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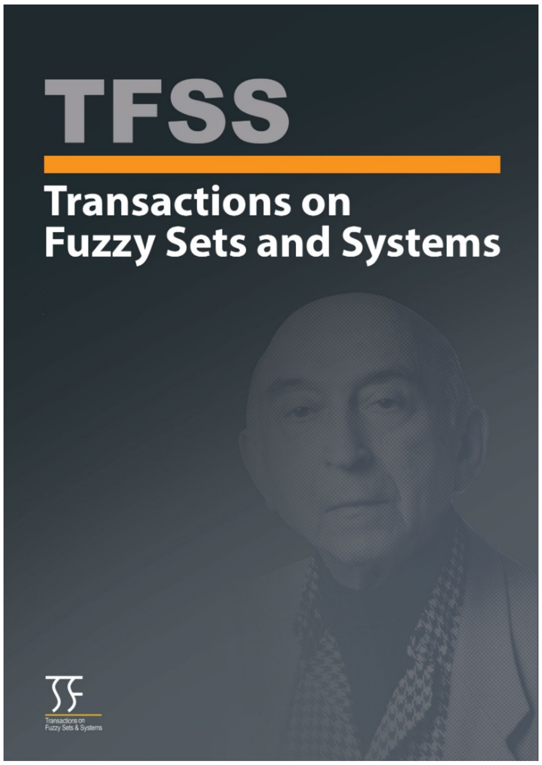

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Approximate Solution of Complex LR Fuzzy Linear Matrix Equation †

Xiaobin Guo*, **Xiangyang Fan**, **Hangru Lin**

Abstract. This paper aims at solving a class of linear LR complex fuzzy matrix equations $A\widetilde{X}B = \widetilde{C}$ using a matrix approach. By using the basic operation of LR fuzzy number matrix, the original complex fuzzy matrix equation is transformed into a clear matrix equation group. Two new and simplified models for calculating fuzzy solutions are designed in detail, and sufficient conditions for strong fuzzy solutions are analyzed. Finally, two examples are given to illustrate the feasibility and effectiveness of the proposed method. Now that the complex fuzzy numbers can describe uncertain factors more vivid and reasonable than the real fuzzy numbers sometimes and the wide application of matrix equations under uncertain conditions, our research work enriches the fuzzy linear systems theory.

AMS Subject Classification 2020: 15A30; 15A24
Keywords and Phrases: Complex fuzzy numbers, Matrix analysis, Fuzzy matrix equations, Approximate solutions.

1 Introduction

There are a large number of phenomena and events in the real world that we can not find a definite classification standard to judgment them. We call this kind of property of things as fuzziness and it is difficult to be accurately measured and described by classical mathematics. In 1965, the American cybernetics expert Professor Zadeh [1] proposed the concept of fuzzy sets wich made the birth of the new subject of fuzzy mathematics. In the past half century, the development of fuzzy mathematics has shown extraordinary vitality, its theoretical research involves fuzzy analysis, fuzzy algebra, fuzzy topology and other disciplines, and its application practice covers many fields such as artificial intelligence, cluster analysis, expert system, fault diagnosis, system evaluation, social sciences, big data processing and so on. As we all know, no matter in statistical analysis or in management science, only the linear system that theory is relatively mature and easy to calculate. If this uncertainty is expressed and calculated by fuzzy numbers, the description of the problem will be more reasonable and accurate, and analysis and decision of the problem will be convenient. Therefore, it is of practical significance to study uncertain linear systems based on fuzzy numbers. Fuzzy number is a special kind of fuzzy set, which is a generalization of one kind of natural real numbers.[2, 3, 4]. In 1998, Friedman[5] et al. proposed a general model for solving fuzzy linear systems, and studied the fuzzy linear system $A\widetilde{x} = \widetilde{b}$ by using the embedding method. After this, T. Allahviranloo et al. and B. Zheng et al. studied some other forms of fuzzy linear systems such as DFLS, GFLS, cffls, DFFLS, and GDFLS

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[6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In recent years, new studies on fuzzy numbers and various types of fuzzy linear systems have emerged in an endless stream. [16, 17, 18].

It is well known that matrix systems play a crucial role in the vast field of scientific computing. These systems often need to deal with situations that contain some or all of the parameter uncertainty, which is particularly common when modeling and predicting complex phenomena. In 2009 and 2018, respectively, Allahviranloo et al.[19] And AmirfakhrianIn et al.[20] used different methods to study and solve the fuzzy linear matrix equation of the form $A\widetilde{X}B = \widetilde{C}$. Different forms of fuzzy matrix equations have been systematically studied by Guo et al. in the last decade. [21] − [22]. For complex fuzzy linear systems, few researchers have proposed research methods in recent decades. The concept of fuzzy complex numbers was first introduced by J.J. Buckley[23] in 1989. In 2000, Qiu et al.[24] restudied fuzzy complex sequences and their convergence properties by studying $n \times n$ fuzzy complex linear systems.In 2009, Rahgooy et al.[25] applied fuzzy composite linear equations to circuit analysis problems. In 2014, Behera and Chakraverty used the embedding method to analyze and discuss fuzzy complex systems of linear equations, and improved the arithmetic operations of complex fuzzy numbers[16].In 2018, Guo et al.[26] introduced the complex fuzzy matrix equation $\widetilde{Z}C = \widetilde{W}$ and proposed a general model for complex LR fuzzy solutions. Recently, Wu et al.[25] established a method for calculating generalized fuzzy solutions of the semi-complex fuzzy matrix equation AX=B by meabs of the MPwg inverse of a crisp matrix.

