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Estimating the Option Pricing of Currency Market Exchanges Using the Adaptive Neural-Fuzzy Network Method with Z-Valuation Language Variables

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Abstract

Today's, financial derivatives are among the most important instruments to decrease and handle financial risks via high efficiency. One of them is the option contracts in several asset areas such as interest rate, channel and currency, etc. Clearly, uncertainty is undesirable and inevitable for investors who have chosen the currency market for investment. As all the investor's effort is to reduce uncertainty, forecasting is one of the tools in such a market. In this matter, one of the novel methods to predict the price of currency derivatives is using adaptive neural-fuzzy networks. However, since the reliability of information is not shown well in fuzzy sets, solving this challenge is of particular importance. This paper utilizes Z-numbers instead of fuzzy ones to mean this type of information. Then, an improved Z-valuation adaptive neural-fuzzy network is developed for currency price prediction. The obtained results reveal that currency price prediction modeling is confirmed using neural networks.

Keywords: Currency Market, Pricing, Adaptive Fuzzy Neural Network, Z-Numbers.

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1. Introduction

Today's, the rate of the common currencies of the world is one of the assets that attracted the attention of the world's masses in the capital market such as the US dollar, the European euro, the Japanese yen, and the British pound. In this matter, the exchange market has prepared the exchange field with incredible speed and simplicity. The neural network (NN) is one of the methods for predicting currency prices, which are the most widely used methods in classification, pattern recognition, and time series prediction. The high power of recognizing all kinds of patterns in the market data, the approximation of complex functions, and its stability and flexibility against data noises are among the obvious and powerful features of the NN to discover the market price generating process.

It is worthwhile to mention that the prediction of time series is the second most used of NN. Most of them include complex non-linear patterns, so that it has received attention to time series forecasting methods. On the other hand, as the parameters of the financial world have been diverse and completely random in the last two decades, many attempts have been made to model exchange rate pricing under uncertainty. One of the most efficient ways to enhance the results of mathematical models under uncertainty is fuzzy concept developed by Zadeh [1]

It should be mentioned that although fuzzy sets initially revolutionized computing, many people developed them due to their distance from the real world. In short, a system based on fuzzy logic (or fuzzy system) is based on "if-then" logic rules. The starting point of a fuzzy system is to define a set of if-then fuzzy rules by an expert in the desired field. Obtaining these rules is the most important and difficult step that requires the necessary knowledge and sufficient power to implement them correctly. If a fuzzy system is successfully implemented using learning capability, then the error of the output values to the lowest value can be minimized using the least squares error method.

By combining the mentioned learning method with the backpropagation method, a combined training method can be achieved. Its performance is that in each training period when moving forward, the outputs of the nodes are calculated normally until the last layer, and then, the result parameters are calculated by the least sum of squared error method. After calculating the error in the backpropagation of the error ratio on the conditionally parameters, their values are corrected using the technique of downward slope of the error.

One of the most common hybrid methods is Adaptive Neural Fuzzy Inference System (ANFIS) developed by Jang including advantages of both fuzzy system and NN. This model implements a fuzzy system of the Takagi-Sugeno type in a neural structure and then employs the backpropagation method or a combination of the error backpropagation and error least squares training methods for the training process. In fact, they use an adaptive network encompassing the general state of the multilayer feedforward NN to solve the problem of identifying the parameters of the fuzzy inference system [2].

It should be noted that an adaptive network is a multi-layer forward structure that its total output behavior can be specified using the value of a set of the modifiable parameters. This tool solves the main problem of the fuzzy inference system by obtaining "if-then" fuzzy rules as well as by optimizing model parameters.

The rules are fixed in ANFIS where the membership functions factors are optimized to determine the factors (the same form of the membership functions) from the process of training views of the NN. It is worth noting that the type of membership functions (such as triangular and Gaussian) as well as the number of membership functions for inputs and outputs can be determined by trial and error. Here, it is essential to specify the type of membership function as well as their numbers in the first layer. Its output is a linear relationship while its parameters can be achieved by combining the methods of least square error and backpropagation method based on gradient reduction.

