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Using the Nash Bargaining Method for Performance Evaluation and Target Setting with Grey Data

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Abstract. There are numerous Data Envelopment Analysis (DEA) applications where the data is not accurate. In many real-world scenarios, data is inaccurate. One type of inaccurate data is grey data, which exists between the fully defined boundaries of structured and unstructured data. This article employs the Nash Bargaining approach for evaluation and target setting. We combine grey data DEA scores with the Nash Bargaining problem to find an equilibrium point between the minimum and maximum efficiency values for each DMU. Based on the bargaining method, the equilibrium point is determined for each of the DMUs as a weighted average or relative equilibrium point between the minimum and maximum efficiency. The proposed approach has been validated on different datasets according to grey data.

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Keywords: Data envelopment analysis; Grey data, Nash bargaining; Grey data envelopment analysis.

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1. Introduction

Game theory was presented in 1944 with the publication of the book "Theory of Games and Economic Behavior" [1] by John von Neumann and Oskar Morgenstern. Game theory has been expanded with the publication of articles by Nash and Shapley, which included both cooperative and non-cooperative game theories. Game theory addresses competition and cooperation among intelligent and rational Decision-Makers (DMs) through mathematical models. Game theory includes two types of games: non-cooperative, which illustrates competition, and cooperative, which illustrates collaboration. The Nash bargaining problem represents one of the earliest and most influential results in cooperative game theory. Considering the fact that two rational and intelligent players exist along with a set of possible allocations, one unique allocation must be chosen among them. The Nash bargaining theory evidently provides an elegant approach to solving this problem.

To evaluate and enhance the relative efficiency of a set of homogenous operational units known as Decision-Making Units (DMU), the non-parametric Data Envelopment Analysis (DEA) is employed. These DMUs consume inputs to generate outputs, which may be either desirable or undesirable [2]. Based on assumptions, convex technologies such as CRS (Constant Returns to Scale) or VRS (Variable Returns to Scale), or non-convex technologies such as FDH (Free Disposal Hull) can be utilized to calculate the efficiency. The observed distance between the DMU and the efficiency frontier indicates the relative

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efficiency of a DMU [2].
DEA is beneficial from another perspective as well. DEA provides an efficient operational target point, that is an efficient model/image, for each DMU. The generated targets depend on the specific DEA model applied, but they are almost always chosen to dominate the projected DMU. There are a lot of methods for determining DEA targets. All methods that calculate efficiency scores also calculate targets. A drawback is that, since they seek the maximum potential improvement, they might be far from the observed DMU. To solve this issue, various methods have been proposed to calculate the minimum distance to the efficient frontier (for instance, see [3]).

Data can be categorized into two classes of structured and unstructured. Structured data are well-organized and can be easily searched through basic algorithms. They follow a precise plan, meaning that they are stored in a predefined frameworks which usually consist of rows and columns. Unstructured data lacks a predefined framework or organization. They are often text-heavy, but they can include data such as dates, numbers, and facts, which cannot be easily organized with a structured approach.

Grey data refers to a type of data that exists between the clearly specified boundaries of structured and unstructured data. It is not as organized as structured data, such as database tables, but it is also not as free-form and disorganized as unstructured data, like text documents or social media posts. Grey data, also known as uncertain data, refers to information that is inaccurate, incomplete, or uncertain. This type of data exists between well-known categories (white data) and completely unknown categories (black data). Grey data are particularly related to situations where data is obtained from subjective judgments, estimates, or measurements that are not entirely precise. Grey data possess features such as being semi-structured, uncertainty, flexibility, and broad information coverage, and it can be derived from various sources such as emails, reports, notes, text messages, and surveys. Grey data has applications in various contexts such as engineering, economics and finance, environmental sciences, healthcare, etc. Here are some examples of grey data: emails, sensor data, unorganized web data, spreadsheets, etc. In DEA, grey data can be employed to handle uncertain or inaccurate inputs and outputs. For instance, Yang and Chen [4] suggested a hybrid method combining AHP and grey relational analysis for supplier evaluation and selection. Chen and Chen [5] employed the DEA and grey model to evaluate the productivity in the agriculture industry of Vietnam. Kucukonder et al. [6] utilized a hybrid DEA approach based on grey relational analysis to conduct a study on egg performance. Toninelli chose his thesis topic "Data Envelopment Analysis: Uncertainty, Undesirable Outputs, and Application in the Global Cement Industry," exploring its use and applications. Wang et al. [7] utilized a combined grey model and DEA to evaluate efficiency in e-commerce markets to support better decision-making. Salami et al. [8] used grey system theory and a multi-stage DEA model for the Malmquist Productivity Index to assess the performance of the electricity supply chain in Iran. Wang et al. [9] employed DEA-Grey integration to improve operational efficiency in industrial systems (blockchain markets as a service).

