 = 4k_B T_0$. Armed with this, we can find the enthalpy as a function of the vector and scalar potentials,

$$w_\alpha^6 + k_\alpha w_\alpha^4 + C = 0 \quad (11)$$

in which

$$k_\alpha = -\gamma_V^2\left[\gamma_V^{-2}a^2 - \left(\gamma_V^{-2}\sigma_\alpha\varphi + \gamma_0 w_0\left(\beta_0 V - 1\right)\right)^2\right],\ C = \gamma_0^2\gamma_V^2\left(\beta_0 - V\right)^2 w_0^6 \quad (12)$$

Equations (9-11) form a closed set of equations for the interaction of an intense laser field with a hot ultra-relativistic electron and positron adiabatic plasma.

3. NUMERICAL ANALYSIS

We searched the profile of the vector potential with zero ($\lambda = 0$) and one ($\lambda = 1$) node. In the computer program, although we should choose a finite domain, but the domain can be choose a large domain for solving the problem.

If the answer with the variation of the domain matches with the meaning of physics, the answer is acceptable. Some people use Runge-Kutta method. Here, the needed numerical computations have been achieved by using the special software by the name of PDE. This software can solve the PDE equations. We define the geometry of the problem and give it the initial values. This software solves the problem and gives us the different plots. The PDE software also determines the grid points and checks the convergence criteria.

To find the $\lambda = 0$ type of solutions, we impose $a_\xi = \varphi_\xi = 0$ at $\xi = 0$ on the derivative of the potentials, while both the scalar and the vector potentials would have finite (non-zero) values. The second types of the solutions $\lambda = 1$ are achieved by imposing the boundary conditions $a = \varphi_\xi = 0$ at $\xi = 0$ while the scalar potential ϕ and a_ξ maintain finite values.

Figs. 1 and 2 show the variations of the scalar and vector potentials for different values of the plasma temperatures. Fixed values of $\beta_0 = 0.8$ and $V = 0.4$ are considered in $\beta_0 > V$ region. As is clear from Figs. 1 and 2, the magnitudes of the scalar and vector potentials increase with the increase of the plasma temperature (w_0). Please note that, the scalar and vector potentials are symmetric with respect to the position of their maxima in Fig.1, while, the vector potential is antisymmetric as shown in Fig. 2. The shortest multi-humped soliton represents a two-humped solution with one positive peak and one negative peak.

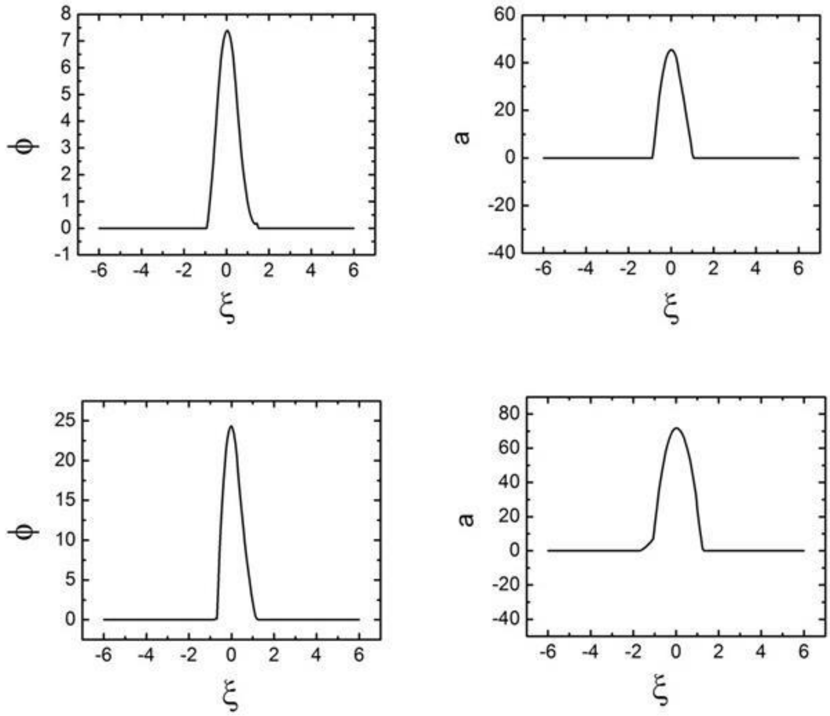

Fig 1. Variation of scalar and vector potentials with $\beta_0 = 0.8$ and $V = 0.4$ in hot electron-positron plasma with no node in the profile of the vector potential. The plasma temperature is $w_0 = 90$ (top row), and $w_0 = 100$ (bottom row).

