Ranking of fuzzy numbers based on angle measure

Mahnaz Barkhordari Ahmadi
Department of Mathematics, Bandar Abbas Branch,
Islamic Azad University, Bandar Abbas , Iran.
mahnazbarkhordarii@gmail.com

ABSTRACT

In this paper, a novel approach for ranking fuzzy numbers based on the angle measure is introduced. Several left and right spreads at each chosen $\alpha-$levels of fuzzy numbers is used to determine center of mass points(CMPs) and then, the angels between the CMPs and the horizontal axis is calculated. The total angle is determined by averaging the computed angles and finally, the novel method is compared with other methods by solving some numerical examples.

Key words: Rankingof fuzzy numbers; center of mass; Angle measure

1. Introduction

In many decision making procedures such as [19, 28], two or more than two quantity must be compared and hence, in fuzzy environment, ranking of fuzzy numbers is a very important decision making procedures. In the first proposed method, Jain[20, 21] employed the concept of maximizing set to order the fuzzy numbers in 1976 and 1978 and after his work, many authors have investigated various ranking methods[1, 3, 4, 5, 7, 10, 11, 12, 13, 14, 16, 17, 18, 22, 24, 25, 26, 27, 31, 32, 33]. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [8], and more recently by Chen and Hwang [9]. Some of important and applicable contributions in this field include: an index for ordering fuzzy numbers defined by Choobineh and Li [10], ranking alternatives using fuzzy numbers studied by Dias [16], automatic ranking of fuzzy numbers using artificial neural networks proposed by Requena et al. [27], ranking fuzzy values with satisfaction function investigated by Lee et al. [22]. Ranking and defuzzification methods based on area compensation presented by Fortemps and Roubens [18], and ranking alternatives with fuzzy weights using maximizing set and minimizing set given by Raj and Kumar [25]. Abbasbandy and Asady give method to rank the fuzzy numbers by' sign distance [1] However, some of these methods are computationally complex and difficult to implement, and others are counterintuitive and not discriminating. Furthermore, many of them produce different ranking outcomes for the same problem[12].

In spite of many ranking methods, no one can rank fuzzy numbers with human intuition consistently in all cases, in this work, we propose a simple ranking method for triangular (trapezoidal) numbers, which associates an intuitive geometrical representation (near to the center of gravity). The proposed ranking method has basic mathematical properties. It does not imply much computational effort and does not require a priori knowledge of the set of all alternatives.

The rest of Paper is organized as follows. Section 2, contains the basic definitions and notations used in the remaining parts of the paper. In Section 3, we introduce the ranking approach based angle measure and describes some useful properties. In Section 4, solving some examples and compare angle measure with other methods. Concluding remarks are finally made in section 5.

2. Basic definitions and notation

A real fuzzy number can be defined as a fuzzy subset of the real line R, which is convex and normal. That is, for a fuzzy number A of R defined by the membership function $\mu_A(x)$, $x \in R$, The following relations exist:

$$\max_x \mu_A(x) = 1$$

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\},$$

Where x_1, $x_2 \in R$, $\forall \lambda \in [0,1]$. A fuzzy number A with the membership function $\mu_A(x)$, $x \in R$ the fuzzy number $A = [a, b, c, d; 1]$ can be defined as

$$\mu_A(x) = \begin{cases} \mu_A^L(x) & a \le x \le b \\ 1 & b \le x \le c \\ \mu_A^R(x) & c \le x \le d \\ 0 & otherwise, \end{cases}$$

Where $\mu_A^L(x)$ is the left membership function that is an increasing function and $\mu_A^L : [a,b] \rightarrow [0,1]$. Meanwhile, $\mu_A^R(x)$ is the right membership function that is an decreasing function and $\mu_A^R : [c,d] \rightarrow [0,1]$. For a fuzzy number A, the $\alpha-$cuts (level sets) $A_\alpha = \{x \in R \mid \mu_A(x) \ge \alpha\}$, $\alpha \in [0,1]$, are convex subsets of R. The lower and upper limits of the $k^{th}\alpha - $ cut for the fuzzy number A_i are defined as

$$l_{ik} = \inf\{x \in R \mid \mu_A(x) \ge \alpha_k\}$$

$$r_{ik} = \sup\{x \in R \mid \mu_A(x) \ge \alpha_k\}$$

respectively, where l_{ik} and r_{ik} are left and right spreads, and $\alpha_k = \dfrac{k}{n}$, for $k = 0,\dots,n$ [13].