In this article, Nash Bargaining is utilized to evaluate efficiency and target setting with grey data. This is done by modeling the problem as a bargaining problem and calculating the Nash Bargaining result (which is unique and Pareto optimal).

2. Prerequisites

2.1 *The classic data envelopment analysis model*

Assume that n units exist (DMUj, j = 1, 2, …, n) that consume m inputs (xi, i = 1, 2, …, m) and generate s outputs (yr, r = 1, 2, …, s). The relative efficiency of each DMU is determined by the following model:

$$\begin{aligned}
&\theta_k = max \sum_{r=1}^{s} u_r y_{rk} \\
&the \ \ the \ \ s.t \\
&\sum_{i=1}^{m} v_i x_{ik} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1,2,\ldots,n \\
&v_i \geq 0, (i = 1,2,\ldots,m), u_r \geq 0, (r = 1,2,\ldots,s)
\end{aligned} \tag{1}$$

In this model, v_i denotes the input weights, and u_r denotes the output weights. DMU_k is only efficient if in the above model $\theta_k = 1$.

2.2 *The grey numbers theory*

The grey numbers theory was suggested by Deng in 1982 and it has been applied in various contexts. In this theory, a grey system is defined as a system containing unknown data, which is denoted by grey numbers and grey variables.

Definition: Imagin X is a reference set, then the grey set G of the reference set is defined as:

$$\begin{cases} \bar{\mu}_G(x): x \to [0,1] \\ \underline{\mu}_G(x): x \to [0,1] \end{cases} \tag{2}$$

$\bar{\mu}_G(x) \geq \underline{\mu}_G(x), x \in X, X = \mathbb{R}$ ، $\bar{\mu}_G(x)$, and $\underline{\mu}_G(x)$ are the upper and lower membership functions of G, respectively. If $\bar{\mu}_G(x) = \underline{\mu}_G(x)$, then the grey set G is a fuzzy set. Meaning that the grey theory also encompasses the fuzzy conditions. In general, a grey number is represented as ⊗G, where:

$$\otimes G = G \begin{cases} \bar{\mu} \\ \underline{\mu} \end{cases}$$

Definition: A grey number with a lower bound and no upper bound is defined as:

$$\otimes G = [G, \infty) \tag{3}$$

Definition: A grey number with a upper bound and no lower bound is defined as:

$$\otimes G = (-\infty, G] \tag{4}$$

Definition: A grey number with lower and upper bounds is referred to as the grey number and is defined as:

$$\otimes G = [\underline{G}, \bar{G}] \tag{5}$$

A grey number can be demonstrated as:

$$[\underline{G}, \bar{G}] = \underline{G} + (\bar{G} - \underline{G}) * \alpha, 0 \leq \alpha \leq 1$$

Mathematical operations on two grey numbers $\otimes G_1 = [\underline{G}_1, \bar{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \bar{G}_2]$ is defined as follows:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \bar{G}_1 + \bar{G}_2]$$

$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \underline{G}_2, \bar{G}_1 - \bar{G}_2]$$

$$\otimes G_1 \times \otimes G_2 = [min(\underline{G}_1\underline{G}_2, \underline{G}_1\bar{G}_2, \bar{G}_1\underline{G}_2, \bar{G}_1\bar{G}_2), max(\underline{G}_1\underline{G}_2, \underline{G}_1\bar{G}_2, \bar{G}_1\underline{G}_2, \bar{G}_1\bar{G}_2)]$$

$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \bar{G}_1] \times \left[\frac{1}{\bar{G}_2}, \frac{1}{\underline{G}_2}\right]$$