The same variations are shown in Figs. 3 and 4 in $\beta_0 < V$ region. We have considered $\beta_0 = 0.7$ and $V = 0.9$ case. In contrast to Figs. 1 and 2, in this region, the magnitudes of the vector and scalar potentials decrease with increasing the plasma temperature. Moreover, it is seen from Figs 1 to 4 that the two humped vector potential ($\lambda = 1$) mode is excited at lower (higher) temperature in region $\beta_0 > V$ $(\beta_0 < V)$ compared with the single humped vector potential ($\lambda = 0$) mode. Furthermore, for the solutions with the antisymmetric profiles of vector potentials (Figs. 2 and 4), the maximum of the vector potential occurs for a negative value of ξ for $\beta_0 > V$, while for $\beta_0 < V$, we observe an opposite behavior.

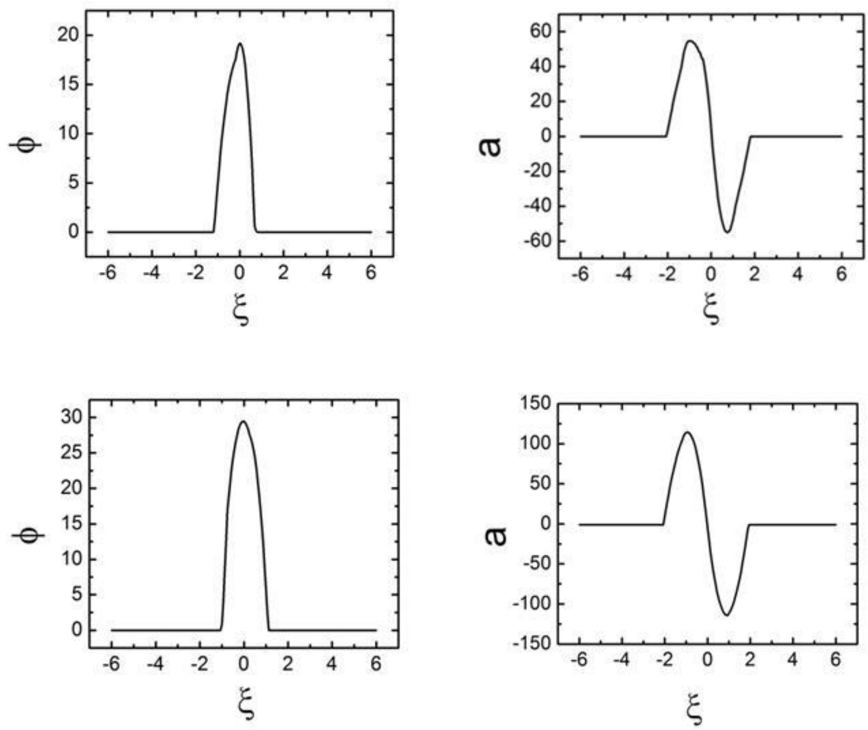

Fig 2. Variation of the scalar and vector potentials with $\beta_0 = 0.8$ and $V = 0.4$ in hot electron-positron plasma with one node in the profile of the vector potential. The plasma temperature is $w_0 = 60$ (top row), and $w_0 = 80$ (bottom row).

This dependence can be interpreted considering the effects of the initial conditions. In other words, the applied initial and boundary conditions to the system for finding the solitary wave solutions, determine whether the first case or second case to occur. We note that both the cases have been seen in other studies. (e.g. see [2] and [19] for the two cases). In the present study these two situations, caused by the initial conditions as well as role of the two velocities. Finally, we note that all the graphs for $\beta_0 > V$ are excited at higher temperature compared with $\beta_0 < V$.

