Profit Efficiency Evaluation: A composed Approach of DEA and multi- objective programming

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Abstract:

Data envelopment analysis (DEA) is a nonparametric method for evaluating the relative efficiency of decision making units (DMUs) described by multiple inputs and multiple outputs. The issue of measuring the cost, revenue and profit efficiency in manufacturing and economic systems is one of the most important issues for managers. In this research, using Data envelopment analysis and multi-objective programming an attempt is made to provide a model for evaluating profit efficiency of banking industry. We apply data envelopment analysis (DEA) and multi-objective programming (MOP) models to measure profit efficiency as cost and revenue scores are as close as possible to their best scores and as far away as possible to their worst scores. The results showed that composing these two models, can directly affect the result and also findings of research distinguished the differences between the efficient DMUs from the point of view of DEA. In this study, Profit efficiency score has been obtained from a fairer perspective than the previous models. A numerical example of Iranian banking industry is used to illustrate the proposed model.

١ - Introduction

Data envelopment analysis (DEA) has been originated for measuring the relative efficiencies of a set of homogeneous decision making units (DMUs) that applies multiple inputs to generate multiple outputs. Currently, there has been a growing interest among decision makers in application of non-parametric techniques like DEA which extends markedly beyond the task of evaluating cost and revenue efficiency. Cost efficiency was first pioneered by Farrell [٤] and then extended by Fare et.al [٥]. The following approach for modeling cost efficiency goes back to Camanho and Dyson [٣]. The authors developed the traditional cost efficiency model into two various situations including precise known prices and incomplete price situations. Their model estimated upper and lower bound for cost efficiency evaluation in presence of price uncertainty. Jahanshahloo et.al [١٠] continued their debate and refined the model with reducing the number of constraints and variables. In this respect, Jahanshahloo et.al [١١] offered an interpretation of cost models and introduced an alternative model foe assessment of cost efficiency assuming that the input prices of each DMU are accessible. In this framework, Amirteimoori et.al [١] improved the cost efficiency interval of a DMU by adjusting its observed inputs and outputs. Camanho and Dyson [٣] contributed to this topic by obtaining cost efficiency from optimistic and pessimistic

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viewpoints including uncertain price. Considering uncertain price, Toloo and Ertay [١٦] applied an alternative cost efficiency model based on DEA approach posits finding the most efficient unit. The concept of revenue efficiency was first debated in Fare et.al [٥]. In their ١٩٩٤ study [٦], the authors improved the overall output price with the goal of maximizing revenue. As another instance, Fukuyama and Matousek [٨] expanded the environmental revenue function based on directional distance function in two-stage network structures. Another recent studies in revenue efficiency, Mogaddas and Vaez Ghasemi [١٢] applied DEA approach to compute a specific set of weights to evaluate cost efficiency in a two-stage network system. A review of the DEA literature demonstrates that Fare et.al's [٧] studies are widely recognized as a seminal reference in profit efficiency research. Fare et.al [٧] investigated two sources of inefficiency in assessing profit efficiency, including technical inefficiency and allocative inefficiency. Portela and Thanassoulis [١٤] highlighted the drawbacks of existing approaches in the literature and suggested another measure of profit efficiency which is grounded in the geometric mean of input/output adjustments to achieve maximum profitability. Several researchers have proposed methods to address profit efficiency. A new indicator of profit inefficiency was suggested by Fukuyama and Weber [٩] emphasizing the choices made by decision-makers regarding the allocation of funds to inputs and the revenue derived from outputs, rather than the physical measurement of input and output quantities. Park and Cho [١٣] introduced a linear programming model, for the evaluation of profit efficiency. The main focus of their paper was on approximation of profit efficiency in the absence of price information. Aparicio et.al [٢] illustrated the utility of DEA for measuring and decomposing revenue inefficiency. Their study considered all sources of technical waste with a specific emphasis on the Spanish quality waste sector. In a current study of all industries, the utilization of profit efficiency is becoming increasingly crucial especially in bank branch activities. In analyzing the literature, it is evident that several studies have advocated profit efficiency from the optimistic perspective. With respect to non-parametric Data Envelopment Analysis (DEA) models, the attention of this study has been given to both optimistic and pessimistic standpoint. This study examines the reasonable and equitable amount for profit regarding to costs incurred and generated revenues. The remainder of this study is organized as follows. Section ٢ provides a brief overview of cost, revenue and profit efficiency. Then Section ٣ formulates an alternative model for assessing profit efficiency as an MOP task. To clarify the details of the proposed method, a real case in banking sector is given in Section ٤. Eventually, Section ٥ concludes the paper.