The length of a grey number is defined as $L(\otimes G) = [\bar{G} - \underline{G}]$

2.2.1 *Linear programming model with grey parameters*

Assume that $X = [x_1 x_2, \dots, x_n]^t, C = [c_1(\otimes), c_2(\otimes), \dots, c_n(\otimes)]^t, b = [b_1(\otimes), b_2(\otimes), \dots, b_n(\otimes)]^t$, and

$$A(\otimes) = \begin{bmatrix} a_{11}(\otimes) & a_{12}(\otimes) & \dots & a_{1n}(\otimes) \\ a_{21}(\otimes) & a_{22}(\otimes) & \dots & a_{2n}(\otimes) \\ \dots & \dots & \dots & \dots \\ a_{m1}(\otimes) & a_{m2}(\otimes) & \dots & a_{mn}(\otimes) \end{bmatrix}$$

$$\underline{c}_j \geq 0, c_j \in [\underline{c}_j, \bar{c}_j], \underline{b}_j \geq 0, b_j \in [\underline{b}_j, \bar{b}_j], \underline{a}_{ij} \geq 0, a_{ij} \in [\underline{a}_{ij}, \bar{a}_{ij}], i = 1,2,\dots,m, j = 1,2,\dots,n$$

Therefore, the following model is a linear programming problem with grey parameters, where X is a grey vector:

$$\begin{aligned} & max\ C(\otimes)X \\ & s.t. \\ & A(\otimes)X = b(\otimes) \\ & X \geq 0 \end{aligned} \tag{6}$$

2.2.2 *Grey data envelopment analysis*

Assume there is *n* decision-making units, each consuming *m* inputs and generating *s* outputs. The relative efficiency of each unit is obtained by solving the following model:

$$\begin{aligned} & max\ \theta_k = \sum_{r=1}^{s} u_r [\underline{y}_{rk}, \bar{y}_{rk}] \\ & s.t. \\ & \sum_{r=1}^{m} v_i [\underline{x}_{ij}, \bar{x}_{ij}] - \sum_{r=1}^{s} u_r [\underline{y}_{rj}, \bar{y}_{rj}] \geq 0, j = 1,2,\dots,n, j \neq k \\ & \textstyle\sum_{r=1}^{m} v_i [\underline{x}_{ik}, \bar{x}_{ik}] = 1 \\ & u_r \geq 0, r = 1,2,\dots,s, v_i \geq 0, i = 1,2,\dots,m \end{aligned} \tag{7}$$

$x_{ij} = [\underline{x}_{ij}, \bar{x}_{ij}]$ *and* $y_{rj} = [\underline{y}_{rj}, \bar{y}_{rj}]$ are grey inputs and outputs, respectively. X_{ij} and y_{ij} are inputs and outputs used by the j-th unit and $u_r, r = 1,2,\dots,s, and\ \ v_i, i = 1,2,\dots,m$ are output and input weights, respectively.

An efficiency score spectrum can be obtained by solving the DEA model for best-case scenarios and worst-case scenarios in grey distances. This spectrum reflects the uncertainty in input and output data. This approach guarantees the efficiency analysis remains realistic and instructive, even when dealing with incorrect or incomplete data.

Maximum efficiency for DMU_k is obtained when it generates the most outputs by using the least inputs, while other DMUs generate the least outputs by using the most inputs. The mathematical model for the maximum efficiency of DMU_k is as follows (best-case scenario):

$$\max \bar{\theta}_k = \sum_{r=1}^{s} u_r \bar{y}_{rk}$$
$$s.t.$$
$$\sum_{r=1}^{m} v_i \bar{x}_{ij} - \sum_{r=1}^{s} u_r \underline{y}_{rj} \geq 0, j = 1,2,\ldots,n, j \neq k \tag{8}$$
$$\sum_{r=1}^{m} v_i \underline{x}_{ik} = 1$$
$$u_r \geq 0, r = 1,2,\ldots,s, v_i \geq 0, i = 1,2,\ldots,m$$

