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Case Study

Optimization of Inventory with Fuzzy Multi-Objective Approach

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ABSTRACT

Resource management is a part of project management and its ultimate goal is to achieve maximum efficiency with the lowest level of inventory. Resource management is built around optimization and increased efficiency. In this paper, inventory optimization is done using fuzzy approach and statistical averaging methods are used to solve fuzzy multi-objective linear programming problems. These methods have been used to form a goal function of fuzzy multi-objective linear programming problems. First, in order to optimize inventory and find the weight of each product in Isfahan Steel Company, a model of fuzzy multi-objective linear programming problem is estimated. The highest weight of products is related to commodity ingots and the lowest weight related to other products. The Fuzzy Multi-Objective Linear Programming Estimation Model has compared the statistical methods with the Chondrasen method. The results show that this method has the capacity to optimize the amount of inventory, reduce storage costs and reduce interest costs due to working capital.

1 Introduction

Companies that excel at inventory management drive the maximum amount of profit and customer satisfaction from the least amount of investment. And as we see from those efficient businesses, managing inventory successfully is all about data: purchases, reorders, shipping, warehousing, storage, receiving, customer satisfaction and more. Fortunately, much of this information is readily available from inventory management systems. Companies that integrate their enterprise resource planning (ERP) systems with inventory management functionality have a competitive advantage. The main competitive advantage, however, comes from being able to share information within different parts of their business in real-time. Enterprise Resource Planning (ERP) inventory management is a system that allows businesses to manage all aspects of

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their business on a single platform, including: inventory, finance, planning, logistics and operations. An ERP inventory management system provides real-time inventory information to the entire organization. This is an important capability for businesses that plan to expand, have complex workflows or supply chains, need advanced automation, operate in "just in time" mode, sell many products or simply wish to maximize their investments in inventory. One of the most important financial and managerial decisions of companies is to decide on determining the desired amount of inventory needed for each financial period. As costs rise and the market evolves, you need advantages to outperform other competitors, and inventory control comes to the aid of your business as a way to improve efficiency. Inventory control is a process that ensures that the items and inventory of the economic unit are provided timely, place, quantity, quality and cost for customers in a timely manner. Inventory expenditure plays an important role in inventory control. All objectives of inventory control should be realized by taking into account the cost constraints that are very significant in the case of items. [1] Many economic units face the problem of working capital shortage in reality, but by carefully examining the documentation of the warehouse, we find that a large amount of their capital is devoted to the purchase and undue accumulation of material and commodity inventory. In this research, we investigate the optimum amount of accumulated and produced inventory at each stage of production to prevent the production of products. If production is done without proper planning, it will reduce working capital and increase the storage and maintenance costs of the products. It should be noted that in the event that the quantity of production and the stored products is not proportional, it may cause the loss of new orders, and therefore determining the optimal amount of inventory at each stage of production and in the packaging and warehouse stage is very important. The optimal inventory should be sufficient to prevent both the reduction of working capital and the increase in storage costs, and not to cause the loss of new customer orders .Real world circumstances are not always deterministic. There exist different kinds of uncertainties in social, industrial and economic systems. Different kinds of uncertainties are defined as stochastic uncertainty and fuzziness [2] A system with a stochastic uncertainty is solved by the stochastic optimization technique using the probability theory. Also, fuzzy programming technique is widely used to solve problem with uncertainty. A system with uncertainty can be optimized to reduce the risk factors in the system with fuzzy conditions. An optimization of a system with uncertainty using fuzzy conditions is called a fuzzy Optimization. Modelling under a fuzzy environment is called fuzzy modelling. Fuzzy linear programming is one of the most frequently applied fuzzy decision making techniques. We get fuzzy linear programming problem (FLPP) by interchanging the parameters of linear programming problem by fuzzy numbers. Several methods have been proposed in the literature to solve FLPP.