٢. Cost, Revenue and Profit efficiency

According to Farrell [٤] efficiency consist of two elements: technical efficiency (TE) and allocative efficiency (AE). TE refers to production where the best available technologies are applied and AE refers to allocation of inputs and products to different producers. Together, these efficiencies are named the economic efficiency, (EE defined. The (EE) is expressed in). The (EE) is expressed is different manners, depending on how the best available production technology is terms of cost minimization, revenue maximization or profit maximization. If cost minimization is assumed, The (EE) is expressed as (CE). In this case, CE constitutes a combination of inputs that generates the minimum possible cost. In a similar manner, the (EE) is expressed as (RE), RE constitutes a combination of outputs that generates the maximum possible revenue and if maximization of profit is of concern, the (EE) is expressed as profit efficiency (PE), that is, the amount of output that maximizes profit [١٥].

٢-١ Cost efficiency

Suppose that there is a set of n decision-making units DMU_j $(\mathrm{j} = ١. \dots . \mathrm{n})$. Let m, s be the numbers of inputs and outputs respectively. The term $\mathrm{x_{ij}} \in \mathrm{R}^{+}(i = ١. \dots . \mathrm{m} ; \mathrm{j} = ١. \dots . \mathrm{n})$ is applied in the input resource i to DMU_j to produce the output$\mathrm{y_{rj}} \in \mathrm{R}^{+}(r = ١. \dots . \mathrm{s} ; \mathrm{j} = ١. \dots . \mathrm{n})$, that is, the output product r fromDMU_j. Also let the unit price of all input be known, and $\mathrm{c_{ij}} \in \mathrm{R}^{+}$ shows the price of inputi fromDMU_j. Given these assumptions, the cost efficiency model can be written as follows:

$$
\begin{aligned}
\min \quad & \sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{c_{io}}\, \bar{\mathrm{x}}_{\mathrm{io}} \\
\mathrm{s.t.} \quad & \sum_{\mathrm{j}=١}^{\mathrm{n}} \lambda_{\mathrm{j}} \mathrm{x_{ij}} \le \bar{\mathrm{x}}_{\mathrm{io}} && \mathrm{i} = ١. \dots . \mathrm{m}; && (١) \\
& \sum_{\mathrm{j}=١}^{\mathrm{n}} \lambda_{\mathrm{j}} \mathrm{y_{rj}} \ge \mathrm{y_{ro}} && \mathrm{r} = ١. \dots . \mathrm{s}; \\
& \lambda_{\mathrm{j}} . \bar{\mathrm{x}}_{\mathrm{io}} \ge ٠ .
\end{aligned}
$$

Model (١) is a constant return to scale (CRS), the observed cost obtained through $\mathrm{DMU_o}$ is presented as $\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{c_{io}}\, \mathrm{x_{io}}$. The cost efficiency of $\mathrm{DMU_o}$ $(\mathrm{CE_o})$ is measured through :

$$
\mathrm{CE_o} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{c_{io}}\, \bar{\mathrm{x}}_{\mathrm{io}}^{*}}{\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{c_{io}} \mathrm{x_{io}}}
$$

Where $\bar{\mathrm{x}}_{\mathrm{io}}{}^{*}$ is the optimal solution of model (١).