Meanwhile, the main problem with these methods is that the reliability of the information is not well provided. For this purpose, Zadeh solved this problem using Z-numbers to effectively deal with uncertain information. Note that $Z(A, B)$ includes two parts of restriction (A) and the confidence of the restriction evaluation (B). Compared with classical fuzzy number, Z-numbers could more ability to describe human knowledge [3].

As the behavior of the currency market is non-linear and chaotic, the main objectives of this paper are to develop a currency price prediction model using fuzzy-adaptive NNs based on Z-numbers, as follows:

1. Proposing some suitable times for buying or selling companies based on the price ratio of Euro to Dollar;
2. Prediction with an acceptable percentage of error to guide Iranian traders in the direction of buying and selling goods and currency;
3. Providing the necessary evidence to answer the proposed problem; is it possible to develop a model in order to determine the exchange rate using NN?

2. Literature review

The literature review for predicting financial data can be classified into two categories: financial econometrics and soft computing. The application of soft computing in economics began in the late 80s with the study of White [4], in the financial market and forecasting the stock price of IBM. Of course, its main purpose was to assess the market efficiency hypothesis test. It should be mentioned that although its results indicated that the minimization algorithms in econometrics were better than the NN algorithms, this study was disputed by several researchers due to the simplicity of the network.

After White [4], several studies have successfully been conducted on soft computing. This success in financial economics attracted the attention of experts in macroeconomics and econometrics. Nevertheless, this method has also been criticized despite its very useful and practical. In the view of statisticians and economists, the strength of NN is considered a weakness in terms of freedom from the constraints in their statistical models. However, soft computing methods have obtained a special place in most recent studies for predicting financial variables via non-linear behaviors.

Particularly, as several factors affect the stock index such as political events, economic conditions, traders' expectations, and other environmental factors, its nature is generally highly volatile, dynamic, nonlinear, complex, and chaotic [5]. Accordingly, such nonlinear dynamic system will cause ambiguity in practice in the variable behavior of the linear models and as a result, these models will not have the required efficiency [6].

Moreover, regarding the previous research for forecasting long-term horizons of currency prices using linear models, the inefficiency of linear models has been confirmed in the long-term forecasting process, while soft computing methods perform better in long horizons [7]. Therefore, the superiority of soft computing methods follows non-linear behavior over other statistical and econometric methods for predicting variables [7]. Therefore, the fuzzy-adaptive NN method is utilized to predict the option pricing of currency market exchanges.

2.1. Neural networks

The conducted studies in the field of NNs were analyzed to predict and relate it to the stock and currency markets. The NN approach as a data-oriented method without considering the assumptions in model-oriented methods is strong and innovative to complex predictions. The real power of NN is their ability to learn. In this way, the fuzzy-adaptive NNs are one of the most widely used methods to recognize time series prediction. On the other hand, the improved fuzzy-adaptive NN based on Z-numbers is better than the fuzzy-adaptive NN under uncertainty. Thus, this paper employed it as the network used in this. Finally, the BP method was chosen for training after investigating the training methods in NNs.

2.2. Neural network features

The features of the NN are as follows:

1. Ability to generalize;
2. Ability to tolerate damage;
3. Ability to repair;
4. Ability to share or associative memory, addressable memory, and storage;
5. Very high speed due to parallel processing

Many different NN models have been developed in the last fifty years or earlier to achieve prediction, classification and clustering tasks [8-14]. This paper focuses only on one main model that is widely and successfully employed in management, accounting and finance. It should be mentioned that ANN method can model the non-linear complexities of financial markets, in which its superiority has been investigated in several studies over statistical and econometric methods [15,16]

2.3. Foreign exchange market forecasting

Lubecke et al. [17] developed some forecasting methods in the currency market into NN and then employed the prediction combination of these methods in the NN. Shin and Han [18] proposed genetic algorithm (GA) to improve the weights in NN of the currency market. Chen and Leung [19] employed regression in NN to decrease the error in determining the price in the currency market. Yu et al. [14] entitled "Adaptive Smoothing Neural Networks in Foreign Exchange Rate Forecasting" have significantly reduced the forecasting error compared to other methods. In this paper, the MSE was significantly reduced compared to the regression models [5].