3. Ranking of fuzzy numbers by the angle measure

In this Section the angle measure method (AMM) is described. AMM rank fuzzy numbers based on the average of angles between horizontal axis and the line that joins the origin and CMPs of each fuzzy number.

Let $A_i \quad i = 0,\dots I$ are fuzzy numbers, to define the angle measure of A_i, the following steps should be done:

- Left and right spreads at $k \quad \alpha-$levels of fuzzy numbers, l_{ik}, r_{ik} is calculated .

- CMPs of each fuzzy numbers A_i which denoted by (x_{ik}, y_{ik}) is determined as bellow :

$$\overline{m}_{A_{ik}} = \int_{l_{ik}}^{r_{ik}} x\alpha_{ik} dx = \frac{1}{2}\alpha_{ik}(r_{ik}^2 - l_{ik}^2)$$

$$m_{A_{ik}} = \int_{l_{ik}}^{r_{ik}} \alpha_{ik} dx = \alpha_{ik}(r_{ik} - l_{ik})$$

$$x_{ik} = \frac{\overline{m}_{A_{ik}}}{m_{A_{ik}}}$$

$$y_{ik} = \alpha_{ik}$$

- θ_{ik} of the $k^{th}\alpha - $ cuts for the fuzzy number A_i is determined as

$$\theta_{ik} = \arctan(\frac{y_{ik}}{x_{ik}})$$

- Total angle is calculated as following

$$\theta'_i = \frac{\sum_{k=0}^{n} \theta_{ik}}{n+1} \tag{1}$$

$$\theta_i = \begin{cases} \theta'_i & \theta'_i \ge 0 \\ \theta'_i + 180 & \theta'_i < 0 \end{cases}$$
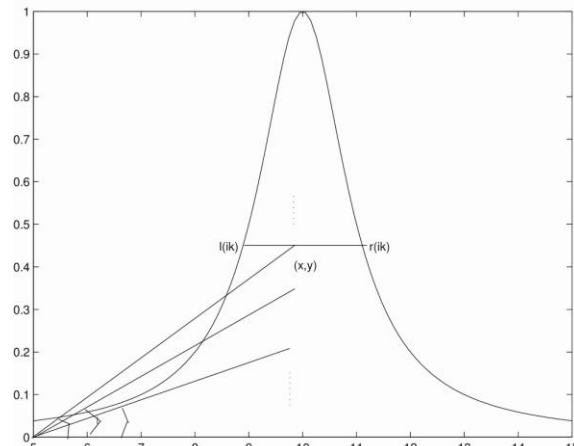

Figure 1. Angles for some different $\alpha-$levels of fuzzy numbers. An arbitrary (x_{ik}, y_{ik}) is determined.

Definition 3.1 For A_i and $A_j \in E$, define the ranking of A_i and A_j by θ_i and θ_j on E, i.e.

(1) if $\theta_i > \theta_j$ then $A_i < A_j$

(2) if $\theta_i < \theta_j$ then $A_i > A_j$

(3) if $\theta_i = \theta_j$ then $A_i : A_j$

The smaller θ_i, the larger the fuzzy number.

3.1. Properties

We consider the following reasonable properties for the ordering approaches, see [30].

A_1 : For an arbitrary finite subset Γ of E and $A \in \Gamma$, $A \pm A$.

A_2 : For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, $A \pm B$ and $B^\circ A$, we should have $A \approx B$.

A_3: For an arbitrary finite subset Γ of E and $(A,B,C) \in \Gamma^3$, $A \pm B$ and $B \pm C$, we should have $A \pm C$.

A_4: For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, $\inf supp(A) \geq \sup supp(B)$, we should have $A \pm B$.

A_4': For an arbitrary finite subset Γ of E and $(A,B) \in \Gamma^2$, $\inf supp(A) > \sup supp(B)$, we should have $A \succ B$.

A_5: Let Γ and Γ' be two arbitrary finite subsets of E in which A and B are in $\Gamma \cap \Gamma'$. We obtain the ranking order $A \succ B$ by (1) on Γ' if and only if $A \succ B$ by (1) on Γ.