٢-٢ Revenue efficiency

Let $\mathrm{p_{ro}}$ be the price of the under evaluated unit (DMU_o) output r, then DEA model of revenue maximization is:

$$
\begin{aligned}
R_o^{*} = \max & \sum_{r=1}^{s} p_{ro} \bar{y}_{ro} \\
s.t. \quad & \sum_{j=١}^{n} \lambda_j x_{ij} \le x_{io} && i = ١. \dots . m; && (٢) \\
& \sum_{j=١}^{n} \lambda_j y_{rj} \ge \bar{y}_{ro} && r = ١. \dots . s; \\
& \lambda_j . \bar{y}_{ro} \ge ٠
\end{aligned}
$$

Model (۲) is a constant return to scale (CRS). The revenue obtained through the DMU_o is equal to $\sum_{r=1}^{s} p_{ro} y_{ro}$. The revenue efficiency of DMU_o (RE_o) is measured through:

$$RE_o = \frac{\sum_{r=1}^{s} p_{ro} y_{ro}}{\sum_{r=1}^{s} p_{ro}\, \bar{y}_{ro}^{*}}$$

۲-۳ Profit efficiency

According to the assumptions of the previous two parts, the profit maximization problem is solved as follows:

$$\begin{aligned}
max \quad & \sum_{r=1}^{r_k} p_{ro}\bar{y}_{ro} - \sum_{i=1}^{m_k} c_{io}\bar{x}_{io} \\
s.t. \quad & \sum_{j=۱}^{n} \lambda_j x_{ij} \leq \bar{x}_{io} \qquad i = ۱. \dots . m \\
& \sum_{j=۱}^{n} \lambda_j y_{rj} \geq \bar{y}_{ro} \qquad r = ۱. \dots . r \qquad\qquad (۳) \\
& \bar{x}_{io} \leq x_{io} \,.\, \bar{y}_{ro} \geq y_{ro} \\
& \lambda_j \geq ۰ .
\end{aligned}$$

Where c_{io} is the price of input i and p_{ro} is the price of output r of DMU_o . The profit obtained by the DMU_o is $\sum_{r=۱}^{r_k} p_{ro} y_{ro} - \sum_{i=۱}^{m_k} c_{io} x_{io}$ and profit efficiency (PE_o) of is measure as follows:

$$PE_o = \frac{\sum_{r=1}^{r_k} p_{ro} y_{ro} - \sum_{i=1}^{m_k} c_{io} x_{io}}{\sum_{r=1}^{r_k} p_{ro}\, \bar{y}_{ro}^{*} - \sum_{i=1}^{m_k} c_{io}\, \bar{x}_{io}^{*}}$$

۳ – Proposed Method

In all previous studies, profit efficiency has been considered from the optimistic point of view. The purpose of this study is to measure the most appropriate amount of cost and revenue simultaneously in a way that yields the best profit to the decision maker. To achieve the fairest and the most appropriate amount of profit, we present a combination of data envelopment analysis (DEA) and multi-objective programming(MOP). Models (۱) and (۲) provide minimum cost (C_o^+) and maximum revenue (R_o^+) as the best values of cost and revenue. With similar perspective, to obtain the worst values, i.e., maximum cost (C_o^-) and minimum revenue (R_o^-), it is sufficient to maximize model (۱) and minimize model (۲) respectively. The aim of this study is to obtain fair and reasonable values of cost and revenue that is as close as possible to the most optimistic values C_o^+ and R_o^+ and sufficiently far from values C_o^- and R_o^-. Considering that the objective function of model (۳) can be rewritten as a two-objective function as follows:

$$\max \quad -\sum_{i=1}^{m} c_{io}\,\bar{x}_{io}$$

(٤)

$$\max \ \sum_{r=1}^{s} p_{ro}\bar{y}_{ro}$$

Therefore, multi-objective planning models can be used to achieve the fairest amount of profit according to the amount of cost and revenue and objective function (٤). The multi-objective model used in this study is the Min-Max weighted model [٢]. Using the multi-objective function (٤) and the limitations of model (٣), the following model is proposed:

$$min\left[max\left\{\mu_{١}\left(C_o^+ - \sum_{i=١}^{m_k} c_{io}x_{io}\right).\mu_{٢}\left(R_o^+ - \sum_{r=١}^{r_k} p_{ro}y_{ro}\right)\right\}\right]$$