In this paper, a matrix model for solving fuzzy matrix equation $A\widetilde{X}B = \widetilde{C}$ is proposed.Compare with the present work, this paper has three mathematical contribution, that is, (1) semi complex LR fuzzy matrix equation $AXB = C$ is firstly investigated by a matrix method. Through giving basic operations of complex LR fuzzy matrices; (2) two new and simple computing models that is a system of linear matrix equations are constructed; (3) Two sufficient condition of strong complex fuzzy solution condition are analyzed and provided. The content structure is as follows:

In Section 2, we review the concept of complex LR fuzzy numbers, based on which we introduce the concept of complex LR fuzzy linear matrix equation. In Section 3, we construct a detailed model of the LR complex fuzzy matrix equation, solve the equation by using the generalized inverse of the coefficient matrix, and at the same time explore the existence conditions of strong fuzzy solutions and their properties in depth. In order to verify the effectiveness and practicability of the method, some numerical examples are given. Section 4 is the summary and conclusion refinement of the whole paper, and Section 5 is the prospect of future research directions, and puts forward the topics and potential research areas for further exploration.

2 Preliminaries

The concepts of fuzzy numbers and fuzzy matrices have the following definitions. [2, 3, 4]

Definition 2.1. *A fuzzy number is a special kind of fuzzy set, denoted as a map $\widetilde{u} : R \to I = [0, 1]$, which has the following four conditions::*

(1) $\widetilde{u}$ is upper semi continuous,

(2) $\widetilde{u}$ is fuzzy convex, i.e., $\widetilde{u}(\lambda x + (1 - \lambda)y) \geq \min\{\widetilde{u}(x), \widetilde{u}(y)\}$ for all $x, y \in R, \lambda \in [0, 1]$,

(3) $\widetilde{u}$ is normal, i.e., there exists $x_0 \in R$ such that $\widetilde{u}(x_0) = 1$,

(4) supp$\widetilde{u} = \{x \in R \mid \widetilde{u}(x) > 0\}$ is the support of the $\widetilde{u}$, and its closure cl(supp$\widetilde{u}$) is compact.

Let E^1 be the set of all fuzzy numbers on R.

Definition 2.2. *We represent an arbitrary fuzzy number $(\underline{u}(r), \overline{u}(r))$, $0 \leq r \leq 1$, by a set of ordered pairs of functions satisfying the following conditions:*

(1) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,

(2) $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function,

(3) r in the interval $[0,1]$,$\underline{u}(r)$ is always less than or equal to overlineu(r).

A crisp number x can be represented as a fuzzy number by setting both $\underline{u}(r)$ and $\overline{u}(r)$ to x with $0 \leq r \leq 1$. By introducing a proper definition, the space of fuzzy numbers $\{(\underline{u}(r), \overline{u}(r))\}$ forms a convex cone E^1. This convex cone can be embedded in a Banach space in an isomorphic and metric consistent manner.

Definition 2.3. *A fuzzy number $\widetilde{M}$ is said to be a LR fuzzy number if*

$$\mu_{\widetilde{M}}(x) = \begin{cases} L(\frac{m-x}{m_\alpha}), & x \leq m, \quad \alpha > 0, \\ R(\frac{x-m}{m_\beta}), & x \geq m, \quad \beta > 0, \end{cases}$$

Here m is the principal mean of $\widetilde{M}$, m_α is the left extension, m_β is the right extension, and the function $L(\cdot)$, we will make it a left-shaped function that satisfies the following conditions:

(1) $L(x) = L(-x)$,

(2) $L(0) = 1$ and $L(1) = 0$,

(3) $L(x)$ is non increasing on $[0, \infty)$.

The definition of a right shape function $R(\cdot)$ is similar to that of $L(\cdot)$.

Clearly, when two LR fuzzy numbers $\widetilde{M} = (m, m_\alpha, m_\beta)_{LR}$ and $\widetilde{N} = (n, n_\alpha, n_\beta)_{LR}$ are equal, if and only if $m = n, m_\alpha = n_\alpha, m_\beta = n_\beta$. Similarly, if $\widetilde{M}$ is positive (negative), if and only if $m - m_\alpha > 0(m + m_\beta < 0)$.