Remark 3.1 *A novel method based on definition 3.1 has the properties $A_1, A_2, A_3, A_4, A_4', A_5$.*
Proof: It is easy to verify that the properties $A_1, A_2, A_3, A_4, A_4', A_5$ are hold.

Remark 3.2 *The simple form of ranking formula for triangular fuzzy number is as follow:*

$$x_{ik} = \frac{1}{2}(a+c)(1-\alpha_{ik}) + b\alpha_{ik} \qquad and \qquad y_{ik} = \alpha_{ik}$$

$$\theta_{ik} = \arctan(\frac{1}{2}(a+c)(\frac{1}{\alpha_{ik}}-1)+b)$$

4. Numerical Examples
Example 4.1.
Consider the following sets, see Yao and Wu [31].
Set 1: A=(0. 4,0. 5,1), B=(0. 4,0. 7,1), C=(0. 4,0. 9,1).
Set 2: A=(0. 3,0. 4,0. 7,0. 9), B=(0. 3,0. 7,0. 9), C=(0. 5,0. 7,0. 9).
Set 3: A=(0. 3,0. 5,0. 7), B=(0. 3,0. 5,0. 8,0. 9), C=(0. 3,0. 5,0. 9).
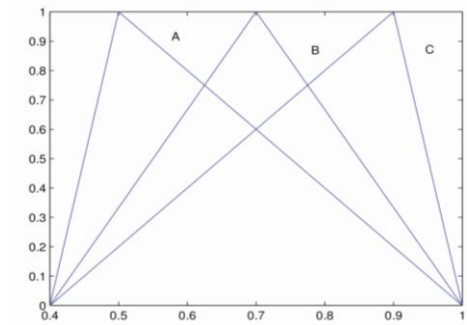Set 4: A=(0,0. 4,0. 7,0. 8), B=(0. 2,0. 5,0. 9), C=(0. 1,0. 6,0. 8).
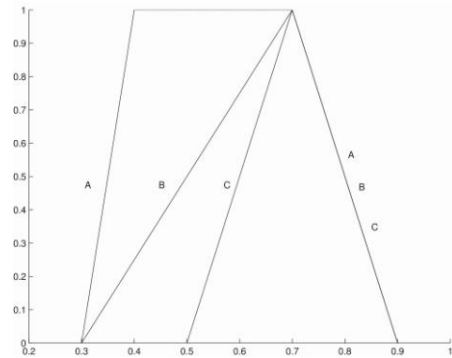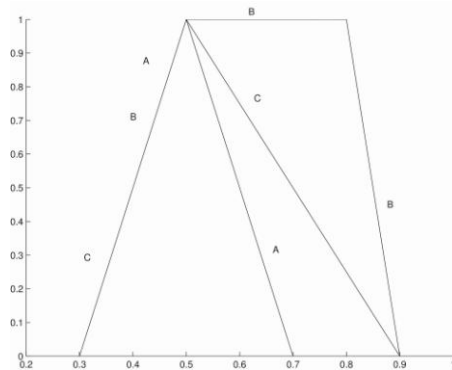

Figure 2. Set 1.

Figure 3. Set 2.

Figure 4. Set 3.
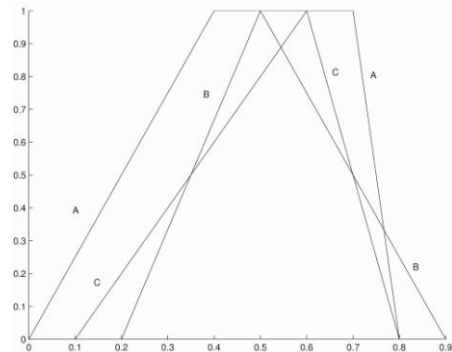
Figure 5. Set 4.