$$s.t. \quad \sum_{j=١}^{n} \lambda_j x_{ij} \le \bar{x}_{io} \qquad i = ١.\dots.m_k$$

$$\sum_{j=١}^{n} \lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = ١.\dots.r_k \qquad (٥)$$

$$\bar{x}_{io} \le x_{io} \,.\, \bar{y}_{ro} \ \ge y_{ro}$$

$$\lambda_j \ \ge ٠.$$

The weights defined in Model (٥) ($\mu_{١},\mu_{٢}$) are positive parameters that generate the search to obtain the most appropriate cost and revenue. To direct the search in the line from the best value to the worst value, we define the weights as follows:

$$\mu_{١} = \frac{١}{C_o^+ - C_o^-}$$

(٦)

$$\mu_{٢} = \frac{١}{R_o^+ - R_o^-}$$

Using the definitions of the above weights $\mu_{١}$ and $\mu_{٢}$, model (٥) is changed to model (٧):

$$min\delta$$

$$s.t$$

$$\frac{C_o^+ - \sum_{i=1}^{m_k} c_{io}x_{io}}{C_o^+ - C_o^-} \le \delta$$

$$\frac{R_o^+ - \sum_{r=1}^{r_k} p_{ro} y_{ro}}{R_o^+ - R_o^-} \le \delta$$

$$\sum_{j=۱}^{n} \lambda_j x_{ij} \le \bar{x}_{io} \qquad i = ۱.\ldots.m_k \qquad\qquad (۷)$$

$$\sum_{j=۱}^{n} \lambda_j y_{rj} \ge \bar{y}_{ro} \qquad r = ۱.\ldots.r_k$$

$$\bar{x}_{io} \le x_{io} \,.\, \bar{y}_{ro} \ge y_{ro}$$

$$\lambda_j \ge ۰$$

The first and second constraints in model (۷) guarantee a fair cost and revenue that is as close as possible to the most optimistic C_o^+ and R_o^+ and sufficiently far from C_o^- and R_o^-. In other words, by integrating these two constrains in Model (۳) and defining appropriate weights $\mu_۱$ and $\mu_۲$, the objective function of Model (۷) provide the most appropriate and fair amount of profit for each decision unit. Parameter δ on the right side of the first and second constrains actually represents the minimum distance between the best value and the worst value. According to the minimization of this parameter in the objective function of model (۷), it can be concluded that parameter δ can be a suitable criterion for determining the difference between efficient DMUs from the point of view of model (۳).

Definition ۱: The unit $\mathrm{DMU_o}$ in the evaluation with model (۷) is considered efficient if the optimal value of the model is equal to ۱.

٤- Numerical Example

We apply the proposed model to ten Iranian banks. Input and output indicators have been considered according to past researches and experts' opinions. The total amount of deposits ($x_۱$), operating expenses ($x_۲$) and facilities ($x_۳$)as input indicators and revenue from commissions ($y_۱$), annual net profit ($y_۲$) and transactions ($y_۳$) are considered as output indicators. The input and output data are given in Table ۱.

Table ۱. ***Data set of inputs and outputs***

DMU	$x_۱$	$x_۲$	$x_۳$	$y_۱$	$y_۲$	$y_۳$
۱	۰,۹٤۸	۱,۰۰۰	۰,۳۳۷	۰,۸۷۹	۰,٤۳۷	۰,٥۳۷
۲	۱,۳۳۰	۰,۹۹۳	۰,۱۸۰	۰,٥۳۸	۰,۲۸۲	۰,۲۸۰
۳	۰,٦۲۱	۰,٦۷٥	۰,۱۹۸	۰,۹۱۱	۰,۰۹۸	۰,٦٥۸