3. Methodology

This paper investigates the currency price to predict it using NN architecture.

3.1. Definition of Z-number

Zadeh [1] proposed Z-numbers along with an unknown variable X. Z-number is an ordered pair of fuzzy numbers $(A, \mathcal{R})$. Note that A is a fuzzy subset of the constraints in which x values can obtain while $\mathcal{R}$ is a fuzzy subset of the confidence scale of the component A. Zadeh [3] developed $(\mathrm{x}, \mathrm{A}, \mathcal{R})$ as a z-value and indicated that this value was equivalent to x being equal to $(A, \mathcal{R})$. Note that Z provides information of variable x. Some Examples of Z are as follows.

Example: (about 45 minutes is very sure whereas about 30 minutes is sure).

This valuation for Z based on Zadeh's suggestion was observed as a limitation in X as follows.

$$prob(x \text{ is } A) \text{ is } R$$

It actually means that

$$R(x): x \text{ is } A \rightarrow \mathrm{P}oss(x = u) = \mu_A(u) \quad (1)$$

$$P(x \text{ is } A) = \int_R \mu_A(u)\, p_x(u)\, du \text{ is } R \quad (2)$$

where μ_A is the membership function of the fuzzy set A while u is a value of x. $p_x(u)$ is the probability density function of y while $\mathrm{P}(x = u)$ is the probability function of x. If the basic probability distribution is unknown, clearly it is a fuzzy number. Few researches have been conducted on Z-numbers [20-23].

3.2. Fuzzy-adaptive NN architecture

Fuzzy systems obtained a special place among modern modeling methods to implement human knowledge using linguistic concepts and expressions and fuzzy rules, non-linear relationships, and the adaptability of these types of systems [24]. In short, a fuzzy system is based on if-then logic rules. However, the reliability of information is not shown well in fuzzy sets, based on which solving this problem is of particular importance. Zadeh [3] developed Z-numbers to mean this type of information .

The starting point of a fuzzy system based on Z-numbers is the definition of a set of Z-number rules by an expert in the desired field. Obtaining these rules is the most important and difficult step that requires the necessary knowledge and sufficient power to implement those rules correctly. As such, the structure of fuzzy-adaptive NN based on Z-numbers is now explained. Adaptive NNs based on Z-numbers, such as adaptive NNs and fuzzy systems, are organized two parts. The first part is the preliminary part and the second part is the following part, which are connected by rules in the form of a network.

The structure of an adaptive neural-fuzzy network based on Z-numbers is developed with two inputs and consists of 3 layers. The first layer performs the fuzzy process (based on Z-numbering) where each node represents a membership function, the learnable parameters of the front part. In the second layer, the firing strength is calculated for each rule. In the third layer, the excitation of each law is normalized based on the excitation power of other laws. In the fourth layer, the output of each rule is obtained. Ultimately, the last layer calculates the output of the Z-number based fuzzy system by summing the outputs of the fourth layer. In this way, Fig. 1 illustrates the structure of the neural-fuzzy inference system based on Z-numbers.

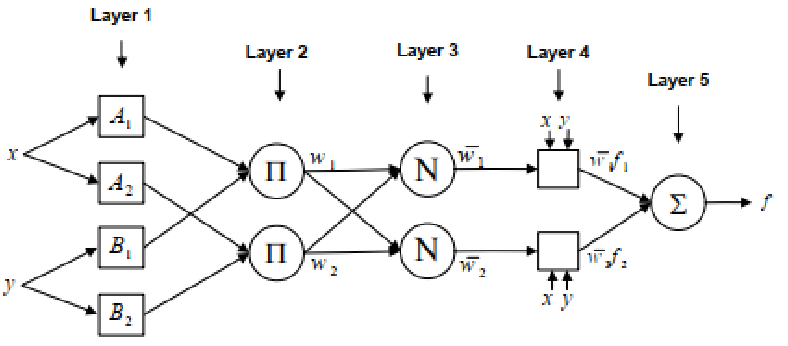


Fig. 1. Structure of adaptive neuro-fuzzy inference system based on Z-numbers

Here, two labels are shown for each input as follows:

$$W_I = \mu_{A_I}^{\alpha}(x) \times \mu_{B_I}^{\beta}(x),\ i = 1,2 \tag{3}$$

where α and β are calculated as follows

where $\mu_{A_I}^{\alpha}(x)$ and $\mu_{B_I}^{\beta}(x)$ are calculated as follows [25]:

$$\mu_{A_I}^{\alpha}(x) = \alpha \mu_{A_I}(x) \tag{4}$$

$$\mu_{B_I}^{\beta}(x) = \beta \mu_{B_I}(x) \tag{5}$$

where α and β are calculated as follows

$$\alpha = \frac{\int_X x \mu_{R_A} dx}{\int_X \mu_{R_A} dx} \tag{6}$$

$$\beta = \frac{\int_X x \mu_{R_B} dx}{\int_X \mu_{R_B} dx} \tag{7}$$

Similar to the adaptive neural-fuzzy network, we will have:

$$\bar{W}_i = \frac{W_1}{W_1 + W_2}, \qquad i = 1,2 \tag{8}$$

$$\left.\begin{aligned} f_1 &= a_1 x + b_1 y + c_1 \\ f_2 &= a_2 x + b_2 y + c_2 \end{aligned}\right\} \Rightarrow f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} = \bar{w}_1 f_1 + \bar{w}_2 f_2 \tag{9}$$

The adaptive neuro-fuzzy inference system based on Z-numbers has a great approximation ability, depending on its ability to partition the input space by determining the membership functions for each input. The Gaussian membership function is as follows:

$$\mu_{A_i}(x) = e^{-\frac{1}{2}\left(\frac{x - m_i}{\sigma^2}\right)^2} \tag{10}$$

$\{m_i, \sigma_i\}$ is the set of parameters in the first section.

3.3. Prediction steps with fuzzy-adaptive NNs based on Z-numbers

The steps of prediction with fuzzy-adaptive NN require careful attention to the following main steps:

Step 1: collecting the daily dollar rate from the central bank's time series bank.
Step 2: obtaining one-day delay data (some data may be lost).
Step 3: calculating the moving average for the past 3 days.
Step 4: selecting a language variable for the input data and classifying using subtractive clustering.
Step 5: mathematical formulating the linguistic variables using Z-numbers
Step 6: considering the output membership function type for linear output data.
Step 7: establishing the fuzzy inference system based on Z-numbers using MATLAB.
Step 8: NN Learning: The data must contain those patterns that the NN is trying to learn, based on which the NN model must infer appropriate weights to represent them.

Step 9: Generalizing NN: The proposed model should be run often when new data is tested to ensure that it simply remembers the features of the training data. It is easy for a NN model to overfit the training data, especially for small datasets. Once these guidelines are implemented, the chances of developing a fuzzy-adaptive NN model that effectively learns training data and generalizes that learning to new data is greatly increased. Most commercially available NN software packages, such as MATLAB 7, have features to facilitate these instructions. MATLAB software has a special toolbox for NNs, which has a variety of learning algorithms, training functions and various other facilities for setting up networks.

Step 10: Obtaining the parameters of the output membership function using the training data
Step 11: Model correction: one-day ahead exchange rate data is obtained using ANFIS, and then the error is calculated using performance evaluation metrics. Afterwards, the best method is achieved by comparing the models.

3.4. Performance evaluation metrics

Commonly, some performance metrics are employed to indicate how the NN learns data relationships. For forecasting problems, these measures are mainly related to the error between predicted outputs and actual desired outputs. Table (1) lists some common metrics for prediction problems. Three of the mean standard error family are the mean squared standard error, the squared mean squared error, and the normalized standard error mean square.

The coefficient of determination (R^2) is normalized regarding the mean square of the standard error, represented as R^2=1-NMSE. Its value is between zero and one. A value of one indicates a perfect match to the data, while a value of zero indicates the performance that can be expected using the mean of the actual output value of *d* as the basis predictions.