Studies on inventory control policies under uncertainty began with the classic newsvendor model [3-5] Because of the daily nature of newspapers, the newsvendor model accounted inventory related costs based on the inventory level at the end of the day. Following this tradition and for reasons of mathematical tractability, subsequent studies on inventory control policies have typically accounted inventory related costs based on the inventory level at the end of each review period (see, for example, [6-9]). However, costs for holding inventory such as costs of capital tied up with stock, costs for storage space and facility, and costs due to spoilage and obsolescence's, etc., usually accrue continuously in time. According to Nahmias [10], the most significant cost component for holding inventory is the opportunity cost for the capital invested in inventory which accrues continuously in time. Similar arguments can also be made for the costs of backlogging shortage. In reality, the cost of stock-out depends on not only the amount but also the duration. Since the inventory level at the end of a review period ignores variations during the period, end-of-period cost accounting does not provide an accurate evaluation of these inventory related costs [11]. As a result, an inventory control policy that minimizes system cost evaluated based on the end-of-period inventory level may not be optimal. This calls for an evaluation of end-of-period cost accounting when inventory related costs actually accrue continuously in time Bellman and Zadeh in 1970 [12], proposed decision making in fuzzy condition. They explained the use of decision making in fuzzy condition by using a controlled system which is either stochastic or deterministic with multistage decision processes, [2] proposed a formulation of FLPP using theory of fuzzy sets. He used FLPP to the linear vector maximum problem and found that the solutions obtained using FLPP are always efficient. Considering parameters ambiguity, Tanaka and Asai also proposed a formulation of FLPP to find a logical solution [13] They highlighted that the FLPP with fuzzy numbers may be considered as a model of decision problems in a system with influential human estimations. Not only linear programming problems (LPP), but also multi-objective LPP (MOLPP) and multi-objective nonlinear programming problems were solved using fuzzy approaches which are available in the literature. The recent applications of 4.0 technologies are gradually leading to the introduction and development of artificial intelligence in production systems. Ivan and Beatrice have discussed the assessment of a Machine Learning model, based on Reinforcement Learning, that may allow the optimization of the inventory level at the machine level, thus improving the ordering system and inventory management [14].it is quite feasible for the latest technologies used for goods inventory and healthcare industry products under time-dependent demand [15]. Nicky in 2023, showed that simple methods to establish the existence and characterization of optimal policies and efficient numerical procedures to compute optimal policy parameters could have been developed much earlier if stochastic inventory problems were looked through the lens of optimal stopping theory, based [16]. the rest of the paper is organized as follows. Section 2 presents the theoretical foundations and background of the research, Section 3 methodology, Section 4 findings, and finally Section 5 concludes the study.