٤	١,٧٨٣	٠,٨٩٧	٠,٤٩١	٠,٥٧٠	٠,٣٩١	٠,٤٦١
٥	١,٨٩٢	١,٢٩٠	٠,٣٧٢	١,٠٨٦	٠,٤٧٢	٠,٣٧٢
٦	٠,٩٩٠	٠,٨٥٦	٠,٢٥٣	٠,٧٢٢	٠,٢٦٣	٠,١٥٣
٧	٠,١٥١	٠,٩٨٧	٠,٢٤١	٠,٥٠٩	٠,١٣١	٠,٤٤١
٨	٠,١٠٨	٠,٢٠٣	٠,٠٩٧	٠,٦١٩	٠,٠٩٧	٠,٢٦٧
٩	١,٣٦٤	٠,٤٣٢	٠,٣٨٠	١,٠٢٣	٠,٣٨٠	٠,٤٧٠
١٠	١,٩٩٢	٠,٩٥٦	٠,١٧٨	٠,٧٦٩	٠,١٧٦	٠,٢٨٨

By implementing the cost and revenue efficiency models, namely models (١) and (٢), the minimum cost (C_o^+) and maximum revenue (R_o^+) values are shown in the second and third columns of Table ٢. To obtain the worst possible value for cost and revenue, models (١) and (٢) are re-solved with maximization and minimization objective functions, respectively. The results of the maximum cost and minimum revenue, which are indicated by C_o^- and R_o^-, are shown in the fourth and fifth columns of table (٢), respectively. Column ٦ of Table ٢ shows profit efficiency using model (٣) and column ٧ shows profit efficiency according to the new constraints in model (٧).

Table ٢: ***Results of Numerical Example***

DMU	C_o^+	R_o^+	C_o^-	R_o^-	profit	Profit new	δ
١	٠,٩٢١	٨,٦٠٢	١,٥٠١	٠	-.٠٣٩	٠,٦٩٨	٠,٠٧
٢	٠,٥٢٣	٤,٥٩٥	٠,٨٠٢	٠	-٠,٨٣٥	٠,٤٩٧	٠,١٢
٣	٠,٩٣٤	٥,٠٥٤	١٣,٣٨٦	٠	١	١	٠,٠٦
٤	١,٢٣١	١٠,٩٤١	١١,٩٣٢	٠	-.٠٦٩٥	٠,٩٩٥	٠,٠٨

٥	١,٠٦٩	٩,٤٩٦	٩,٦٢٨	٠	-.٠٤١١	٠,٩٣٧	٠,٠٦
٦	٠,٧٢٠	٦,٤٥٨	٣,٩٦٠	٠	-٠,٢٠١	١	٠,٠٩
٧	٠,٦٠٤	٣,٤٦٢	١١,٤١٤	٠	١	١	٠,٠٧
٨	٠,٤٣٢	٢,٤٧٦	٦,٩١١	٠	١	١	٠,٠٠
٩	١,٠٥٣	٥,٢٦٩	١٢,١٦٥	٠	١	١	٠,٠٥
١٠	٠,٦٢٥	٤,٥٤٤	٧,٤٥٤	٠	-١,٢٣٠	٠,٥٦٧	٠,٠٧

The last column of table ٢ shows the value of the parameter δ, the minimum distance between C_o^+ and C_o^- and also between R_o^+ and R_o^-. According to the results of model (٧) in table (٢), the seventh column of table (٢), five units ٣, ٦, ٧, ٨ and ٩ have profit efficiency equal to one. In other words, these units are efficient. But in the evaluation with model (٣), only four units ٣, ٧, ٨ and ٩ have been evaluated as efficient. But the strength of model (٧) is the existence of the criterion δ to identify which unit has performed better than the others among the efficient units. Considering that the parameter δ is the smallest distance between C_o^+ and C_o^- and also between R_o^+ and R_o^-, so by referring to the results of the last column of table (٢) and its zero value, it can be concluded that among the efficient units Unit ٨ has performed better.

٥ - *Conclusions*

Although many studies have been proposed for profit efficiency in data envelopment analysis, but all studies are based on the best profit or optimistic profit. In this study, by combining data envelopment analysis models and multi-objective models, a model to obtain the best amount of profit efficiency is suggested. The basis of this model is based on obtaining the smallest distance between the minimum cost and the maximum cost, as well as the smallest distance between the maximum income and the minimum income. One of the strengths of the proposed model is to identify units with better performance among profit efficient units according to the amount of cost and income. Also, according to the amount of cost and income, we can identify the factors of profit inefficiency and try to solve them.

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