Definition 2.4. *We have for any LR fuzzy numbers $\widetilde{M} = (m, m_\alpha, m_\beta)_{LR}$ and $\widetilde{N} = (n, n_\alpha, n_\beta)_{LR}$, the following.*

(1) Addition

$$\widetilde{M} \oplus \widetilde{N} = (m, m_\alpha, m_\beta)_{LR} \oplus (n, n_\alpha, n_\beta)_{LR} = (m + n, m_\alpha + n_\alpha, m_\beta + n_\beta)_{LR}.$$

(2) Subtraction

$$\widetilde{M} - \widetilde{N} = (m, m_\alpha, m_\beta)_{LR} - (n, n_\alpha, n_\beta)_{LR} = (m - n, m_\alpha + n_\beta, m_\beta + n_\alpha)_{LR}.$$

(3) Scalar multiplication

$$\lambda \otimes \widetilde{M} = \lambda \otimes (m, m_\alpha, m_\beta)_{LR} \cong \begin{cases} (\lambda m, \lambda m_\alpha, \lambda m_\beta)_{LR}, & \lambda \geq 0, \\ (\lambda m, -\lambda m_\beta, -\lambda m_\alpha)_{RL}, \lambda < 0. \end{cases}$$

Definition 2.5. *The LR complex fuzzy number consists of real part and imaginary part. An arbitrary complex LR fuzzy number could be represented as $\widetilde{x} = \widetilde{p} + i\widetilde{q}$,* where $\widetilde{p} = (p, p^l, p^r), \widetilde{q} = (q, q^l, q^r)$. In this case, $\widetilde{x}$ can be written as

$$\widetilde{x} = \widetilde{p} + i\widetilde{q} = (p, p^l, p^r) + i(q, q^l, q^r).$$

Definition 2.6. *Each element of A complex LR fuzzy matrix $\widetilde{A} = (\widetilde{a}_{ij})$ is a matrix constructed from complex fuzzy numbers. Let $\widetilde{A} = (\widetilde{a}_{ij}) = ([m, m^l, m^r] + i[n, n^l, n^r])_{ij}, i, j = 1, 2, \cdots, n$, the complex LR fuzzy matrix $\widetilde{A}$ can be represented by $\widetilde{A} = (M, M^l, M^r) + i(N, N^l, N^r)$.*

Definition 2.7. *Given two arbitrary complex LR fuzzy matrices $\widetilde{X}$ and $\widetilde{Y}$, consisting of real parts $\widetilde{P}$ and $\widetilde{U}$, and imaginary parts $\widetilde{Q}$ and $\widetilde{V}$, respectively, where these real and imaginary parts are LR fuzzy number matrices. The arithmetic operation rules between these two complex LR fuzzy matrices are defined as follows.*

(1) $\widetilde{X} + \widetilde{Y} = (\widetilde{P} + \widetilde{U}) + i(\widetilde{Q} + \widetilde{V})$,

(2) $\widetilde{X} - \widetilde{Y} = (\widetilde{P} - \widetilde{U}) + i(\widetilde{Q} - \widetilde{V})$,

(3) $k\widetilde{X} = k\widetilde{P} + ik\widetilde{Q}, k \in R$,

(4) $\widetilde{X} \times \widetilde{Y} = (\widetilde{P} \times \widetilde{U} - \widetilde{Q} \times \widetilde{V}) + i(\widetilde{P} \times \widetilde{V} + \widetilde{Q} \times \widetilde{U})$.

Definition 2.8. *The matrix system*

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{12} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{pmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{x}_{n1} & \widetilde{x}_{n2} & \cdots & \widetilde{x}_{nn} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{12} & \cdots & b_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix}$$

$$= \begin{pmatrix} \widetilde{c}_{11} & \widetilde{c}_{12} & \cdots & \widetilde{c}_{1m} \\ \widetilde{c}_{21} & \widetilde{c}_{12} & \cdots & \widetilde{c}_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{c}_{n1} & \widetilde{c}_{n2} & \cdots & \widetilde{c}_{nn} \end{pmatrix}, \tag{2.1}$$

where $a_{ij}, b_{ij}, 1 \leq i, j \leq n$ are crisp numbers and $\widetilde{c}_{ij}, 1 \leq i, j \leq n$ are complex LR fuzzy numbers, is called a LR complex fuzzy linear matrix equations(CLRFLMEs).

Using matrix notation, we have

$$A\widetilde{X}B = \widetilde{C}, \tag{2.2}$$

A complex LR fuzzy numbers matrix

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r)$$

$$= (m, m^l, m^r) + i(n, n^l, n^r), 1 \leq i, j \leq n$$

is said to the solution of the general dual complex fuzzy matrix equation (2.1) *if $\widetilde{X}$ satisfies the Eqs.*(2.2).

3 Solving the CLRFLMEs

Theorem 3.1. *Given a complex fuzzy linear matrix system $A\widetilde{X}B = \widetilde{C}$, it can be equivalently expressed as a series of linear matrix equations, as follows.*

$$\begin{cases} A[MN] \bigotimes B = [UV], \\ A \bigotimes \begin{pmatrix} M^l & M^r \\ N^l & N^r \end{pmatrix} \begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix} = \begin{pmatrix} U^l & U^r \\ V^l & V^r \end{pmatrix}, \end{cases} \tag{3.1}$$

where

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r), \widetilde{C} = (U, U^l, U^r) + i(V, V^l, V^r). \tag{3.2}$$

And the elements b^+_{ij} of matrix B^+ and b^-_{ij} of matrix B^- are determined by this way: if $b_{ij} \geq 0, b^+_{ij} = b_{ij}$ else $b^+_{ij} = 0, 1 \leq i, j \leq n$; if $b_{ij} < 0, b^-_{ij} = b_{ij}$ else $b^-_{ij} = 0, 1 \leq i, j \leq n$.