2 Theoretical Fundamentals and Research Background

The Considering that this research is in the field of optimization of inventory with a fuzzy multi-objective approach in the Esfahan steel company, there is no similar research in the world, so the following is only research that is somewhat similar to this research. Technically, the echelon-stock method can be used under any cost accounting scheme provided that the (limiting) distributions for all echelon inventory levels can be characterized and the instantaneous expected inventory related costs can be integrated accordingly over a system reorder cycle. These tasks, however, seem to be cumbersome when inventory holding and backordering costs accrue continuously in time. In view of the literature, [17] started the recent interest in periodic-review inventory control policies under continuous-time cost accounting. Laganathan and Lalitha [18], solved a multi-objective nonlinear programming problem using α-cut method in fuzzy approach and compared the solution with the solution obtained by Zimmermann [19] who used membership function. Beher et al [20]. also used α-cut method to solve multi-objective linear programming problem (MOLPP) in fuzzy approach and compared the solutions to the solution obtained by Zimmermann [19] ‹who used membership function. Thakre et al [21]. solved an objective function with constraint matrix and cost coefficients which are fuzzy in nature. They used MOLPP with the constraints to solve FLPP and proved that the solutions are independent of weights. [22] discussed Chandra Sen's approach, statistical averaging method and new statistical averaging method to solve MOLPP. For MOLFPP, a new geometric average technique was proposed by Nahar and Alim [23]. Veeramani and Sumathi [24] solved fuzzy linear fractional programming problem (FLFPP) where the cost, resources and technological coefficients of the objective function were triangular fuzzy numbers. In the solution procedure, they converted the FLFPP into a multi-objective linear fractional programming problem. There are a lot of methods for solving fully fuzzy linear programming problems in the literature. Ebrahim nejad and Tavana [25] converted the FLPP into an equivalent crisp linear programming problem and solved by simplex method. Here they proposed a new concept in which the coefficients of objective function and the values of the right-hand side are represented by trapezoidal fuzzy numbers and other parts are represented by real numbers. A general form of fuzzy near fractional programming problem with trapezoidal fuzzy numbers is proposed by Das [26]. Lotfi et al [27]. proposed a method to obtain the approximate solution of fully fuzzy linear programming problems. A method to solve fully fuzzy linear programming problems is proposed by Amit Kumar et al [28]. using idea of crisp linear programming and ranking function. FLPP is solved by S. Nahar, S. et al [29]. To solve FLPP, Nahar, S. et al [29]. used weighted sum method. Here both equal and unequal weight has been used. In this paper there is a discussion for ranking function. Here they used triangular and trapezoidal fuzzy number. Sen, C. [30] developed averaging technique of multi-objective optimization for rural development planning. Akter, M. et al [31]. proposed fuzzy synthetic evaluation method for risk assessment on the natural hazard. Nishad, A. [32] used alpha cut fuzzy number for solving fractional programming problem in fuzzy field. Pitam, S., [33] developed goal programming approach for fuzzy multi-objective linear and fuzzy multi-objective linear programming problem .Nazemi et al [34], In their study, according to ABC's method, 77 items of raw materials were divided into three groups. In this research, a fusion approach of fuzzy and ABC models was used which presented a significant achievement in clarifying the work with nominal and non-nominal variables. Since the lack or absence of raw materials in the market causes problems for production units, accurate identification of goods in the mentioned three groups by considering quality criteria can help planning in purchasing and inventory control system of companies. Esmaeilzadeh and Olfat [35] proposed three new approaches to determine the level of importance of inventory items. These approaches can reduce the cost of inventory control. The results show that the number of high-importance items decreases and the number of items of low importance increases. Given that low-key control policies require lower costs, it can be said that these results will reduce inventory control costs.

The main objective of this study was to make managers more accurate in classification and reduce inventory control costs. To achieve maximum profit from the sale of perishable products, the price reduction policy has been implemented. Perishable products decline after a period of time, and demand for these products decreases. In this case, the policy of lowering prices or offering discounts is one of the ways to increase demand. The main goal is to find the optimal discount amounts and the initial order quantity. Safarzadeh et al [36]. In order to achieve the optimal order

quantity in the retail industry, paying attention to shelf space is necessary for controlling inventory of corrupt goods, regardless of warehouse space. Math et al [37]. Supply chain management and distribution network design have attracted the attention of many researchers in recent years. Nasiri et al [38]. developed a complex nonlinear integer mathematical programming model that reports the performance of the proposed algorithm in terms of different indices. Fazli and Faraji [39]. By examining decisions related to cost allocation, inventory and routing in a three-tier supply chain including suppliers, warehouses and payment. They used exploratory and meta-heuristic methods to achieve optimal results. An inappropriate change in inventory growth has a negative impact on the company's risk-taking in the future [40]. System dynamics is an approach to study and investigate systems. System dynamics acts as a soft tool to study behavior based on cause-and-effect relationships and its goal is to design policies that can change the behavior of the system in a desirable way [41]. Bo and Tian, an approximate optimal strategy derived from a stochastic linear quadratic (SLQ) optimal control problem have considered, and a piecewise parameterization and optimization (PPAO) method have proposed. Firstly, using the principle of dynamic programming, the control form of SLQ optimal control problem is relevant to a Riccati differential equation. they presented a PPAO method for finding an approximate optimal strategy for stochastic control problems. Finally, the inventory control problems with different dimensions were used to justify the feasibility of PPAO method, and the results show that parametric control greatly simplifies the control form [42].The rapid expansion of the cold chain market is a key supply chain trend, but its high energy consumption conflicts with low-carbon goals. Yong Wang [43] in 2025 proposes a multi-objective lot sizing procurement (LSP) model for managing the procurement of perishable products in the cold chain. This model, constrained by limited inventory and transportation capacity, aims to optimize multi-period procurement plans and order allocation and minimize total costs and carbon emissions. The proposed multi-objective LSP model adopts a posteriori mode, which contributes to enhancing the model's applicability. Jianhua in 2024 extended the novel concept of meta-inventory by using a theoretical Newsvendor model for original equipment manufacturing (OEM) and own brand manufacturing factories.
Analytical and numerical results showed that a factory achieves better performance by using a separate model since it clarifies the responsibility of digital twins [44]. Finally, according to the literature, in order to optimize the products available in Isfahan Zob Ahan Company, a model of fuzzy multi-objective linear programming was evaluated. Other findings research, through a regular and logical process based on the judgment method in a survey of 14 experts in the field of capital market investment and a quantitative and multivariate model of fuzzy network analysis, to assess the level of importance, ranking and refining the effective factors. Portfolio optimization was undertaken. Based on the analysis, the variables of profit volatility, return on capital, company value, market risk, stock profitability, financial structure, liquidity and survival index can be introduced as the most important factors affecting the optimization of the stock portfolio [45].