Minimum efficiency for DMU$_k$ is obtained when it generates the most outputs by using the least inputs, while other DMUs generate the least outputs by using the most inputs. The mathematical model for the maximum efficiency of DMU$_k$ is as follows (worst-case scenario):

$$\max \underline{\theta}_k = \sum_{r=1}^{s} u_r \underline{y}_{rk}$$
$$s.t.$$
$$\sum_{r=1}^{m} v_i \underline{x}_{ij} - \sum_{r=1}^{s} u_r \bar{y}_{rj} \geq 0, j = 1,2,\ldots,n, j \neq k \tag{9}$$
$$\sum_{r=1}^{m} v_i \bar{x}_{ik} = 1$$
$$u_r \geq 0, r = 1,2,\ldots,s, v_i \geq 0, i = 1,2,\ldots,m$$

The efficiency of DMU$_k$ is denoted as $\theta_k = [\underline{\theta}_k, \bar{\theta}_k]$. There are three classes of DMU efficiency:

i. $E^* = \{DMU_j | \underline{\theta}_j \geq 1, j = 1,2,\ldots,n\}$, in this case, DMU is efficient.

ii. $E = \{DMU_j | \underline{\theta}_j \leq 1, \bar{\theta}_j \geq 1, j = 1,2,\ldots,n\}$, in this case, DMU is relatively efficient.

iii. $F = \{DMU_j | \bar{\theta}_j \leq 1, j = 1,2,\ldots,n\}$, in this case, DMU is inefficient.

There are different ranking methods for grey data. One of these methods is known as the Average Method, which is defined as:

$$[\underline{\theta}_j, \bar{\theta}_j] \leq [\underline{\theta}_l, \bar{\theta}_l] \Leftrightarrow \frac{\underline{\theta}_j + \bar{\theta}_j}{2} \leq \frac{\underline{\theta}_l + \bar{\theta}_l}{2}$$

Where $\theta_k^{disagree}$ is equal to the efficiency of each unit, considering the average of each input and each output in representing the grey number. That is:

$$x_k^{ave} = \frac{\underline{x}_{ik} + \underline{x}_{ik}}{2}, y_k^{ave} = \frac{\bar{y}_{rk} + \underline{y}_{rk}}{2}$$

Each of $\theta_k^{disagree}$ values are obtained using these inputs and outputs.

2.3 *Nash bargaining method*

We denote the set of all individuals as N = {1, 2, …, n}. A bargaining problem consists of a set of N players, each possessing a real-valued utility function; a utility space, in which each point is a vector $u \in R^N$, and its members represent the utilities of each player according to specific agreements among them. A feasible set $S \subset R^N$ contains the vectors corresponding to all possible agreements between the players. This feasible set is assumed to be closed, bounded, convex, and comprehensive. There is a disagreement (failure) point $d \in S$, where the components of d represent the utility of each player in case of a disagreement. If a player chooses not to bargain with another player, the disagreement point reflects the possible payoff pairs. It is assumed that there is at least one point in S that dominates d, meaning $\exists u \in S: u > d$. This indicates that there exists a feasible agreement in which all players gain higher utility than if they fail to reach an agreement. Nash argued that a rational solution must meet the following four criteria:

i. Pareto efficiency
ii. Invariance to Affine Transformation
iii. Independence of irrelevant options
iv. Symmetry

According to Nash, the solution to the Nash Bargaining problem is obtained by solving the following optimization model:

$$\begin{aligned} & max \prod_{r \in N} (u_r - d_r) \\ & s.t. \\ & u \in S \\ & u \geq d \end{aligned} \tag{10}$$

The maximization objective function is the product of the utility gain of all players (according to the difference point). The solution to the Nash Bargaining problem can be specified through various sets of properties that are uniquely identified.

Integrating the Nash Bargaining method in DEA includes formulating the efficiency evaluation as a bargaining problem between several DMUs. The goal is to find a solution that maximizes the bargaining of each DMU by considering the inaccuracies in the data. In this method, the difference point can be the efficiency scores received by DMUs under certain basic conditions or existing conditions.

2.4 *Formulating the proposed Nash bargaining problem*

We combine the DEA scores with the Nash Bargaining problem. This method strives to find an equilibrium point among the minimum and maximum efficiency values of each DMU. In our approach, we consider the difference point as the efficiency of players without bargaining. For each DMU, the equilibrium point has been determined based on the bargaining method as either a weighted average or relative equilibrium point among the minimum and maximum efficiency.

