Original Research .

Completion of the paper ranking Efficient DMUs based on single virtual inefficient DMU in DEA

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Received: 05 Aug 2024 / Accepted: 01 Sep 2024 / Published online: 07 Sep 2024

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Abstract

This paper builds upon the foundation laid by Shetty's research, aiming to enhance our understanding of decision-making unit (DMU) efficiency. In doing so, we introduce a novel approach that offers a more comprehensive method for ranking DMUs. Unlike traditional methods that assess DMUs individually, our proposed methodology centers on the creation of a virtual DMU. This virtual entity serves as a composite representation, synthesizing the inputs and outputs of all DMUs within the study. By aggregating this information, we establish a benchmark against which the efficiency of individual DMUs can be assessed. This approach not only simplifies the evaluation process but also provides a more holistic perspective, enabling researchers to discern patterns and trends across the entire dataset. As is widely acknowledged, the efficiency frontier is delineated by the efficient decision-making units (DMUs). The method proposed in the aforementioned paper proved to be efficacious particularly in scenarios where the number of efficient DMUs was limited, enabling the model to accurately rank them. However, challenges may arise as the population of efficient DMUs increases. This is due to the necessity of excluding these efficient DMUs from the efficiency frontier to obtain their efficiency ranks. Consequently, their ranking criterion would be determined by the efficiency scores of virtual DMUs generated by the revised efficiency frontier. In instances where the number of efficient DMUs expands, the process of excluding them from the efficiency frontier becomes more intricate. Furthermore, the reliance on virtual DMUs for ranking purposes introduces a layer of complexity, as the efficiency scores of these virtual entities are contingent upon the composition of the new efficiency frontier. Therefore, as the number of efficient DMUs escalates, the effectiveness of the proposed methodology in accurately ranking them may diminish. This underscores the importance of ongoing refinement and adaptation of methodologies to accommodate evolving datasets and analytical requirements in the assessment of DMU efficiency. If, following the removal of the efficient DMU, the efficiency score of a remaining DMU within the possibility production set is higher, it suggests that the respective DMU exhibits greater efficiency. Efficient Decision Making Units (DMUs) construct the defining hyperplane; therefore, the exclusion of these contributing efficient DMUs in an attempt to derive their ranking, amidst an increase in their numbers, will impede the acquisition of efficiency scores for virtual DMUs. Hence, achieving a comprehensive ranking of all DMUs is unattainable unless those positioned precisely on the defining hyperplane are included. In this complementary method, we delineate an **anti-ideal** virtual DMU encompassing all DMUs situated on the corresponding defining hyperplane, which may be oriented in various directions. Then, we use this method for ranking efficient DMUs. In this method, DMUs located on different intersecting defining hyperplanes may hold multiple ranks, from which the highest rank is deemed the most relevant. As the proposed method aligns with the aforementioned study, it incorporates all the advantages, including simplicity and stability, and notably eliminates the identified flaw.

Keywords- Data Envelopment Analysis (DEA); Decision Making Units (DMU); Ranking; Virtual DMU; Anti Ideal Point (AIP)

INTRODUCTION

In recent years, a notably appropriate approach has emerged in the intellectual, cultural, and social domains for assessing performance and productivity. Among the evaluation methods, Data Envelopment Analysis (DEA) is extensively employed to assess the relative performance of a group of production processes known as decision-making units (DMUs). This non-parametric technique evaluates DMUs by employing various models that generate multiple outputs utilizing multiple inputs. DEA was initially introduced by Charnes et al. [1] as a method for evaluating the relative efficiency of DMUs with multiple inputs and outputs. Subsequently, Banker et al. [2] advanced the fundamental DEA models under the assumption of variable returns to scale. A DMU is deemed efficient if its performance, relative to other DMUs, cannot be enhanced. Basic DEA models often identify more than one DMU as efficient upon evaluating the comparative efficiency of DMUs. Hence, when DEA models are employed to compute the efficiency of DMUs, several of them may attain an efficiency score of 1. To select a superior candidate among these efficient DEA candidates, various methods have been proposed (see [3]).