3 Methodology

Each The concept of a fuzzy set is an extension of the concept of a crisp set. A crisp set on a universal set U is defined by its characteristic function from U to $\{0, 1\}$. A fuzzy set on a domain U is defined by its membership function from *U* to [0, 1]. Let *U* be a nonempty set, to be called the universal set or the universe of discourse or simply a domain. Then by a fuzzy set on *U* is meant a function A: U: → [0,1]. A is called the membership function, A(x) is called the membership grade of x in (U, A). We also write A= x U = {(A, A (x)): x∈U}.

A linear programming problem with fuzzy values is called fuzzy linear programming problem. In this paper, any fuzzy number is denoted by using ~ above fuzzy number, e.g., $\tilde{C}_j$, $\tilde{a}_{ij}$ etc. Consider a fuzzy linear programming problem as in Equation (1).

$$(\tilde{c}\,, x) = f_i(x_j) = f_i(x) = \min \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{1}$$

subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq \tilde{b}_i \;;\; 1 \leq i \leq m,\ \exists\, x_i > 0$$

The membership functions of a_{ij} a and b_i have been expressed as in Equation (2) and (3), respectively.

$$\mu_{aij}(x) = \begin{cases} 1; & x < a_{ij} \\ \dfrac{a_{ij} + d_{ij} - x}{d_{ij}} & a_{ij} \leq x \leq a_{ij} + d_{ij} \\ 0; & x \geq a_{ij} + d_{ij} \end{cases} \tag{2}$$

$$\mu_{bij}(x) = \begin{cases} 1; & x < b_{ij} \\ \dfrac{b_i + p_i - x}{p_i} & b_i \leq x \leq b_i + p_i \\ 0; & b_i + p_i \leq x \end{cases} \tag{3}$$

Let $\tilde{a}_{ij} = (m_{ij}\,, l_{ij}\,, r_{ij})$ and $\tilde{b}_1 = (d_i\,,\, e_i\,, f_i)$ be fuzzy numbers. Therefore, the constraints in Equation (1) can be modified as in Equation (4).

$$\min \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j \tag{4}$$

subject to

$$\sum (m_{ij}\,, l_{ij}\,, r_{ij}) x_{ij} \leq (d_i\,, e_i\,, f_i) \forall i = 1\text{-}m$$

$$X_j \geq 0\,,\ j = 1\text{-}n$$

Theorem: For any two triangular fuzzy numbers Thakre et al. [8] $A = (s_1, l_1, r_1)$ and $B = (s_2, l_2, r_2)$

$A \leq B$ iff $s_1 \leq s_2$

$$\begin{cases} S_1 - l_1 \leq S_2 - l_2 \\ S1 + r1 \leq S2 + r2 \end{cases} \tag{5}$$

$$\min \tilde{z} = \sum_{j=1}^{n} \tilde{c}_j x_j$$

subject to

$$\begin{cases} \sum m_{ij} x_j \le d_i \\ \sum (m_{ij} - l_{ij}) x_i \le d_i - e_i \quad \forall i \\ \sum (m_{ij} + r_{ij}) x_i \le d_i + f_i \\ \qquad x_i \ge 0 \ (\forall i) \end{cases} \tag{6}$$

where membership function of $\tilde{c}_j$ (x) is

$$\mu_{Cj} = \begin{cases} \dfrac{x - \alpha_i}{\beta_i - \alpha_i} & \alpha_i \le x \le \beta_i \\ \dfrac{\gamma_i - x}{\gamma_i - \beta} & \beta_i \le x \le \gamma_i \\ 0; & \text{elsewhere} \end{cases} \tag{7}$$

4 Findings

This research pertains to the Esfahan Steel Company. The relevant data was obtained from the financial statements of this company in the Codal system between 1398 and 1401 as well as the budget data of 1402.