Proof. Let $\widetilde{C} = [U, U^l, U^r] + i[V, V^l, V^r] = ([u, u^l, u^r] + i[v, v^l, v^r])_{n\times n}$ and the unknown complex fuzzy matrix $\widetilde{X} = [M, M^l, M^r] + i[N, N^l, N^r] = ([m, m^l, m^r] + i[n, m^l, m^r])_{n\times n}$. We also let $A = A^+ + A^-$ in which the elements a^+_{ij} of matrix A^+ and a^-_{ij} of matrix A^+ are determined by the following way: if $a_{ij} \geq 0, a^+_{ij} = a_{ij}$ else $a^+_{ij} = 0, 1 \leq i, j \leq n$; if $a_{ij} < 0, a^-_{ij} = a_{ij}$ else $a^-_{ij} = 0, 1 \leq i, j \leq n$ and also let $B = B^+ + B^-$ in the same way.

For complex fuzzy matrix equation $A\widetilde{X}B = \widetilde{C}$, i.e.,

$$A([M, M^l, M^r] + i[N, N^l, N^r])B = [U, U^l, U^r] + i[V, V^l, V^r].$$

Supposing $A = A^+ + A^-$ and $B = B^+ + B^-$, we have

$$(A^+ + A^-)([M, M^l, M^r] + i[N, N^l, N^r])(B^+ + B^-) = [U, U^l, U^r] + i[V, V^l, V^r]. \tag{3.3}$$

Since

$$\widetilde{m}_{ij}k = \begin{cases} (k\underline{m}_{ij}(r), k\overline{m}_{ij}(r)), & k \geq 0, \\ (k\overline{m}_{ij}(r), k\underline{m}_{ij}(r)), & k < 0, \end{cases}$$

and

$$\widetilde{M}B = \begin{cases} (\underline{M}(r)B, \overline{M}(r)B), & B \geq 0, \\ (\overline{M}(r)B, \underline{M}(r)B), & B < 0, \end{cases}$$

so the Eqs.(3.3) can be rewritten as

$$\begin{gathered} A^+[M, M^l, M^r]B^+ + A^+[M, M^l, M^r]B^- + A^-[M, M^l, M^r]B^+ + A^-[M, M^l, M^r]B^- + \\ i(A^+[N, N^l, N^r]B^+ + A^+[N, N^l, N^r]B^- + A^-[N, N^l, N^r]B^+ + A^-[N, N^l, N^r]B^-) \\ = [U, U^l, U^r] + i[V, V^l, V^r]. \end{gathered}$$

In comparison with the coefficients of i, we get

$$A^+[M, M^l, M^r]B^+ + A^+[M, M^l, M^r]B^- + A^-[M, M^l, M^r]B^+ + A^-[M, M^l, M^r]B^- = [U, U^l, U^r],$$

and

$$A^+[N, N^l, N^r]B^+ + A^+[N, N^l, N^r]B^- + A^-[N, N^l, N^r]B^+ + A^-[N, N^l, N^r]B^- = [V, V^l, V^r],$$

i.e.,

$$\begin{cases} A^+MB^+ + A^+MB^- + A^-MB^+ + A^-MB^- = U, \\ A^+M^lB^+ - A^+M^rB^- - A^-M^rB^+ + A^-M^lB^- = U^l, \\ A^+M^rB^+ - A^+M^lB^- - A^-M^lB^+ + A^-M^rB^- = U^l, \\ A^+NB^+ + A^+NB^- + A^-NB^+ + A^-NB^- = V, \\ A^+N^lB^+ - A^+N^rB^- - A^-N^rB^+ + A^-N^lB^- = V^l, \\ A^+N^rB^+ - A^+N^lB^- - A^-N^lB^+ + A^-N^rB^- = V^l, \end{cases} \tag{3.4}$$

Denoting in matrix form, they can be written as

$$\begin{cases} AMB = U, \\ A\left(\; M^l, M^r \; \right)\begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix} = \left(\; U^l, U^r, \; \right), \end{cases} \tag{3.5}$$

and

$$\begin{cases} ANB = V, \\ A\left(\; M^l, M^r \; \right)\begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix} = \left(\; V^l, V^r, \; \right). \end{cases} \tag{3.6}$$

From Eqs.(3.5) and (3.6), we obtain the Eqs.(3.1). and (3.2) as follows:

$$\begin{cases} A[MN] \bigotimes B = [UV], \\ A \bigotimes \begin{pmatrix} M^l & M^r \\ N^l & N^r \end{pmatrix}\begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix} = \begin{pmatrix} U^l & U^r \\ V^l & V^r \end{pmatrix}, \end{cases}$$

where $\bigotimes$ is the Kronecker product of matrices and

$$\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r),\widetilde{C}=(U,U^l,U^r)+i(V,V^l,V^r).$$

Similarly, we can derivd another model for solving the Eqs.(2.2). □

Theorem 3.2. *The complex LR fuzzy linear matrix system $A\widetilde{X}B=\widetilde{C}$ can be converted into the following system of linear matrix equations*

$$\begin{cases} A\bigotimes\begin{pmatrix} M \\ N \end{pmatrix}B=\begin{pmatrix} U \\ V \end{pmatrix}, \\ \begin{pmatrix} A^+ & -A^- \\ -A^- & A^+ \end{pmatrix}\begin{pmatrix} M^l & N^l \\ M^r & N^r \end{pmatrix}\bigotimes B=\begin{pmatrix} U^l & V^l \\ U^r & V^r \end{pmatrix}, \end{cases} \tag{3.7}$$

where

$$\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r),\widetilde{C}=(U,U^l,U^r)+i(V,V^l,V^r). \tag{3.8}$$

And the elements a^+_{ij} of matrix A^+ and a^-_{ij} of matrix A^- are determined by the following way: if $a_{ij}\geq 0, a^+_{ij}=a_{ij}$ else $a^+_{ij}=0, 1\leq i,j\leq n$; if $a_{ij}<0, a^-_{ij}=a_{ij}$ else $a^-_{ij}=0, 1\leq i,j\leq n$.