Table 1. Comparative results of Example 4.1

Authors	Fuzzy number	set 1	set 2	set 3	set 4
A angle measure method for n=10	A	36.3347^o	36.6467^o	39.5242^o	40.2134^o
	B	32.1914^o	33.4474^o	34.4644^o	38.6373^o
	C	28.7557^o	32.1914^o	37.8154^o	37.9847^o
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Choobineh and Li	A	0. 333	0. 458	0. 333	0. 50
	B	0. 50	0. 583	0. 4167	0. 5833
	C	0. 667	0. 667	0. 5417	0. 6111
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Yager	A	0.60	0. 575	0. 5	0. 45
	B	0.70	0. 65	0. 55	0. 525
	C	0. 80	0. 7	0. 625	0. 55
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chen	A	·.3375	·.4315	·.375	·.52
	B	·.50	0. 5625	0. 425	0. 57
	C	0. 667	0. 625	0. 55	0. 625
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Baldwin and Guild	A	0. 30	0. 27	0. 27	0. 40
	B	0. 33	0. 27	0. 37	0. 42
	C	0. 44	0. 37	0. 45	0. 42
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$	$A \prec B \prec C$
Chu and Tsao	A	0.299	0. 2847	0. 25	0. 24402
	B	0. 350	0. 32478	0. 31526	0. 26243
	C	0. 3993	0. 350	0. 27475	0. 2619
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Yao and Wu	A	0.6	0. 575	0. 5	0. 475
	B	0.7	0. 65	0. 625	0. 525
	C	0.8	0. 7	0. 55	0. 525
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Sign Distance Method p=1	A	1.2	1. 15	1	0. 95
	B	1.4	1.3	1.25	1.05
	C	1.6	1.4	1.1	1.05
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Sign Distance Method p=1	A	0.8869	0. 8756	0. 7257	0. 7853
	B	1. 0194	0. 9522	0. 9416	0. 7958
	C	1. 1605	1. 0033	0. 8165	0. 8386
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec B \prec C$
Cheng Distance	A	0.79	0. 7577	0. 7071	0. 7106
	B	0. 8602	0. 8149	0. 8037	0. 7256
	C	0. 9268	0. 8602	0. 7458	0. 7241
Results		$A \prec B \prec C$	$A \prec B \prec C$	$A \prec C \prec B$	$A \prec C \prec B$
Cheng CV uniform distribution	A	0.0272	0. 0328	0. 0133	0. 0693
	B	0.0214	0. 0246	0. 0304	0. 0385
	C	0.0225	0. 0095	0. 0275	0. 0433
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$
Cheng CV proportional distribution	A	0.0183	0.026	0.008	0.0471
	B	0. 0128	0. 0146	0.0234	0.0236
	C	0. 0137	0. 0057	0.0173	0.0255
Results		$B \prec C \prec A$	$C \prec B \prec A$	$A \prec C \prec B$	$B \prec C \prec A$

Example 4.2 *Consider the following sets, see L-H Chen and H-W Lu [13].*

Set 1: $A_1 = (0.3, 0.5, 0.7), A_2 = (0.1, 0.2, 0.3)$.

Set 2: $E_1 = (0.2, 0.5, 0.8), E_2 = (0.3, 0.4, 0.9)$.

Set 3: $B_1 = (0.7, 0.8, 0.9), B_2 = (0, 0.4, 1)$.

Set 4: $F_1 = (0.2, 0.5, 0.8), F_2 = (0.3, 0.3, 0.9)$.

Set 5: $C_1 = (0.4, 0.7, 1), C_2 = (0.1, 0.4, 0.9)$.

Set 6: $I_1 = (0.4, 0.6, 0.8), I_2 = (0.3, 0.6, 0.8)$.

Set 7: $G_1 = (0.3, 0.5, 0.7), G_2 = (0.1, 0.5, 0.9)$.

Set 8: $D_1 = (0, 0.5, 1), D_2 = (0.3, 0.4, 1)$.