The Nash Bargaining solution aims to maximize the product of usefulness against the difference points. Therefore, the problem can be stated as follows:

$$\begin{aligned} & Max \prod_{k=1}^{n} \left(E_k - \theta_k^{disagree}\right) \\ & s.t. \\ & \underline{\theta}_k \leq E_k \leq \bar{\theta}_k \end{aligned} \tag{11}$$

Where E_k is the negotiated efficiency and it is within the $[\underline{\theta}_k, \bar{\theta}_k]$ interval.

If necessary, grey data can be converted to usable data for DEA models using statistical

techniques and Machine Learning.
When there are lost or uncertain values in the problem, their values depend on several factors: the model's complexity, ease of implementation, prediction accuracy, and value of existing data.
Averaging methods, linear regression, decision tree, and neural networks are some of the suitable approaches to address this issue.

3. Practical example

Let us consider a service company as an example. This company has 20 decision-making units, three grey inputs, and two grey outputs (customer satisfaction being one of these grey outputs).
Inputs include:
Operation Costs ($K): Denotes the total operating costs, presented as a range to account for uncertainty or variations in the costs (in thousands of dollars).
Labor Hours (K): Denotes the total number of work hours spent, presented as a range to take into account the possible fluctuations (in thousands of hours).
Service Facilities (Unit): Denotes the number of service facilities or operational units involved, presented as a range to reflect potential alterations in service availability.
Outputs include:
Service Deliveries (Unit): Measures the number of completed service deliveries, provided as an interval to account for discrepancies or estimations.
Customer Satisfaction (Score): Represents the level of customer satisfaction, reported as a score range to reflect variations in customer feedback or survey outcomes.

Table 1. Specifications of grey inputs and grey outputs.

DMU	I1(Operating Costs)	I2(Labor Hours)	I3(Service Facilities)	O1(Service Deliveries)	O2(Customer Satisfaction)
DMU1	(87.49, 119.01)	(52.44, 79.90)	(47.26, 62.47)	(216.15, 237.92)	(97.36, 122.65)
DMU2	(94.64, 111.97)	(50.69, 88.19)	(36.62, 51.27)	(206.36, 222.20)	(102.67, 120.72)
DMU3	(83.12, 103.12)	(55.18, 83.25)	(36.22, 56.50)	(204.56, 228.54)	(91.81, 126.71)
DMU4	(81.16, 117.32)	(56.23, 80.40)	(44.59, 62.75)	(216.36, 237.21)	(96.42, 113.73)
DMU5	(92.02, 114.16)	(60.93, 73.70)	(47.74, 59.44)	(200.14, 230.21)	(90.82, 121.82)
DMU6	(81.53, 107.44)	(58.19, 80.36)	(42.60, 55.23)	(202.92, 234.83)	(97.74, 127.22)
DMU7	(97.69, 114.24)	(61.08, 73.44)	(37.64, 61.08)	(201.49, 230.63)	(95.83, 112.99)
DMU8	(88.31, 110.77)	(52.05, 89.61)	(46.80, 57.88)	(203.36, 238.70)	(91.70, 120.82)
DMU9	(85.61, 109.96)	(59.23, 71.24)	(47.08, 63.61)	(208.92, 232.99)	(102.44, 113.95)
DMU10	(85.71, 101.23)	(67.95, 80.74)	(37.22, 61.73)	(211.62, 239.55)	(93.21, 127.85)
DMU11	(89.45, 116.36)	(60.55, 72.21)	(42.11, 59.48)	(202.73, 239.99)	(90.96, 116.73)
DMU12	(98.83, 105.27)	(61.63, 77.52)	(40.21, 67.65)	(200.17, 223.53)	(96.52, 114.24)