Proof. The proof is similar with the above Theorem 3.1. □

Theorem 3.3. [27]*Given a matrix S belong to $R^{m\times n}$, T belong to $R^{p\times q}$, and C belong to $R^{m\times q}$, there exists a minimal solution X^* to the matrix equation $SXT=C$, which can be expressed as follows.*

$$X^*=S^\dagger CT^\dagger.$$

In order to find a solution to the fuzzy matrix equation (2.2)*, we first need to compute the system of linear equations* (3.1) *or* (3.7)*. We obtain the minimum solution of the linear system*(2.2) *as follows.*

$$\begin{cases} [MN]=A^\dagger[UV]\bigotimes B^\dagger, \\ \begin{pmatrix} M^l & M^r \\ N^l & N^r \end{pmatrix}==A^\dagger\bigotimes\begin{pmatrix} U^l & U^r \\ V^l & V^r \end{pmatrix}\begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix}^\dagger \end{cases} \tag{3.9}$$

or

$$\begin{cases} \begin{pmatrix} M \\ N \end{pmatrix}=A^\dagger\bigotimes\begin{pmatrix} U \\ V \end{pmatrix}B^\dagger, \\ \begin{pmatrix} M^l & N^l \\ M^r & N^r \end{pmatrix}=\begin{pmatrix} A^+ & -A^- \\ -A^- & A^+ \end{pmatrix}^\dagger\begin{pmatrix} U^l & V^l \\ U^r & V^r \end{pmatrix}\bigotimes B^\dagger, \end{cases} \tag{3.10}$$

where $(.)^\dagger$ is the Moore-Penrose generalized inverse of matrix $(.)$.

It seems that we obtained the complex fuzzy solution matrix $\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r)$ as the above expression (3.9) *or* (3.10)*. However, the solution matrix may still not be an appropriate LR fuzzy numbers matrix except for that both $\widetilde{M}=(M,M^l,M^r)$ and $\widetilde{N}=(N,N^l,N^r)$ are appropriate LR fuzzy matrices. So we give the definition of LR fuzzy solution to the Eq.*(2.2) *as follows.*

Definition 3.4. *Let $\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r)$. If $((M,M^l,M^r)$ and (N,N^l,N^r) is the minimal solution of Eqs.*(3.1) *or* (3.7)*, such that $M^l\geq O, M^r\geq O$ and $N^l\geq O, N^r\geq O$, we said that $\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r)$ is a strong LR complex fuzzy minimal solution of fuzzy matrix equation*(2.2)*. Otherwise, the $\widetilde{X}=(M,M^l,M^r)+i(N,N^l,N^r)$ is said to a weak LR complex fuzzy fuzzy minimal solution of fuzzy matrix equation*(2.2) *given by*

$$\widetilde{X}=\widetilde{m}_{ij}+i\widetilde{n}_{ij}$$

where

$$\widetilde{m}_{ij} = \begin{cases} (m_{ij}, m_{ij}^l, m_{ij}^r), & m_{ij}^l > 0, \quad m_{ij}^r > 0, \\ (m_{ij}, 0, \max\{-m_{ij}^l, m_{ij}^r\}), & m_{ij}^l < 0, \quad m_{ij}^r > 0, \\ (m_{ij}, \max\{m_{ij}^l, -m_{ij}^r\}, 0,), & m_{ij}^l > 0, \quad m_{ij}^r < 0, \\ (m_{ij}, -m_{ij}^l, -m_{ij}^r), & m_{ij}^l < 0, \quad m_{ij}^r < 0. \end{cases} \quad i, j = 1, \ldots, n. \tag{3.11}$$

and

$$\widetilde{n}_{ij} = \begin{cases} (n_{ij}, n_{ij}^l, n_{ij}^r), & n_{ij}^l > 0, \quad n_{ij}^r > 0, \\ (n_{ij}, 0, \max\{-n_{ij}^l, n_{ij}^r\}), & n_{ij}^l < 0, \quad n_{ij}^r > 0, \\ (n_{ij}, \max\{n_{ij}^l, -n_{ij}^r\}, 0,), & n_{ij}^l > 0, \quad n_{ij}^r < 0, \\ (n_{ij}, -n_{ij}^l, -n_{ij}^r), & n_{ij}^l < 0, \quad n_{ij}^r < 0. \end{cases} \quad i, j = 1, \ldots, n. \tag{3.12}$$