In the DEA literature, several ranking methods have been proposed. Alder [4] has reviewed six groups of methods to comprehensively rank both efficient and inefficient DMUs. The first group involves evaluating cross efficiency (CE), where DMUs are assessed both internally and against their peers. The second group employs the super efficiency (SE) method, wherein the evaluated unit is excluded from the reference set. The third group is based on benchmarking, ranking a DMU highly if it serves as a benchmark for many other inefficient DMUs. The fourth group utilizes multivariate statistical techniques, such as discriminant analysis and canonical correlation analysis. The fifth group ranks DMUs using the proportional measure of inefficiency, while the sixth group ranks them based on multiple criteria decision methodologies using the DEA approach. However, it should be noted that while each of these techniques may be valuable in specific contexts, no single ranking methodology serves as a comprehensive solution on its own. Andersen introduced a super-efficiency DEA model, also referenced by Banker, designed to rank efficient DMUs. The input-oriented (output-oriented) super-efficiency DEA model excludes the DMU under evaluation from the reference set, allowing efficient DMUs to have efficiency scores greater (smaller) than or equal to one. The original super-efficiency DEA model is presented under the assumption of constant returns to scale (CRS) and is applicable if all inputs and outputs of DMUs are positive. Nevertheless, the issue of infeasibility may arise in variable returns to scale (VRS) super-efficiency DEA models.

Several modified variable returns to scale (VRS) radial super-efficiency DEA models, such as those proposed by Chen, have been introduced to tackle the infeasibility problem. Among these models, the VRS Nerlove-Luenberger super-efficiency DEA model, Ray, utilizes the directional distance function (DDF) and is frequently feasible when operating with non-negative datasets. However, this model encounters limitations in two exceptions. To overcome these limitations, two DDF-based variable returns to scale (VRS) super-efficiency DEA models are selected carefully. The model proposed by Chen may become infeasible if zero data exist in outputs. Furthermore, the majority of these methods are incapable of ranking non-extreme efficient DMUs (refer to [5] for details).Since research on the ranking of non-extreme efficient units is limited, incomplete, and fraught with difficulties, Gholam Abri [6] proposed a method for ranking non-extreme efficient DMUs. The proposed method in this research also has the ability to rank non-extreme efficient units.

After the generation of older studies about ranking, in this section, we mention some new studies in this regard . Liu et al. [7] utilized the cross-evaluation method to rank unfavorable outputs. Wu et al. [8] proposed a model to enhance cross-evaluation results when the solution was not Pareto optimal. Chen et al [5] introduced a method for ranking efficient units in the presence of negative data. To encompass more diverse and comprehensive methods, research by Zulfaqari et al. can also be considered [9]. Additionally, Shahmirzadi [10] presented a method for ranking efficient units by altering the reference set. Zhang et al. [11] and Sojoodi et al. [12] introduced models that address the infeasibility issues of envelopment models and the unboundedness of multiplier models. Ghaem Nasab et al. [13] Utilized game theory and Shapley value methods as equitable approaches to rank units in real-world scenarios. Yu and Rakshit [14] articulated input and output goals using a bargaining approach and data envelopment analysis to rank applied samples, such as global airlines.

One approach to rank efficient DMUs is by introducing a single virtual DMU, as presented by Shetty et al. [15]. This method utilizes a virtual DMU to rank efficient DMUs, where the input and output levels of the virtual DMU are set as the averages of inputs and outputs of all DMUs. In comparison to other methods, this approach is characterized by its simplicity, robustness, and effectiveness in certain scenarios. However, as the number of efficient DMUs increases, the model encounters flaws and struggles to rank them accurately. This paper endeavors to address this issue by proposing a method to comprehensively rank all efficient DMUs in the aforementioned scenario. This paper is organized as follows: Section 2 briefly introduces the background of DEA. Section 3 presents our proposed method. A numerical example is provided in Section 4, and Section 5 presents our concluding remarks.