The variables related to the aforementioned model for the Esfahan Steel Company are as follows:

X_1	:	Beam
X_2	:	Other products
X_3	:	Ingot
X_4	:	Rebar

Objective functions are calculated using the available variables, which are as follows:

Z_1	:	Total Minimum Storage Costs and Working Capital Interest
Z_2	:	Minimum Costs of Lost Orders
Z_3	:	Total Maximum Storage Costs and Working Capital Interest
Z_4	:	Maximum Costs of Lost Orders

The limitations associated with the model include:

1. Limitation on annual production of iron ore requirements up to 4200 thousand tons
2. Limits on the annual supply of required coal to a maximum of 1,400,000 tonnes.
3. The limit on the manufacturing time required to produce products includes 24 million direct hours.

Consider the Fuzzy Multi-Objective Linear Programming Problem (FMOLPP) as in Equation (8).

$$\begin{cases} \text{Min } \tilde{z}^1 = (1248, 1489, 1723)x_1 + (1154, 1363, 1577)x_2 + (993, 1204, 1410)x_3 + (1260, 1494, 1728)x_4 \\ \text{Min } \tilde{z}^2 = (497, 692, 790)x_1 + (328, 369, 390)x_2 + (260, 419, 499)x_3 + (435, 619, 712)x_4 \end{cases} \quad (8)$$

subject to

$$(133, 117, 110)\, x_1 + (68, 23, 20)\, x_2 + (146, 126, 117)\, x_3 + (87, 80, 71)\, x_4 \leq (420, 402, 380)$$

$$(44, 36, 32)\, x_1 + (23, 18, 12)\, x_2 + (49, 30, 25)\, x_3 + (29, 10, 7)\, x_4 \leq (150, 136, 120)$$

$$(10, 9.5, 9)\, x_1 + (6, 5.6, 5.2)\, x_2 + (4, 3.8, 3.5)\, x_3 + (8, 7.6, 7.2)\, x_4 \leq (250000, 242000, 220000)$$

$$X_1, X_2, X_3, X_4 \geq 0$$

where the membership function of $\tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \tilde{C}_4, \tilde{C}_5, \tilde{C}_6, \tilde{C}_7, \tilde{C}_8$ are

$$\mu_{C2}(X) = \begin{cases} \dfrac{x - 1154}{209} & 1154 < x \leq 1363 \\ \dfrac{1577 - x}{214} & 1363 < x \leq 1577 \\ 0; & \text{elsewhere} \end{cases} \quad (10)$$

$$\mu_{C1}(X) = \begin{cases} \dfrac{x - 1248}{241} & 1248 < x \leq 1489 \\ \dfrac{1723 - x}{234} & 1489 < x \leq 1723 \\ 0; & \text{elsewhere} \end{cases} \quad (9)$$

$$\mu_{C4}(X) = \begin{cases} \dfrac{x - 1260}{234} & 1260 < x \leq 1494 \\ \dfrac{1728 - x}{234} & 1494 < x \leq 1728 \\ 0; & \text{elsewhere} \end{cases} \quad (12)$$

$$\mu_{C3}(X) = \begin{cases} \dfrac{x - 993}{211} & 993 < x \leqslant 1204 \\ \dfrac{1410 - x}{206} & 1204 < x \leqslant 1410 \\ 0; & \text{elsewhere} \end{cases} \quad (11)$$

$$\mu_{C6}(X) = \begin{cases} \dfrac{x-328}{41} & 328 < x \leq 369 \\ \dfrac{390-x}{21} & 369 < x \leq 390 \\ 0; & \text{elsewhere} \end{cases} \quad (14)$$