DMU13	(92.82, 110.64)	(66.04, 81.93)	(40.58, 60.32)	(210.63, 223.75)	(98.38, 127.46)
DMU14	(92.34, 109.25)	(52.53, 74.07)	(42.92, 63.32)	(217.28, 228.14)	(97.95, 122.33)
DMU15	(96.35, 108.68)	(66.34, 88.42)	(41.07, 60.83)	(215.78, 237.47)	(91.64, 123.69)
DMU16	(90.63, 110.28)	(68.54, 83.98)	(47.47, 65.34)	(201.08, 230.16)	(103.82, 118.92)
DMU17	(91.31, 105.36)	(52.93, 87.61)	(41.15, 64.18)	(212.78, 224.35)	(92.89, 118.98)
DMU18	(89.97, 102.59)	(67.79, 82.64)	(37.34, 51.73)	(217.48, 237.21)	(100.04, 112.42)
DMU19	(92.28, 116.96)	(64.13, 84.06)	(38.92, 59.12)	(200.21, 221.85)	(98.57, 121.38)
DMU20	(82.47, 100.49)	(63.65, 86.48)	(47.18, 62.04)	(210.63, 234.86)	(92.92, 122.74)

The aforementioned models are utilized for this dataset. The obtained results are presented in Table 2.

Table 2. Results obtained from the models.

	$\underline{\theta}_k$	$\bar{\theta}_k$	$\theta_k^{disagree}$	E_k	Ranking	Average Calculation	Ranking (Average)
DMU1	0.63593	1.62652	0.98816	0.63593	16	1.13123	9
DMU2	0.65328	1.84225	1.00000	0.65328	14	1.24777	3
DMU3	0.67910	1.85714	1.00000	0.67910	11	1.26812	2
DMU4	0.64929	1.49578	1.00000	0.62929	15	1.07254	18
DMU5	0.62571	1.48675	0.93407	0.48675	5	1.05623	19
DMU6	0.65500	1.68052	1.00000	0.65500	13	1.31776	1
DMU7	0.62743	1.54099	0.97136	0.62743	19	1.08421	14
DMU8	0.63462	1.61426	0.96132	0.63462	18	1.12444	10
DMU9	0.67203	1.48781	1.00000	0.67203	12	1.07992	15
DMU10	0.71525	1.71532	1.00000	1.71532	1	1.21529	4
DMU11	0.63499	1.51827	0.98956	0.63499	17	1.07663	17
DMU12	0.65101	1.50323	0.91430	1.50323	4	1.07712	16
DMU13	0.65240	1.61024	0.95115	1.61024	2	1.13132	8
DMU14	0.68789	1.69569	1.00000	0.68789	10	1.19179	5
DMU15	0.67931	1.54935	0.94180	1.54935	3	1.11433	12
DMU16	0.62958	1.36908	0.93152	0.62958	9	0.99933	20
DMU17	0.69098	1.67802	0.95984	0.69098	8	1.18450	6
DMU18	0.73797	1.57833	1.00000	0.73797	6	1.15815	7
DMU19	0.59656	1.58955	0.92668	0.59656	20	1.09306	13
DMU20	0.71714	1.54096	1.00000	0.71714	7	1.12905	11

In this table, the focus is on minimum (pessimistic) and maximum (optimistic) efficiency, and the equilibrium efficiency is calculated using the average. It is observed that based on the equilibrium average, units 2, 3, 4, 6, 9, 10, 14, 18, and 20 are relatively efficient. The Negotiated Efficiency column includes efficiency scores for each DMU after applying the Nash Bargaining method, considering grey data in inputs and outputs. These scores are fair and efficient, reflecting the uncertainty in input and output data. By reviewing the ranking columns, we notice the differences between the methods, indicating the advantages of our proposed method. This example illustrates how grey data can be managed in a service company using DEA and bargaining methods. This approach enables you to accurately assess the efficiency and performance of various units despite data uncertainty, leading to more informed decision-making.

The Nash bargaining problem aims to maximize the profit for each party involved. For DEA with grey data, efficiency scores are calculated for each DMU based on the ratio of weighted outputs to weighted inputs. The grey intervals indicate the uncertainty in measurements, and the Nash bargaining problem guarantees that the chosen efficiency score balances this uncertainty to maximize profit or efficiency for both inputs and outputs. This approach symmetrically considers the contribution (output) and resource usage (input) of each DMU during the negotiation process, without favoring one DMU over another based on whether it is a provider of inputs or a consumer of outputs. Dependent transformations include scaling and achieving feasible agreements (inputs and outputs). The Nash bargaining solution is consistent with changes in scale or shifts in the input and output distances, without altering the relative efficiency scores obtained from the negotiation process. Modifications in unrelated options (such as the addition or removal of DMUs without directly impacting the negotiation) do not affect the negotiated efficiency score. The focus is on negotiating between pairs of DMUs to determine efficient allocation based on the given grey distance data, therefore, the specifications of the Nash solution hold.