Theorem 3.5. *Let*

$$S^{\dagger} = \begin{pmatrix} E & F \\ F & E \end{pmatrix},$$

where

$$S = \begin{pmatrix} A^+ & -A^- \\ -A^- & A^+ \end{pmatrix}.$$

Then

$$\begin{cases} E = \frac{1}{2}((A^+ - A^-)^{\dagger} + (A^+ + A^-)^{\dagger}), \\ F = \frac{1}{2}((A^+ - A^-)^{\dagger} - (A^+ + A^-)^{\dagger}), \end{cases} \tag{3.13}$$

where $(A^+ + A^-)^{\dagger}, (A^+ - A^-)^{\dagger}$ *are Moore-Penrose inverses of matrices* $A^+ + A^-$ *and* $A^+ - A^-$*, respectively.*

For the model Eqs.(3.7)*, we have the following result.*

Theorem 3.6. *If*

$$A^{\dagger} \geq O,$$

$$(B^+ - B^-)^{\dagger} + (B^+ + B^-)^{\dagger}) \geq O, (B^+ - B^-)^{\dagger} - (B^+ + B^-)^{\dagger}) \geq O,$$

the fuzzy matrix equation (2.2) *has a strong LR complex fuzzy minimal solution as follows:*

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r],$$

where

$$\begin{cases} [MN] = A^{\dagger}[UV] \bigotimes B^{\dagger}, \\ M^l = A^{\dagger}U^l E + A^{\dagger}U^r F, \\ M^r = A^{\dagger}U^l F + A^{\dagger}U^r E, \\ N^l = A^{\dagger}V^l E + A^{\dagger}V^r F, \\ N^r = A^{\dagger}V^l F + A^{\dagger}V^r E, \\ E = \frac{1}{2}((B^+ - B^-)^{\dagger} + (B^+ + B^-)^{\dagger}), \\ F = \frac{1}{2}((B^+ - B^-)^{\dagger} - (B^+ + B^-)^{\dagger}). \end{cases} \tag{3.14}$$

Proof. Since U^l and U^r are the left and right extensions of the fuzzy matrix $\widetilde{U}$, respectively, $C^l \geq O$ and $C^r \geq O$, this indicates that $\begin{pmatrix} U^l, U^r \end{pmatrix}$ is a nonnegative matrix. Similarly, for the fuzzy matrix $\widetilde{\widetilde{V}} = (V, V^l, V^r)$, the properties are the same.

Let

$$S^{\dagger} = \begin{pmatrix} E & F \\ F & E \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (B^+ - B^-)^{\dagger} + (B^+ + B^-)^{\dagger} & (B^+ - B^-)^{\dagger} - (B^+ + B^-)^{\dagger} \\ (B^+ - B^-)^{\dagger} - (B^+ + B^-)^{\dagger} & (B^+ - B^-)^{\dagger} + (B^+ + B^-)^{\dagger} \end{pmatrix}.$$

The nonnegativity of the condition $S^{\dagger}$, is equivalent to the fact that both matrices E and F satisfy the nonnegativity condition.

Now that $E \geq O$ and $F \geq O$, the product of three non negative matrices

$$\begin{pmatrix} M^l, M^r \\ N^l, N^r \end{pmatrix} = A^\dagger \bigotimes \begin{pmatrix} U^l, U^r \\ V^l, V^r \end{pmatrix} \begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix}^\dagger$$

$$= A^\dagger \begin{pmatrix} C^l, C^r \end{pmatrix} \begin{pmatrix} E & F \\ F & E \end{pmatrix} = \begin{pmatrix} A^\dagger U^l E + A^\dagger U^r F, A^\dagger U^l F + A^\dagger U^r E \\ A^\dagger V^l E + A^\dagger V^r F, A^\dagger V^l F + A^\dagger V^r E \end{pmatrix} \geq O$$

is non negative in nature. It means that $M^l \geq O, M^r \geq O$ and $N^l \geq O, N^r \geq O$.

For the model Eqs.(3.1), we have the following result similarly. □

Theorem 3.7. *If*

$$B^\dagger \geq O,$$
$$(A^+ - A^-)^\dagger + (A^+ + A^-)^\dagger) \geq O, (A^+ - A^-)^\dagger - (A^+ + A^-)^\dagger) \geq O,$$

the fuzzy matrix equation (2.2) *has a strong LR fuzzy minimal solution as follows:*

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r],$$

where

$$\begin{cases} \begin{pmatrix} M \\ N \end{pmatrix} = A^\dagger \bigotimes \begin{pmatrix} U \\ V \end{pmatrix} B^\dagger, \\ M^l = EU^l B^\dagger + FU^r B^\dagger, \\ M^r = FU^l B^\dagger + EU^r B^\dagger, \\ N^l = EV^l B^\dagger + FV^r B^\dagger, \\ N^r = FV^l B^\dagger + EV^r B^\dagger, \\ E = \frac{1}{2}((A^+ - A^-)^\dagger + (A^+ + A^-)^\dagger), \\ F = \frac{1}{2}((A^+ - A^-)^\dagger - (A^+ + A^-)^\dagger). \end{cases} \tag{3.15}$$