BACKGROUND

DEA is a technique extensively employed in the supply chain management literature. This non-parametric multi-factor approach enhances our capability to capture the multidimensionality of performance, as discussed previously. More formally, DEA is a mathematical programming technique for measuring the relative efficiency of decision-making units, wherein each DMU utilizes a set of inputs to produce outputs. Suppose observed input and output vectors of DMU_j are $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})$ respectively, and $X_j \geq 0$, $X_j \neq 0$, $Y_j \geq 0$, $Y_j \neq 0$.
The production possibility set TC is defined as:

Tc = $\{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j \,, Y \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \lambda_j \geq 0, j = 1, \dots, n\}$.

By the definition stated, the CCR model is as follows:

$$
\begin{aligned}
& Min\ \theta \\
& \text{s.t.} \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ijo} && i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j y_{ij} \geq y_{ro} && r = 1, \dots, s \\
& \lambda_j \geq 0 && j = 1, \dots, n \qquad (1)
\end{aligned}
$$

Moreover, the production possibility set T_v is defined as:

$$T_v = \{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j \,, Y \leq \sum_{j=1}^{n} \lambda_j Y_j \,, \sum_{j=1}^{n} \lambda_j = 1 \,, \lambda_j \geq 0, j = 1, \dots, n\}.$$

By the above definition, the BCC model is as follows:

$$
\begin{aligned}
& Min\ \theta \\
& \text{s.t.} \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ijo} && i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j y_{ij} \geq y_{ro} && r = 1, \dots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 && j = 1, \dots, n \qquad (2)
\end{aligned}
$$

The model below is employed to define Pareto-efficient DMU_o ($o \in \{1, \dots, n\}$).

$$Max\ Z = \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

s.t.

$$\begin{array}{ll}
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta^* x_{io} & i = 1, \dots, m \\
\sum_{j=1}^{n} \lambda_j y_{ij} - s_r^+ = y_{ro} & r = 1, \dots, s \\
s_i^- \geq 0 & i = 1, \dots, m \\
s_r^+ \geq 0 & r = 1, \dots, s \\
\lambda_j \geq 0 & j = 1, \dots, n \qquad (3)
\end{array}$$

Model could be attained by incorporating $\sum_{j=1}^{n} \lambda_j = 1$ as a constraint.

$$Max\ Z = \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

s.t.

$$\begin{array}{ll}
\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta^* x_{io} & i = 1, \dots, m \\
\sum_{j=1}^{n} \lambda_j y_{ij} - s_r^+ = y_{ro} & r = 1, \dots, s \\
\sum_{j=1}^{n} \lambda_j = 1 & \\
s_i^- \geq 0 & i = 1, \dots, m \\
s_r^+ \geq 0 & r = 1, \dots, s \\
\lambda_j \geq 0 & j = 1, \dots, n \qquad (4)
\end{array}$$

In the above models, θ^* serves as the objective function for models 1 and 2.

$DMU_o(o \in \{1, \dots, n\})$ is *Pareto efficient* if and only if:
i) $\theta^* = 1$ (the optimal value of model 1 or 2).
ii) $Z^* = 0$ (the optimal value of model 1 of 2).

For DMU_o its reference set, E_0 is defined by:

$E_0 = \{ j \mid \lambda_j^* > 0\ in\ some\ optimal\ of\ (1)\ or\ (2)\} \subseteq \{1, \dots, n\}$

When evaluating the relative efficiency of each DMU using DEA models, efficient scores ranging between zero and one are obtained. Consequently, in DEA models, it is common for more than one unit to be efficient, with their efficiency scores being 1. Furthermore, it is worth noting that the number of efficient units in variable return to scale (VRS) models is not less than that in constant return to scale (CRS) models. Therefore, researchers have proposed various methods to discriminate among these efficient units. This concept is referred to as ranking efficient units in DEA. There are numerous methods available, each with its unique qualities and properties for ranking efficient units. The foremost model for ranking, introduced by Andersen and Petersen [16], is notable, yet it exhibits certain drawbacks such as instability and infeasibility. The AP model is as follows:

$$Min\ \theta$$

s.t.

$$\begin{array}{ll}
\sum_{j=1, j\neq o}^{n} \lambda_j x_{ij} \leq \theta x_{io} & i = 1, \dots, m \\
\sum_{j=1, j\neq o}^{n} \lambda_j y_{rj} \geq y_{ro} & r = 1, \dots, s \\
\lambda_j \geq 0 & j = 1, \dots, n \qquad (5)
\end{array}$$

Andersen and Petersen [16] have ranked extreme efficient units by omitting them from possibility production set (PPS). By introducing a new constraint, $\sum_{j=1,j\neq o}^{n} \lambda_j = 1$, the model could be regarded as variable returns to scale.