$$\mu_{C5}(X) = \begin{cases} \dfrac{x-497}{195} & 497 < x \leq 692 \\ \dfrac{790-x}{98} & 692 < x \leq 790 \\ 0; & \text{elsewhere} \end{cases} \quad (13)$$

$$\mu_{C8}(X) = \begin{cases} \dfrac{x-435}{184} & 435 < x \leq 619 \\ \dfrac{712-x}{93} & 619 < x \leq 712 \\ 0; & \text{elsewhere} \end{cases} \quad (16)$$

$$\mu_{C7}(X) = \begin{cases} \dfrac{x-260}{159} & 260 < x \leq 419 \\ \dfrac{499-x}{80} & 419 < x \leq 499 \\ 0; & \text{elsewhere} \end{cases} \quad (15)$$

From Equation (8) we get,

$$\begin{cases} \text{Min } z_1 = 1248x_1 + 1154x_2 + 993x_3 + 1260x_4 \\ \text{Min } z_2 = 497x_1 + 328x_2 + 260x_3 + 435x_4 \\ \text{Min } z_3 = 1723x_1 + 1577x_2 + 1410x_3 + 1728x_4 \\ \text{Min } z_4 = 790x_1 + 390x_2 + 499x_3 + 712x_4 \end{cases} \quad (17)$$

Subject to

$$\begin{cases} 133x_1 + 68x_2 + 146x_3 + 87x_4 \leq 420 \\ 16x_1 + 45x_2 + 20x_3 + 7x_4 \leq 18 \\ 243x_1 + 88x_2 + 263x_3 + 158x_4 \leq 800 \\ 44\, x_1 + 23\, x_2 + 49\, x_3 + 29\, x_4 \leq 150 \\ 8\, x_1 + 5\, x_2 + 19\, x_3 + 19\, x_4 \leq 14 \\ 76\, x_1 + 35\, x_2 + 74\, x_3 + 36\, x_4 \leq 270 \end{cases} \quad (18)$$

For the first objective function in Equation (17) with constraints in Equation (18), by applying simplex algorithm, we get:

(0.1992, 0.3681, 0.0800. 0.3527) with φ_1= 0.08703

Similarly, for the second objective function in Equation (17) with constraints in Equation (18), we get

(0.1992, 0.3681, 0.0800. 0.3527) with φ_1= 0. 26049

Similarly for the third objective function in Equation (13) with constraints in Equation (14), we get

(0.1992, 0.3681, 0.0800. 0.3527) with φ_1= 0. 06261

And for last objective function in Equation (13) with constraints in Equation (14), we get

(0.1992, 0.3681, 0.0800. 0.3527) with φ_1= 0. 15739

4.1 Chandra Sen's Method

Chandra Sen's method is a multi-objective optimization technique which is used for making a single objective from multi-objectives. In the last three decades several new multi-objective optimization techniques have been developed.

Applying Chandra Sen's method [17] for making single objective function from a multi objective function, we get:

$$\text{Min } z = \frac{Z1}{\varphi} + \frac{Z2}{\varphi} + \frac{Z3}{\varphi} + \frac{Z4}{\varphi}$$

Min z =0.08703($1248x_1 + 1154x_2 + 993x_3 + 1260x_4$) + 0.26049 ($497x_1 + 328x_2 + 260x_3 + 435x_4$) + 0.06261 ($1723x_1 + 1577x_2 + 1410x_3 + 1728x_4$) + 0.15739 ($790x_1 + 390x_2 + 499x_3 + 712x_4$) = x_1(108.62 + 129.45 + 107.88 + 124.34) + x_2(100.43 + 85.44 + 98.74 + 61.38) + x_3(86.42 + 67.73 + 88.26 + 78.54) + x_4(109.66 + 113.31 + 108.18 + 112.07) = $470.31x_1 + 346x_2 + 320.98x_3 + 443.24x_4$

Thus, the single objective function becomes:

$$\text{Min } z = 470.31x_1 + 346x_2 + 320.98x_3 + 443.24x_4 \quad (19)$$

For this objective function in Equation (19) with constraints in Equation (18), we get