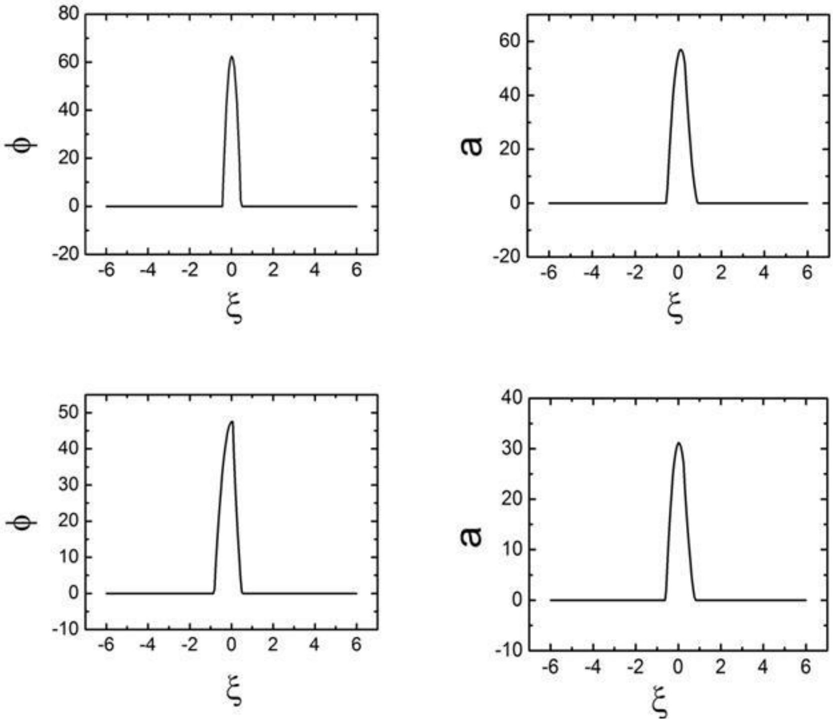

Fig 3. Variation of scalar and vector potentials with $\beta_0 = 0.7$ and $V = 0.9$ in hot electron-positron plasma with no node in the profile of the vector potential. The plasma temperature is $w_0 = 40$ (top row), and $w_0 = 45$ (bottom row).

4. CONCLUDING REMARKS

The formation of solitary waves in adiabatic hot electron- positron plasma in ultra-relativistic regime is investigated. We used the fully relativistic hydrodynamics model to find the governing equations for the scalar and vector potentials and the enthalpy of the system. Some numerical results for single peak scalar potential and single hump and two humps for the vector potential are illustrated. An important result of our investigation was that the soliton velocity and the plasma fluid velocity both play an essential role in the solitary wave properties. It is shown, as the plasma temperature increases, the soliton amplitude increases for the region that the plasma fluid velocity is larger than the group velocity of soliton and vice versa. It is also found that the solitons with one node are excited at lower (higher) temperatures in region where relative to the soliton modes with zero node.

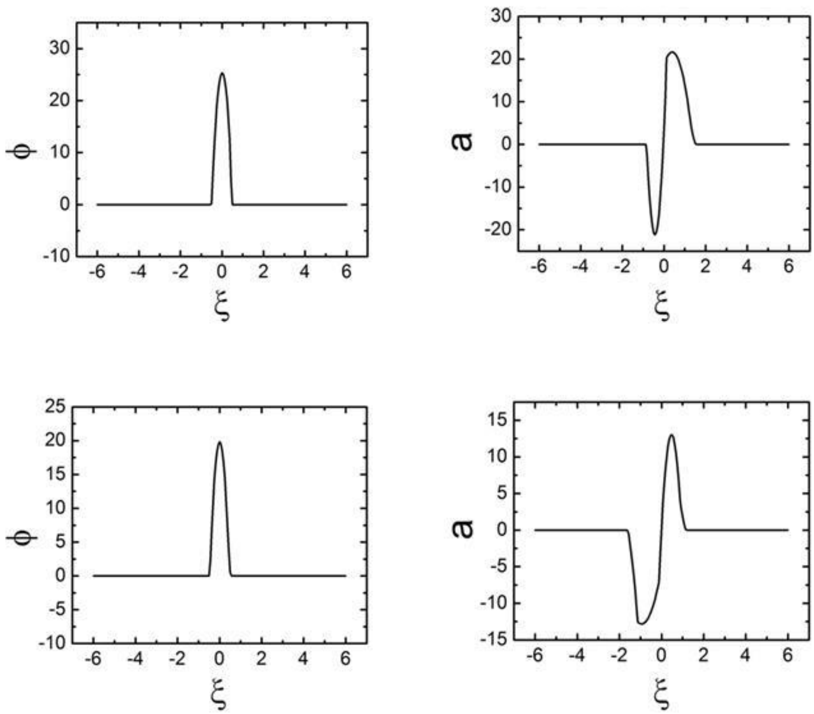

Fig 4. Variation of scalar and vector potentials with $\beta_0 = 0.7$ and $V = 0.9$ in hot electron-positron plasma with one node in the profile of the vector potential. The plasma temperature is $w_0 = 50$ (top row), and $w_0 = 55$ (bottom row).

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