5. Conclusion

It is obvious that the conventional DEA method is not valid when there is a lack of certain data. The proposed method in this article involves formulating a bargaining problem with the Grey Data Envelopment Analysis model. The main contribution of this article is indeed the application of a bargaining method. Incorporating the bargaining method into the DEA model with grey data can improve the process of evaluating the efficiency of decision-making units. More accurate and comprehensive results can be obtained by using statistical techniques and machine learning to convert grey data into usable data and employing optimization algorithms for solving the bargaining model. This approach guarantees robust analysis by integrating uncertainty directly into the DEA framework, combined with the Nash bargaining method to guarantee a fair and optimal allocation of efficiency scores. The presented example shows how to manage grey data in a service company using DEA and bargaining methods. Despite the uncertainty of the data, this approach allows us to accurately evaluate the efficiency and performance of various units and adopt better decisions. Additionally, Nash's proposed solution is not only Pareto optimized (i.e. efficient), but also possesses other specifications of Nash's solution.

References

[1] E. Abdali, and R. Fallahnejad, A Bargaining Game Model For Measuring Efficiency Of Two-Stage Network DEA With Non-Discretionary Inputs, International journal of computer mathematics: computer systems theory, **5 (1)** (2020) 48–59, doi:10.1080/23799927.2020.1723708.

[2] W. W. Cooper, L. M. Seiford, K. Tone, and J. Zhu, Some Models and Measures for Evaluating Performances with DEA: Past Accomplishments and Future Prospects, Journal of Productivity Analysis, **28** (2007) 151–163, doi: https://doi.org/10.1007/s11123-007-0056-4%0A%0A.

[3] H. Fukuyama, and W. L. Weber, A Directional Slacks-Based Measure of Technical Inefficiency, Socio-Economic Planning Sciences, **43 (4)** (2009) 274–287, doi:10.1016/j.seps.2008.12.001.

[4] C. C. Yang, and B. S. Chen, Supplier Selection Using Combined Analytical Hierarchy Process and Grey Relational Analysis, Journal of Manufacturing Technology Management, **17 (7)** (2006) 926–941, doi: https://doi.org/10.1108/17410380610688241.

[5] Y. S. Chen, and B. Y. Chen, Applying DEA, MPI, and Grey Model to Explore the Operation Performance of the Taiwanese Wafer Fabrication Industry, Technological Forecasting and Social Change, **78 (3)** (2011) 536–546, doi: https://doi.org/10.1016/j.techfore.2010.04.013.

[6] H. Küçükönder, P. Ç. Demirarslan, A. Burgut, and M. Boğa, A Hybrid Approach of Data Envelopment Analysis Based Grey Relational Analysis: A Study on Egg Yield, Pakistan Journal of Zoology, **51 (3)** (2019) 903–912, doi:10.17582/journal.pjz/2019.51.3.903.912.

[7] C. N. Wang, T. T. Dang, N. A. T. Nguyen, and T. T. H. Le, Supporting Better Decision-Making: A Combined

Grey Model and Data Envelopment Analysis for Efficiency Evaluation in E-Commerce Marketplaces, Sustainability (Switzerland), **12 (24)** (2020) 1–24, doi:10.3390/su122410385.

[8] M. Salami, F. M. Sobhani, and M. S. Ghazizadeh, Multistage Data Envelopment Analysis Model for Malmquist Productivity Index Using Grey ' s System Theory to Evaluate Performance of Electric Power Supply Chain in Iran, International Journal of Economics and Management Engineering, **13 (9)** (2019) 1181–1192.

[9] C. N. Wang, F. C. Yang, N. T. M. Vo, C. T. Duong, and V. T. T. Nguyen, Enhancing Operational Efficiency in Industrial Systems: A DEA-Grey Integration. IEEE Access, **12** (2024) 58532–58550. doi:10.1109/ACCESS.2024.3374335.