(0.1992, 0.3681, 0.0800. 0.3527) with Z = 400.0001

4.2 Statistical Averaging Method

There are three means known as arithmetic mean, geometric mean and harmonic mean. For ungrouped raw data, the mean is defined as the sum of the objectives divided by the number of observations. It is easy to understand and easy to calculate. If the number of items is sufficiently large, it is more accurate and more reliable. The Geometric mean of a series containing n observations is the nth root of the product of the values. The Harmonic mean of a set of observations is defined as the reciprocal of the arithmetic average of the reciprocal of the given values .Applying the arithmetic mean, geometric mean and harmonic mean among $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ we proceed as follows:

$$\text{A.M} = \frac{0.08703+0.26049+0.06261+0.15739}{4} = 0.1419$$

$$G.M = \sqrt[4]{0.08703 * 0.26049 * 0.06261 + 0.15739} = 0.1223$$

$$H.M = \frac{4}{\frac{1}{0.08703}+\frac{1}{0.26049}+\frac{1}{0.06261}+\frac{1}{0.15739}} = \frac{4}{11.49+3.84+15.97+6.35} = 0.1062$$

Arithmetic averaging method:

$$\text{Min } z = \frac{\mathbf{A.M}}{\mathbf{1}} (z_1+z_2+z_3+z_4) \tag{20}$$

Min z =0.1419(1248x_1 + 1154x_2 + 993x_3 + 1260x_4 + 497x_1 + 328x_2 + 260x_3 + 435x_4 + 1723x_1 + 1577x_2 + 1410x_3 + 1728x_4 + 790x_1 + 390x_2 + 499x_3 + 712x_4) = 0.1419 (4258x_1 + 3449x_2 + 3162x_3 + 4135x_4) = 604x_1 + 489x_2 + 449x_3 + 587x_4

Thus, the single objective function becomes:

Min z = 604x_1 + 489x_2 + 449x_3 + 587x_4

For this objective function in Equation (16) with constraints in Equation (18), we get

(0.1992, 0.3681, 0.0800. 0.3527) with Z = 534.2369

Geometric averaging method:

$$\text{Min } z = \frac{\boldsymbol{G.M}}{\mathbf{1}} (z_1+z_2+z_3+z_4) \tag{21}$$

Min z = 0.1223 (4258x_1 + 3449x_2 + 3162x_3 + 4135x_4) =521x_1 + 422x_2 + 387x_3 + 506x_4

Thus, the single objective function becomes:

Min z = 521x_1 + 422x_2 + 387x_3 + 506x_4

For this objective function in Equation (21) with constraints in Equation (18), we get:

(0.1992, 0.3681, 0.0800. 0.3527) with Z = 460.3483

Harmonic averaging method:

$$\text{Min } z = \frac{\boldsymbol{H.M}}{\mathbf{1}} (z_1+z_2+z_3+z_4) \tag{22}$$

Min z =0.1062 (4258x_1 + 3449x_2 + 3162x_3 + 4135x_4) =452x_1 + 366x_2 + 336x_3 + 439x_4

Thus, the single objective becomes:

Min z = 452x_1 + 366x_2 + 336x_3 + 439x_4

For this objective function in Equation (22) with constraints in Equation (18), we get:

(0.1992, 0.3681, 0.0800. 0.3527) with Z = 400.0001

Table 1 shows the comparison between the Chandra Sen's method and the statistical averaging methods. Here, the statistical averaging method consists of arithmetic mean, geometric mean and harmonic mean. The statistical averaging method gives a better result than Chandra Sen's method. These methods are used for obtaining a single objective function from multi-objective functions.

4.3 New Statistical Averaging Method

By Choosing the minimum from the optimal values of the maximum type in Chandra Sen's method we get

$$\text{Min } z = 0.26049\,(z_1 + z_2 + z_3 + z_4) \quad (23)$$

$$\text{Min } z = 0.26049\,(4258x_1 + 3449x_2 + 3162x_3 + 4135x_4) = 1109x_1 + 898x_2 + 824x_3 + 1077x_4$$

Thus the single objective becomes

$$\text{Min } z = 1109x_1 + 898x_2 + 824x_3 + 1077x_4$$

For this objective function in Equation (23) with constraints in Equation (18), we get

(0.1992, 0.3681, 0.0800. 0.3527) with Z = 980.8376

We can find a single objective function from multi objective functions by using any method among Chandra Sen's method, the statistical averaging method, and the new statistical averaging method.