Table 1. Common performance metrics for forecasting problems [15].

$\frac{1}{n}\sum_{i=1}^{n}\left(x_i - z_i\right)^2$	mean square standard error (MSE)
$\sqrt{\frac{\sum_{i=1}^{n}\left(x_i - z_i\right)^2}{n}}$	root mean square error (RMSE)
$\frac{\sum_{i=1}^{n}\left(x_i - z_i\right)^2}{\sum_{i=1}^{n}\left(x_i - \overline{x}_i\right)^2}$	Normalized Mean Square Standard Error (NMSE)
$1-\frac{\sum_{i=1}^{n}\left(x_i - z_i\right)^2}{\sum_{i=1}^{n}\left(x_i - \overline{x}_i\right)^2}$	R^2 (coefficient of determination)

3.5. Literature literature in the field of currency market

The currency market is taken from the beginning of the two words “foreign” and “exchange” to mean the foreign exchange, which is also called spot. In fact, the foreign exchange market is the largest financial market in the world, with more than a trillion dollars traded daily. By investigating and examining currency markets and forecasting methods, it is found that artificial intelligence has been employed among these methods and some indicators, which is one of the powerful software. After studying currency forecasting methods based on NN and adaptive neural systems, it was decided to develop a method to forecast currency rates.

3.6. Dollar price predicting

The dollar price is now predicted using fuzzy-adaptive NNs. The fuzzy-adaptive NNs and then the presented model is first investigated. Here, it should be noted that one of the important issues in the architecture of fuzzy-adaptive networks is the number of lags and hidden neurons. As such, to develop a network via the best predictive ability, several lags and neurons are utilized. During the investigation, it was found that the best number of lags is 2 while the best number of hidden neurons is 10.

3.7. Collecting data

The required data are gathered to compare the mentioned models including the daily time series of the rial-dollar exchange rate parity, from the Central Bank of the Islamic Republic of Iran website for the periods of 2020-2022. In all cases, each year is chosen as a sample year, while April to January are selected as training samples, and the months of February and March are the testing months. The software used is MATLAB.

3.8. Calculating the moving average

As the calculation of the moving average (MA) is obtained from the price changes, it is a function of the price and the market trend. In terms of structure, it is always a bit behind the trend for predicting the future price. How to calculate the moving average is different depending on the formula. Its simplest method is the simple moving average [26], which takes data from the latest price changes. As in all the price, changes (tick charts) are not in the currency market or other markets, the closing price of the candlestick or bar chart is employed to obtain the moving average.

To calculate it, the opening price of the candlestick or the highest and the lowest price of the candlestick can also be employed other than the closing price of the candlestick. Here, there are various methods to calculate it, and the simplest and most practical one is now explained. Nonetheless, other moving averages give a different value to price data such as exponential moving averages where the user determines the value of each price data in this average. Surface moving average and weighted linear moving average are other types that generally give new data more value and weight for calculation. However, what is important is to investigate different moving averages in different markets. The general formula of the moving average is as follows:

$$y_t = e_t - \theta_1 e_{t-1} - \ldots - \theta_q e_{t-q} \tag{11}$$

Such series are called moving averages of order q and are denoted as MA(q) [27]. where y_t is the state of the time series at *t* and $\left(1, -\theta_1, -\theta_2, \ldots, -\theta_q\right)$ are the weights applied to the previous values of the time series. This pattern was first investigated by Slatsky and Wald. Note that this process employs the predict errors to improve the predicts.

3.9. Autoregressive

Autoregressive processes represent the regressions on themselves, which is achieved if the residuals of the regression are modeled by the residual values with an interval. Its order is equal to the number of intervals used to model the residuals. The autoregressive model of order p (AR(p)), is defined as follows:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + e_t \tag{12}$$

where e_t is pure disturbance [27]. If equation (12) is expressed in terms of the backward transfer operator, it is as follows:

$$\theta_p\left(B\right) y_t = \left(1 - \phi_1 B + \phi_2 B^2 + \ldots + \phi_p B^p\right) y_t = e_t \tag{13}$$

This model is actually the regression of y_t in relation to y in the past; that is why it is called autoregressive.