Proof. The proof is straight forward. □

4 Numerical Examples

Example 4.1. Consider the following complex LR fuzzy linear matrix equation

$$\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} \\ \widetilde{x}_{21} & \widetilde{x}_{22} \end{pmatrix} \begin{pmatrix} -3 & 4 \\ 1 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} (3,2,1)_{LR} & (4,1,1)_{LR} \\ (5,2,2)_{LR} & (3,1,2)_{LR} \end{pmatrix} + i \begin{pmatrix} (4,1,2)_{LR} & (2,1,1)_{LR} \\ (5,3,2)_{LR} & (3,2,1)_{LR} \end{pmatrix}.$$

By the Theorem 3.2., the original fuzzy matrix equation is extended into the following a system of linear matrix equations (3.7)

$$\begin{cases} A \bigotimes \begin{pmatrix} M \\ N \end{pmatrix} B = \begin{pmatrix} U \\ V \end{pmatrix}, \\ \begin{pmatrix} A^+ & -A^- \\ -A^- & A^+ \end{pmatrix} \begin{pmatrix} M^l & N^l \\ M^r & N^r \end{pmatrix} \bigotimes B = \begin{pmatrix} U^l & V^l \\ U^r & V^r \end{pmatrix}, \end{cases}$$

where

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r), \widetilde{C} = (U, U^l, U^r) + i(V, V^l, V^r).$$

and

$$A^{+}=\begin{pmatrix}1&0\\0&2\end{pmatrix},A^{-}=\begin{pmatrix}0&-1\\0&0\end{pmatrix},$$

$$U=\begin{pmatrix}3&4\\5&3\end{pmatrix},U^{l}=\begin{pmatrix}2&1\\2&1\end{pmatrix},U^{r}=\begin{pmatrix}1&1\\2&2\end{pmatrix},$$

$$V=\begin{pmatrix}4&2\\5&3\end{pmatrix},V^{l}=\begin{pmatrix}1&1\\3&2\end{pmatrix},V^{r}=\begin{pmatrix}2&1\\2&1\end{pmatrix},$$

From the Eqs. (3.14), the solution of the computing model is

$$\begin{pmatrix}M\\N\end{pmatrix}=A^{\dagger}\bigotimes\begin{pmatrix}U\\V\end{pmatrix}B^{\dagger}$$

$$=\begin{pmatrix}1&-1\\0&2\end{pmatrix}^{\dagger}\begin{pmatrix}3&4\\5&3\\4&5\\2&3\end{pmatrix}\begin{pmatrix}-3&4\\1&0\end{pmatrix}^{\dagger}=\begin{pmatrix}1.3750&9.6250\\0.3750&3.6250\\0.8570&9.1250\\0.3750&3.6250\end{pmatrix}$$

and

$$\begin{pmatrix}M^{l}&N^{l}\\M^{r}&N^{r}\end{pmatrix}=\begin{pmatrix}A^{+}&-A^{-}\\-A^{-}&A^{+}\end{pmatrix}^{\dagger}\begin{pmatrix}U^{l}&V^{l}\\U^{r}&V^{r}\end{pmatrix}\bigotimes B^{\dagger}$$

$$=\begin{pmatrix}1&0&0&1\\0&2&0&0\\0&1&1&0\\0&0&0&2\end{pmatrix}^{\dagger}\begin{pmatrix}2&1&1&1\\2&1&3&2\\1&1&2&1\\1&2&2&1\end{pmatrix}\begin{pmatrix}-3&4\\1&0\end{pmatrix}^{\dagger}$$

$$=\begin{pmatrix}0.3750&1.5000&0.1250&0.8750\\0.1250&0.2500&0.2500&2.2500\\0.5000&0.8750&0.0000&0.5000\\0.2500&0.6250&0.1250&0.8750\end{pmatrix}.$$

It means

$$\widetilde{M}=\begin{pmatrix}(1.3750,0.3750,0.5000)&(9.6250,1.5000,0.8750)\\(0.3750,0.1250,0.2500)&(3.6250,0.2500,0.6250)\end{pmatrix}$$

and

$$\widetilde{N}=\begin{pmatrix}(0.8570,0.1250,0.0000)&(9.1250,0.8750,0.5000)\\(0.3750,0.2500,0.1250)&(3.6250,2.2500,0.8750)\end{pmatrix}.$$

Since M^{l},M^{r} and N^{l},N^{r} are nonnegative matrices and $M-M^{l}>O,N-N^{l}>O$, the solution we obtained is an appropriate LR complex fuzzy matrix

$$\widetilde{X}=(M,M^{l},M^{r})+i(N,N^{l},N^{r})$$

$$=\begin{pmatrix}(1.3750,0.3750,0.5000)&(9.6250,1.5000,0.8750)\\(0.3750,0.1250,0.2500)&(3.6250,0.2500,0.6250)\end{pmatrix}$$

$$+i\begin{pmatrix}(0.8570,0.1250,0.0000)&(9.1250,0.8750,0.5000)\\(0.3750,0.2500,0.1250)&(3.6250,2.2500,0.8750)\end{pmatrix},$$

which admits a nonnegative strong LR complex fuzzy solution of the original fuzzy matrix system.