Table2 shows that the statistical averaging method and the new statistical averaging method provide better optimization than Chandra Sen's method.

Table 1: Comparison between Chandra Sen's method and Statistical Averaging Method.

Chandra Sen's method	Arithmetic mean	Geometric mean	Harmonic mean
400.0001	534.2369	460.3483	400.0001

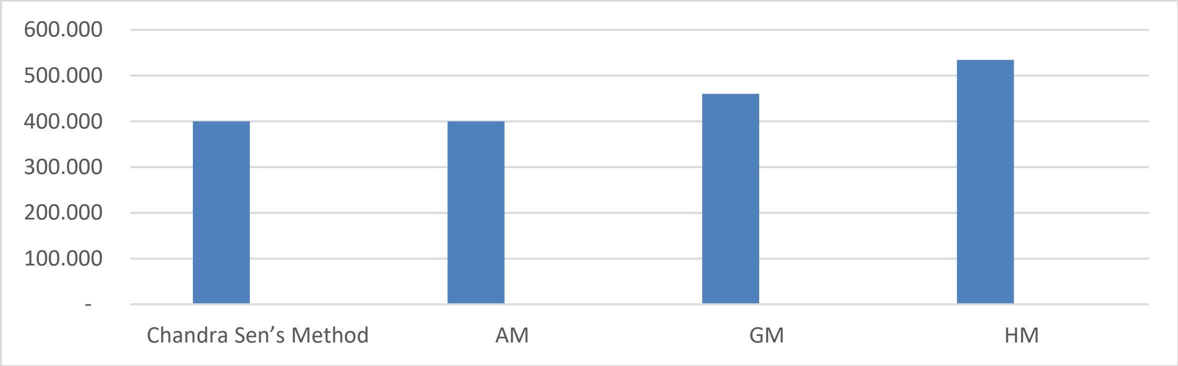

Fig. 1: Optimal values of FLPP in Chandra Sen's Method and Statistical Averaging Method.

Researcher's findings

Table 2: Comparison among Chandra Sen's Method, Statistical Averaging Method, and New Statistical Averaging Method.

Chandra Sen's Method	Statistical Averaging Method (SAM)	New Statistical Averaging Method (NSAM)
$Z_{MIN} = 400.0001$ with (0.1992 , 0.3681 , 0.0800 . 0.3527)	$Z_{MIN} = 534.2369$ with (0.1992 , 0.3681 , 0.0800 . 0.3527)	$Z_{MIN} = 980.8376$ with (0.1992 , 0.3681 , 0.0800 . 0.3527)
	Researcher's findings	

The models are solved using the LINGO software

5 Discussion and Conclusions

In this paper, a multi-objective fuzzy linear programming problem is solved using the multi-age method, statistical averaging method, and the new statistical method of averaging. Our goal was to find the optimum weight of each of the products of the Esfahan Steel Company under conditions of uncertainty. By optimizing the amount of inventory, storage costs and working capital gains are reduced, although the risk of doing so can affect the economic unit's flexibility against unexpected orders. For the production of products, the Zob Ahan Company has three basic limitations, which are: 1. Limitation of Iron Ore 2. Limitation on Flea, 3. Limitations on Manufacturing Specialist Force. After estimating the model, the results of the studies show that the highest weight of production for the ingot product is the product that has a good domestic and foreign sales market and the lowest weight is for other products. In order to reduce storage costs and working capital interest, and also to reevaluate the production processes, warehousing and production planning in order to reduce storage costs and working capital interest and also to provide appropriate flexibility against unexpected orders. This research is based on dynamic model and using fuzzy multi-objective method and is a case study specific to the Isfahan Steel Company. It should be noted that in order to optimize the inventory value, we have determined data on storage costs, working capital interest, lost orders, iron ore and coke rate and time of production of products as fuzzy parameters which are triangular fuzzy numbers. The solutions obtained by using statistical and new statistical averaging method have performed better than the Chandrasen method and lead to further reduction in expenditures. For future research, it is suggested that this research be reviewed for order-based companies and evaluate the optimal amount of inventory according to project production and non-uniformity of products.

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