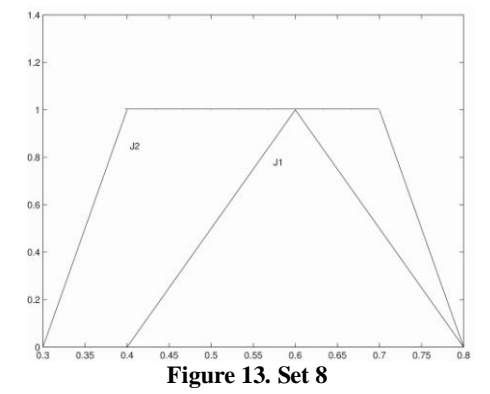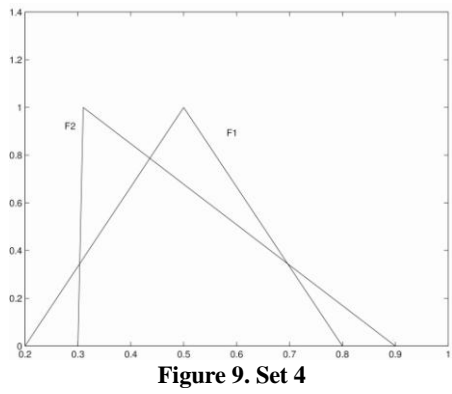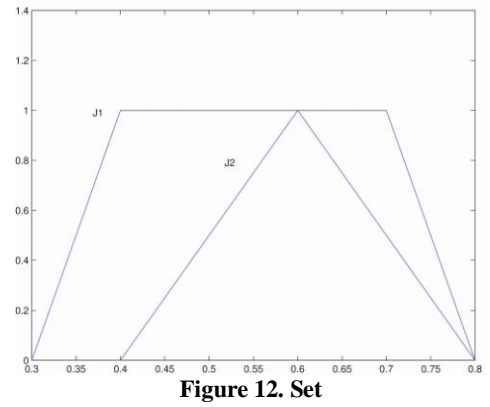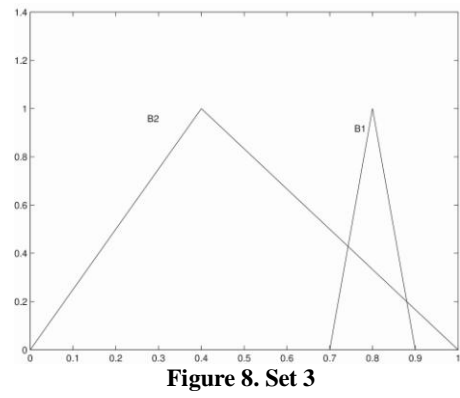Set 9: $J_1 = (0.4, 0.6, 0.8), J_2 = (0.3, 0.4, 0.7, 0.8)$.

Figure 6. Set 1

Figure 10. Set 5

Figure 7. Set 2

Figure 11. set 6

Figure 8. Set 3

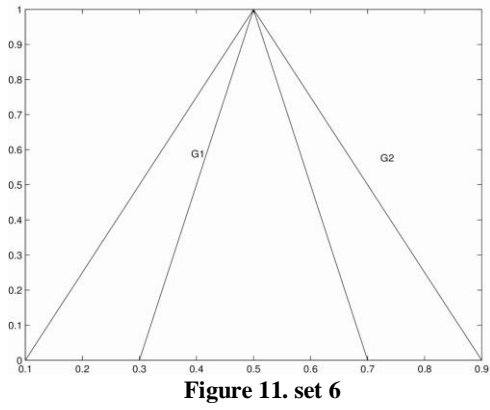
Figure 12. Set

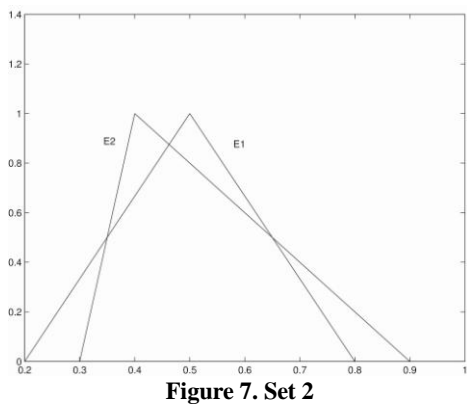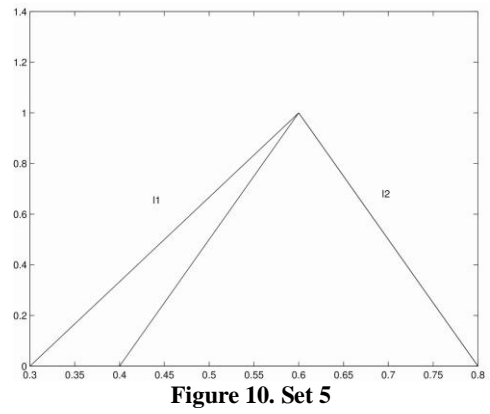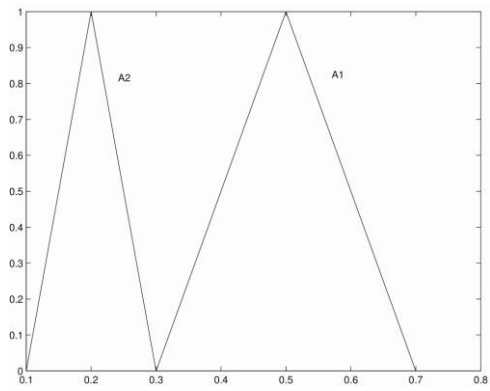**Figure 9. Set 4**