METHODOLOGY

I. The problem of the mentioned method

In this section, we provide commentary on the paper by Shetty et al. [15], where they propose an approach to rank efficient DMUs utilizing a virtual DMU. The input and output levels of this virtual DMU are derived as the averages of inputs and outputs of all DMUs. Based on our understanding, the efficiency frontier is delineated by efficient DMUs. When the number of contributing DMUs is limited, their proposed method adeptly ranks the DMUs without errors. However, challenges emerge when the number of DMUs increases. In the process of ranking the efficient DMUs, the model systematically removes them one by one from the reference set, and their ranking criteria become the efficiency score of the virtual DMU derived from the new efficiency frontier. A DMU's ranking improves as its score increases within the new possibility production set (after eliminating efficient DMUs). Now, as the number of efficient DMUs constructing the defining hyperplane increases, excluding them for the purpose of determining their ranking does not alter the score of the virtual DMU created. Therefore, all the efficient DMUs cannot be ranked unless only those DMUs precisely located on the defining hyperplane are considered. Therefore, in order to solve this problem, the present research was presented. Moreover, the proposed method in this research also has the ability to rank non-extreme efficient units. In the numerical illustration section, we depicted what we previously discussed. Nonetheless, we explain our method in the proposed method section.

II. Proposed Method

To address the issue outlined in the previous section, we introduce a new method. Initially, the Pareto-efficient DMUs are identified using model 1 and 3 (or 2 and 4). Next, utilizing the method proposed by Hamed et al. [17], we pinpoint the strong defining hyperplanes of the production possibility set. Subsequently, all the DMUs positioned on each defining hyperplane are recognized. Assuming A_t represents the index set of all DMUs positioned on the defining hyperplane H_t, the **anti-ideal** virtual DMU corresponding to A_t is defined as follows:

$$AIP = (x_{i\,AIP}, y_{r\,AIP}) = \left(Max\left(x_{ij}\right) \mid j \in A_t, Min\left(y_{rj}\right) \mid j \in A_t\right) \quad i=1,\dots,m, \quad r=1,\dots,s.$$

Actually, the virtual (x_{AV}, y_{AV}) is replaced by the **anti-ideal point** (AIP), defined as above. In this approach, rather than utilizing a virtual DMU with averaged input and output, we employ a virtual DMU with anti-ideal input and output. This adjustment is made because the virtual DMU is efficient with the average inputs and outputs of the set A_t (the convex composition of efficient DMUs remains efficient) rendering it challenging to obtain an index for ranking efficient DMUs. Therefore, for each strong defining hyperplane that constructs the production possibility set, we have corresponding anti-ideal DMUs. Eventually, we sequentially eliminate efficient DMUs belonging to set A_t (t=1,...,k), one by one. Subsequently, we calculate the efficiency score of the anti-ideal DMU consisting of this set, relative to the new efficiency frontier. A higher score indicates that the rank of the eliminated DMU was greater. As stated, the main goal of this research is to solve the problem of the article presented by Shetty et al. [15].

The method proposed in the aforementioned paper proved to be efficacious particularly in scenarios where the number of efficient DMUs was limited, enabling the model to accurately rank them. However, challenges may arise as the population of efficient DMUs increases. In this research, we have solved the problem by introducing anti-ideal points as follows. As we know, the average of inputs and outputs is considered a special case of convex composition. On the other hand, the convex combination of a number of efficient units is itself an efficient unit, so the new unit is located on the previous strong hyperplane and does not provide a criterion for ranking those units. Therefore, to solve this problem, anti- ideal points was introduced. The details of the method are described in section 2-3. In addition, the advantage of the presented method is that it has the ability to rank non-extreme efficient units.