3.9.1. Stability condition

Stability in autoregressive

The equation $\theta_p(B) = 0$ is called characteristic equation including real or complex roots. If the absolute value of all polynomial roots of $\theta_p(B)$ is greater than unity, then the process is stable. In this paper, the average of the last 3 days is calculated using the equation. A part of the output of the 1st order moving average and autoregressive calculation are reported in Table 1.

Table 2. Part of the calculation output of AR and MA

Date	Dollar(T)	AR (1)	MA (3)	Dollar(T+1)
07/22/2020	50353	50672	50479	50657
08/22/2020	50412	50657	50580	50551
09/22/2020	50673	50550	50626	50615
10/22/2020	50657	50615	50607	50651
11/22/2020	50550	50650	5060	50531
12/22/2020	50615	50529	50598	50486
01/22/2021	50650	50486	50555	50651
02/22/2021	50530	50650	50555	51021
03/22/2021	50486	51020	50718	50986
04/22/2021	50650	50986	50885	50836
05/22/2021	51020	50836	50947	51212
06/22/2021	50986	51212	51011	51281
07/22/2021	50836	51279	51110	51426
08/22/2021	51212	51426	51306	51932

Then, the lowest prediction error is predicted using least squares and training data, and thus the parameters are estimated.

3.9.2. Choice of membership function type for output variables

The fuzzy rules based on Z-numbers are now defined. As there are two linguistic variables, the following two Z-valuation rules can be expressed as a result:

- **Rule One:**

if $x_1\ (Dollar(t)) = L_{(Low,\text{Likely})}(Dollar(t))$ **and** $x_2\ (MA-3(t)) = L_{(Low,\text{Likely})}(MA-3(t))$
Then

$$F_{(Low,\text{Likely})}\ \big(Dollar(T+1)\big) = P_{(Low,\text{Likely})}\big(Dollar(t)\big)x_1 + q(MA-3(t)\ )\ x_2 + 2r_{(Low,\text{Likely})}$$

- **Rule Two:**

if $x_1\ (Dollar(t)) = L_{(High,\text{Likely})}(Dollar(t))$ **and** $x_2\ (MA-3(t)) = L_{(High,\text{Likely})}(MA-3(t))$
Then

$$F_{(High,\text{Likely})}\ \big(Dollar(T+1)\big) = P_{(High,\text{Likely})}\big(Dollar(t)\big)x_1 + q(MA-3(t)\ )\ x_2 + 2r_{(High,\text{Likely})}$$

where $x_1\ (Dollar(t))$ and $x_2\ (MA-3(t))$ are linguistic variables. $L_{(Low,\text{Likely})}(Dollar(t))$ and $L_{(High,\text{Likely})}$ are linguistic variable values based on Z-scores. $P\big(Dollar(t)\big)$ and $q(MA-3(t)\ )$ are the parameters to present the forecast value of a coefficient of the input values in addition to a constant value. Here, the output variable contains the input variables in addition to a bias. Accordingly, the coefficient of x_1 is x_2 while the coefficient of x_2 is q. $F_{(Low,\text{Likely})}$ and $F_{(High,\text{Likely})}$ represent the i-th output value.

To predict the value of $Dollar(T+1)$ in the test data as well as training data by the proposed method, the parameters of the improved fuzzy inference system obtained in the previous step are added to the proposed model so that the output of the model is obtained for the training data. Then, $Dollar(T+1)$ **is** approximated using the test data and parameters of improved fuzzy inference systems based on Z-numbers.

In this paper, two linguistic variables with two titles (low, probable) and (high, probable) are defined. Based on Rule One and Rule Two, if the daily price is 53220 Rials and the average of 3 days is 51240 Rials, the next day's price will be 51300 Rials. We obtain the optimal weights of the NN using the least squared error and the parameters are estimated. According to the previous sections by considering Gaussian membership function, the input parameters of Gaussian membership function can be observed in Table 3 along with the output parameters listed in Table 4.