Example 4.2. Consider another complex LR fuzzy linear matrix equation

$$\begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} \\ \widetilde{x}_{21} & \widetilde{x}_{22} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (2,2,1)_{LR} & (3,2,1)_{LR} \\ (3,1,1)_{LR} & (2,1,2)_{LR} \\ (1,1,1)_{LR} & (3,2,1)_{LR} \end{pmatrix} + i \begin{pmatrix} (5,1,3)_{LR} & (2,1,2)_{LR} \\ (3,2,1)_{LR} & (3,1,2)_{LR} \\ (2,1,2)_{LR} & (1,1,1)_{LR} \end{pmatrix}.$$

Suppose

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r),$$

$$A = A^+ + A^- = \begin{pmatrix} 2 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix},$$

$$B = B^+ + B^- = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

and

$$\widetilde{U} = \left(U, U^l, U^r\right) = \left(\begin{pmatrix} 2 & 3 \\ 3 & 2 \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 1 \end{pmatrix}\right),$$

$$\widetilde{V} = \left(V, V^l, V^r\right) = \left(\begin{pmatrix} 5 & 2 \\ 3 & 3 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 3 & 2 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}\right).$$

By the Theorem 3.1., the original fuzzy matrix equation is extended into the following a system of linear matrix equations(3.5)

$$\begin{cases} A[MN] \bigotimes B = [UV], \\ A \bigotimes \begin{pmatrix} M^l & M^r \\ N^l & N^r \end{pmatrix} \begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix} = \begin{pmatrix} U^l & U^r \\ V^l & V^r \end{pmatrix}, \end{cases}$$

where

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r), \widetilde{C} = (U, U^l, U^r) + i(V, V^l, V^r).$$

From the Eqs.(3.13), the solution of the computing model is

$$[MN] = A^\dagger [UV] \bigotimes B^\dagger$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}^\dagger \begin{pmatrix} 2 & 3 & 5 & 2 \\ 3 & 2 & 3 & 3 \\ 1 & 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^\dagger$$

$$= \begin{pmatrix} 0.0179 & -0.0179 & 0.2500 & 0.2500 \\ -0.3393 & 0.3393 & 0.2500 & 0.2500 \end{pmatrix},$$

$$\begin{pmatrix} M^l & M^r \\ N^l & N^r \end{pmatrix} = A^\dagger \bigotimes \begin{pmatrix} U^l & U^r \\ V^l & V^r \end{pmatrix} \begin{pmatrix} B^+ & -B^- \\ -B^- & B^+ \end{pmatrix}^\dagger$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}^{\dagger} \begin{pmatrix} 2 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}^{\dagger}$$

$$= \begin{pmatrix} 0.4464 & 0.3571 & 0.3571 & 0.4464 \\ 0.0179 & 0.2143 & 0.2143 & 0.0179 \\ 0.5179 & 0.4464 & 0.4464 & 0.5179 \\ -0.0893 & 0.2679 & 0.2679 & -0.0893 \end{pmatrix}.$$

It means

$$\widetilde{M} = \begin{pmatrix} (0.0179, 0.4464, 0.3571)_{LR} & (-0.0179, 0.3571, 0.4464)_{LR} \\ (-0.3393, 0.0179, 0.2143)_{LR} & (0.3393, 0.4464, 0.0179)_{LR} \end{pmatrix}$$

and

$$\widetilde{N} = \begin{pmatrix} (0.2500, 0.5179, 0.4464)_{LR} & (0.2500, 0.4464, 0.5179)_{LR} \\ (0.2500, -0.0893, 0.2679)_{LR} & (0.2500, 0.2679, -0.0893)_{LR} \end{pmatrix}.$$

Since $\widetilde{M}$ is an appropriate LR complex fuzzy matrix, but $\widetilde{N}$ is not an appropriate one, the solution we obtained is

$$\widetilde{X} = (M, M^l, M^r) + i(N, N^l, N^r)$$

$$= \begin{pmatrix} (0.0179, 0.4464, 0.3571)_{LR} & (-0.0179, 0.3571, 0.4464)_{LR} \\ (-0.3393, 0.0179, 0.2143)_{LR} & (0.3393, 0.4464, 0.0179)_{LR} \end{pmatrix}$$

$$+i \begin{pmatrix} (0.2500, 0.5179, 0.4464)_{LR} & (0.2500, 0.4464, 0.5179)_{LR} \\ (0.2500, 0.0000, 0.2679)_{LR} & (0.2500, 0.2679, 0.0000)_{LR} \end{pmatrix},$$

which admits a weak complex LR fuzzy solution of the original fuzzy matrix system by the by Definition 3.5.

5 Conclusion

In this paper, two models are proposed to solve the LR complex fuzzy linear matrix equation $A\widetilde{X}B = \widetilde{C}$, where A and B are crisp matrices for $m \times m$ and $n \times n$, respectively, and $\widetilde{C}$ is an arbitrary matrix of LR fuzzy numbers for $m \times n$.We obtained the complex fuzzy approximate solutions of fuzzy linear matrix equations by solving a crisp linear matrix equation system. In addition, we also discussed the two existence conditions of strongly complex fuzzy solutions. We demonstrated two numerical examples to show effectiveness of the proposed method. Next, we will consider the case where matrices A and B are complex matrices, and apply the algorithm to other types of linear matrix equations. This method is not limited to a specific type of fuzzy matrix equations, it has a wide range of applicability. To some extent, our study enriches the computational theory of fuzzy linear systems.

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