III. Numerical Example

To describe the problem of the previously presented method and show the performance of the proposed method, we present the following example. A system with ten decision making units including one input and one output is considered in figure 1. Data is given in table 1. Initially, employing the proposed method and utilizing the outcomes of models 2 and 4 , we designate DMUs A, B, C, D, E, and F as *Pareto-efficient* DMUs. In this example DMUs G, H, I, J are inefficient DMUs and their efficiency scores and ranks are $\boldsymbol{\theta_G^* = 0.4999}$, $\boldsymbol{\theta_H^* = 0.6250}$, $\boldsymbol{\theta_I^* = 0.7500}$, $\boldsymbol{\theta_J^* = 0.333}$, $\boldsymbol{rank_G = 9}$, $\boldsymbol{rank_H = 8}$, $\boldsymbol{rank_I = 7}$, and $\boldsymbol{rank_J = 10}$, respectively. For ranking efficient DMUs we follow the method. Utilizing the method proposed by Hamed et al. [17], we identify strong defining hyperplanes as follows:

TABLE 1
A SYSTEM WITH 10 DMUS

DMUs	A	B	C	D	E	F	G	H	I	J
Input	1	1.5	3	4	5.5	8	2	2	4	3
Output	2	4	7	8	9	10	1	3	7	2

$\mathrm{F} = \{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$

$F_A = \{A, B\}$; $\overline{F_A} = \{C, D, E, F\}$

$F_B = \{A, B, C\}$; $\overline{F_B} = \{D, E, F\}$

$F_C = \{B, C, D\}$; $\overline{F_C} = \{A, E, F\}$

$F_D = \{C, D, E\}$; $\overline{F_D} = \{A, B, F\}$

$F_E = \{D, E, F\}$; $\overline{F_E} = \{A, B, C\}$

$F_F = \{E, F\}$; $\overline{F_F} = \{A, B, C, D\}$

$$\overline{DMU}\left|\begin{matrix}\bar{x} = 3.4 \\ \bar{y} = 5.3\end{matrix}\right.$$

Strong hyperplanes are:

$H_1: D_1 = \{A, B\}$; $\begin{vmatrix} x-1 & y-2 \\ 1.5-1 & 4-2 \end{vmatrix} = 0$

That yields to $4x - y = 2$

$H_2: D_2 = \{B, C\}$; $\begin{vmatrix} x-1.5 & y-4 \\ 3-1.5 & 7-4 \end{vmatrix} = 0$

That yields to $6x - 3y = -3$

$H_3: D_3 = \{C, D\}$; $\begin{vmatrix} x-3 & y-7 \\ 4-3 & 8-7 \end{vmatrix} = 0$

That yields to $x - y = -4$

$H_4: D_4 = \{D, E\}$; $\begin{vmatrix} x-4 & y-8 \\ 5.5-4 & 9-8 \end{vmatrix} = 0$

That yields to $2x - 3y = -16$

$H_5: D_5 = \{E, F\}$; $\begin{vmatrix} x-8 & y-10 \\ 8-5.5 & 10-9 \end{vmatrix} = 0$

That yields to $2x - 5y = -34$

As illustrated, there are 5 strong defining hyperplanes, each with one corresponding **anti-ideal** virtual DMU. Therefore, we consider the **anti-ideal** DMU from each defining hyperplane.

$$DMU_{AB}^{Anti}\begin{vmatrix}1.5\\2\end{vmatrix} \;;\; DMU_{BC}^{Anti}\begin{vmatrix}3\\4\end{vmatrix} \;;\; DMU_{CD}^{Anti}\begin{vmatrix}4\\7\end{vmatrix} \;;\; DMU_{DE}^{Anti}\begin{vmatrix}5.5\\8\end{vmatrix} \;;\; DMU_{EF}^{Anti}\begin{vmatrix}8\\9\end{vmatrix}$$

Now, to obtain the ranking of A, B, C, D, E, and F, we follow the proposed method. As an example, and for illustration purposes, we describe one of them. As depicted in Figure 1, DMU E is positioned at the intersection of hyperplanes H4 and H5. Therefore, to determine its rank, we proceed as follows:

Since DMU E is situated on the H4 hyperplane, we designate the **anti-ideal** DMU of H4, denoted as DE, in the two-dimensional space.

$$DMU_{DE}^{Anti}\begin{vmatrix}5.5\\8\end{vmatrix}$$

Next, we remove DMU E from the PPS and calculate the distance of this **anti-ideal** DMU to the new frontier after elimination, resulting in $\theta_{1E}^{*} = 0.7272$. Since E is also situated on H5, we consider the anti-ideal DMU of H5, denoted as EF, in the two-dimensional space:

$$DMU_{EF}^{Anti}\begin{vmatrix}8\\9\end{vmatrix}$$

the obtained distance from new frontier is $\theta_{2E}^{*} = 0.7500$. So following the proposed method:

$$\theta_{E}^{*} = Max\{\theta_{1E}^{*}, \theta_{2E}^{*}\} = Max\{0.7272, 0.7500\} = 0.7500$$

Following as the above example, the efficiency score of all Pareto-efficient DMUs is:

$\theta_{A}^{*} = 1$

$\theta_{1B}^{*} = 0.6667$, $\theta_{2B}^{*} = 0.6000$, $\theta_{B}^{*} = Max\{\theta_{1B}^{*}, \theta_{2B}^{*}\} = 0.6667$

$\theta_{1C}^{*} = 0.5000$, $\theta_{2C}^{*} = 0.8437$, $\theta_{C}^{*} = Max\{\theta_{1C}^{*}, \theta_{2C}^{*}\} = 0.8437$

$\theta_{1D}^{*} = 0.7500$, $\theta_{2D}^{*} = 0.7727$, $\theta_{D}^{*} = Max\{\theta_{1D}^{*}, \theta_{2D}^{*}\} = 0.7727$

$\theta_{1E}^{*} = 0.7126$, $\theta_{2E}^{*} = 0.7275$, $\theta_{E}^{*} = Max\{\theta_{1E}^{*}, \theta_{2E}^{*}\} = 0.7275$