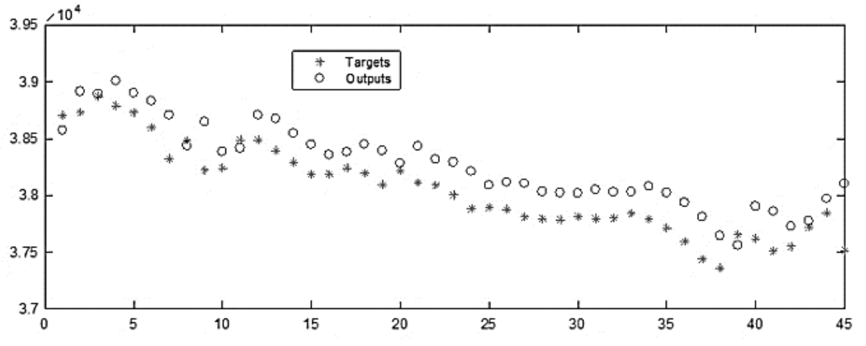
Table 3. The input parameters of Gaussian membership function

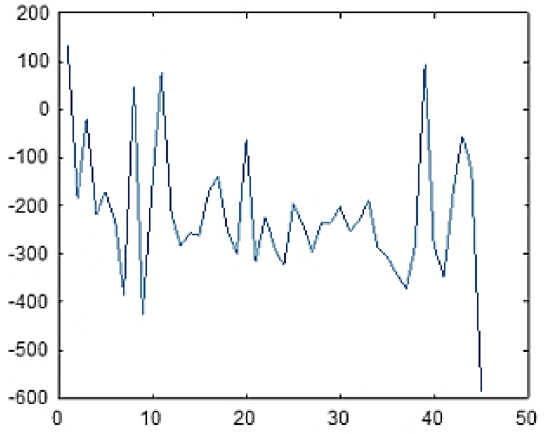
Methods	Average	Variance
Autoregressive in 2020	52149.9	2255.4
Moving Average	532114.9	2220.2
Autoregressive in 2021	533428.9	2224.4
Moving Average	53436.9	2135.6
Autoregressive in 2022	55291985271	21620599340
Moving Average	5352279405331	2584813461

Table 4. output parameters

p	q	R	
0.959	0.00159	1169.3	2020
0.959	0.00159	1169.3	2021
0.8179	0.176	216.485	2022

The parameters of the output membership function reveal that the value of p has a significant effect on estimating every three years. That is, the price of the previous day has a greater impact than the average price of three days in the forecast to predict one day ahead. The prediction price chart for the test data is depicted in Fig. 2.





RMS=256.1, MSE=65493.8

Fig. 2. Dollar price forecast for test data with error in 2022

The comparison of the error squared using the fuzzy-adaptive NN method based on Z-numbers and the autoregressive method are listed in Table 5.

Table 5. Comparison of square error of two autoregressive models and fuzzy-adaptive NN model based on Z-numbers

2022	2021	2020	Method
275	265	195	Fuzzy-adaptive NN based on Z-numbers
457	479	268	autoregressive

The comparison of the above table reveals that the neural-fuzzy network model based on Z-numbers performs much better than autoregressive. Furthermore, the obtained results indicate that the fuzzy-adaptive NN model based on Z-numbers has a closer price estimation to reality.

4. Conclusion and discussion

Estimating the option pricing of the currency exchange market as an economic evaluation index and forecasting it using an advanced mathematical method are essential in the financial market, which was chosen as the objective. Based on the obtained results, the prediction error rate by the fuzzy-adaptive NN model based on Z-numbers has a better performance than the autoregressive one.

Accordingly, the research question of "Is it possible to provide a model for determining the exchange option rate of the currency market using fuzzy-adaptive NNs based on Z-numbers?" It is confirmed. In this paper, a model was proposed to predict the exchange rate using fuzzy-adaptive NNs based on Z-numbers. The obtained results reveal that NNs have the unique characteristics of fast convergence, high accuracy, and strong function approximation ability, which are suitable for currency price prediction.

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