$\theta_{F}^{*} = 0.6875$

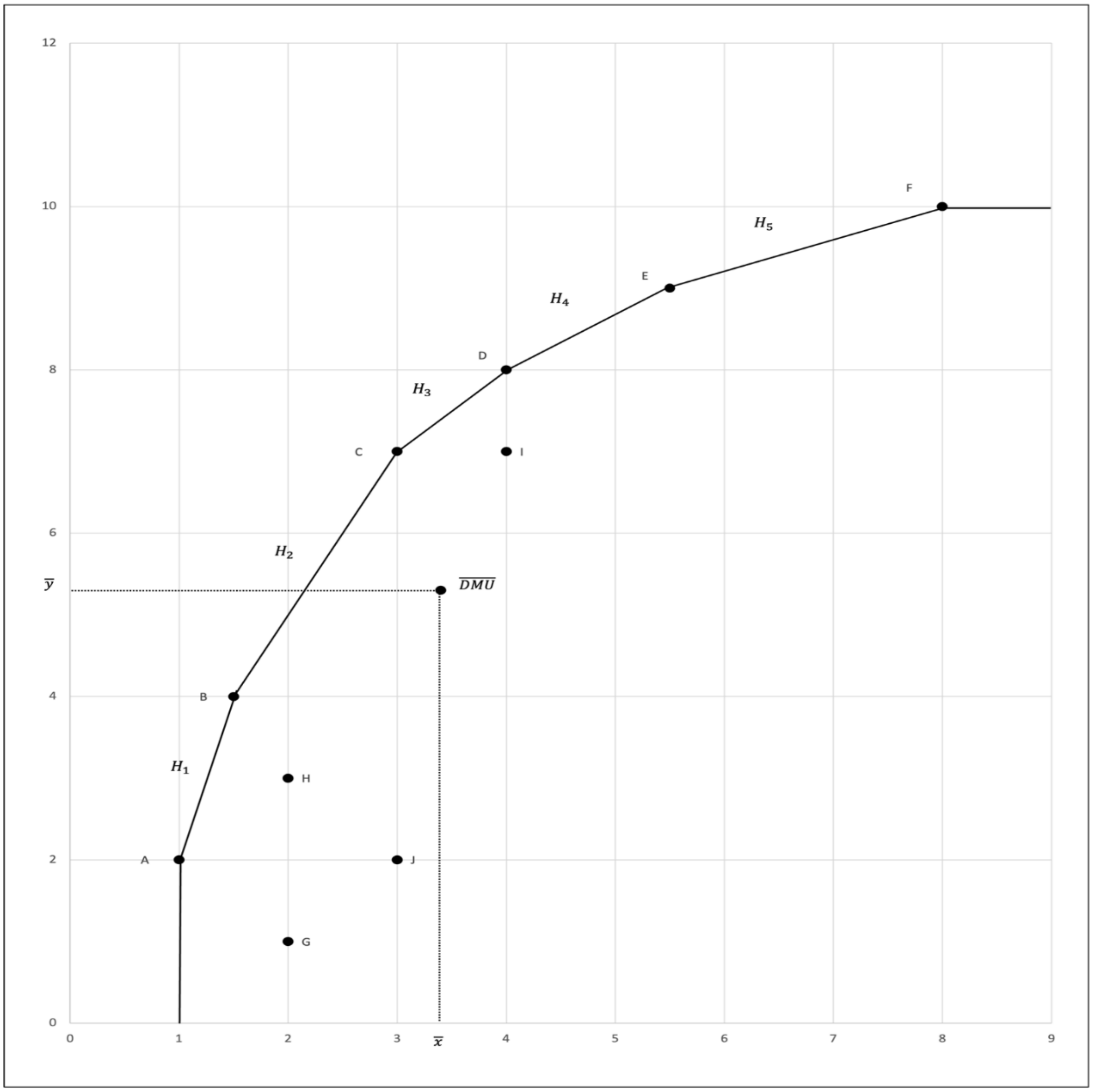

FIGURE 1
DMUS AND THE EFFICIENCY FRONTIER OF DMUS A, B, C, D, E, F, G, H, I,J AND THE VIRTUAL DMU

So, the rank for each of these DMUs are $rank_A = 1, rank_C = 2, rank_D = 3, rank_E = 4, rank_F = 5, rank_B = 6$.
As we can observe, following Shetty's proposed method, the DMU would be calculated as $DMU_{AV}\left|\begin{matrix}3.4\\5.3\end{matrix}\right.$.
In this scenario, if we opt to utilize input-oriented models, we would only rank DMUs C and B, which are positioned on the H_2 hyperplane. Since eliminating A, D, E, and F DMUs would not alter the ranking of DMU_{AV}, the ranking process would remain unchanged. Additionally, if we opt to use output-oriented models, we would only rank DMUs C and D, which are situated on the H_3 hyperplane. Even if we solely utilize one **anti-ideal** DMU for the PPS, the outcome would remain consistent, and ranking all the efficient DMUs would not be attainable.

CONCLUSION

This paper focuses on Shetty's method, which proposes a technique for ranking efficient DMUs based on a single virtual inefficient DMU in DEA. The input and output levels of this virtual DMU are the averages of inputs and outputs of all DMUs. The fundamental concept of the proposed approach is to compare efficient units by evaluating the efficiency of the virtual DMU with the linear combination of all efficient units, excluding one efficient unit at a time. However, as demonstrated, Shetty's method becomes ineffective as the number of efficient DMUs increases, failing to rank all DMUs. Therefore, this paper introduces a new method to address this issue. The primary idea of this method is to first obtain all anti-ideal DMUs corresponding to the hyperplanes of the PPS. In other words, for each defining hyperplane and efficient DMUs on them, one anti-ideal DMU will be created. Then, by sequentially removing efficient DMUs on the hyperplane and calculating the efficiency score of the anti-ideal DMU to the new efficiency frontier, a criterion for ranking efficient DMUs will be established. A higher efficiency score indicates a better condition of the efficient unit. In this scenario, it is possible that there are multiple efficiency scores for DMUs placed on different hyperplane intersections. In such cases, the maximum obtained score for each efficient DMU will be utilized. This new method also incorporates all the advantages of the previous method, namely simplicity and robustness.

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