Figure 13. Set 8

Table 2. Comparative results of Example 4.2

Fuzzy number	Tseng and Klein	Kolodziejczyk a R1´	Kerre	Baldwin-Guid	Chen-Lu $\beta = 1$	Angle measure
A_1	1	1	1	0.46	0. 350	39.5242^o
A_2	0.0	0.0	0.70	0.0		57.9321^o
Results	$A_2 \prec A_1$	$A_2 \prec A_1$	$A_2 \prec A_1$	$A_2 \prec A_1$	$A_2 \prec A_1$	$A_2 \prec A_1$
B_1	0. 87	0. 87	0.99	0. 56	0. 100	29.3682^o
B_2	0. 13	0. 13	0. 54	0. 19		42.1744^o
Results	$B_2 \prec B_1$	$B_2 \prec B_1$	$B_2 \prec B_1$	$B_2 \prec B_1$	$B_2 \prec B_1$	$B_2 \prec B_1$
C_1	0. 87	0. 87	1. 0	0. 56	0. 300	32.1914^o
C_2	0. 13	0. 13	0. 55	0. 19		42.1744^o
Results	$C_2 \prec C_1$	$C_2 \prec C_1$	$C_2 \prec C_1$	$C_2 \prec C_1$	$C_2 \prec C_1$	$C_2 \prec C_1$
D_1	0. 47	0. 47	0.89	0. 44	0. 050	39.5242^o
D_2	0. 53	0. 53	0. 95	0. 48		39.4824^o
Results	$D_1 \prec D_2$ $D_1 \prec D_2$	$D_1 \prec D_2$ $D_1 \prec D_2$	$D_1 \prec D_2$			
E_1	0. 49	0. 49	0. 95	0. 36	0. 00	39.5242^o
E_2	0. 51	0. 51	0. 96	0. 39		40.3145^o
Results	$E_1 \prec E_2$	$E_1 \prec E_2$	$E_1 \prec E_2$	$E_1 \prec E_2$	$E_1 \prec E_2$	$E_1 \prec E_2$
F_1	0. 56	0. 56	0. 93	0. 40	0. 100	39.5242^o
F_2	0. 44	0. 44	0. 87	0. 36		43.0344^o
Results	$F_2 \prec F_1$	$F_2 \prec F_1$	$F_2 \prec F_1$	$F_2 \prec F_1$	$F_2 \prec F_1$	$F_2 \prec F_1$
G_1	0. 50	0. 50	0. 90	0. 38	-0. 100	39.5242^o
G_2	0. 50	0. 50	0. 90	1. 38		39.5242^o
Results	$G_1 = G_2$	$G_1 = G_2$	$G_1 = G_2$	$G_1 = G_2$	$G_1 = G_2$	$G_1 = G_2$
I_1	0. 56	0. 56	1. 0	0. 33	0. 000	35.5303^o
I_2	0. 44	0. 44	0. 95	0. 29		36.2861^o
Results	$I_2 \prec I_1$	$I_2 \prec I_1$	$I_1 \prec I_2$	$I_1 \prec I_2$	$I_2 = I_1$	$I_2 \prec I_1$
J_1	0. 64	0. 64	1. 0	0. 38	0. 000	35.5303^o
J_2	0. 36	0. 36	0. 85	0. 29		37.4348^o
Results	$J_2 \prec J_1$	$J_2 \prec J_1$	$J_2 \prec J_1$	$J_2 \prec J_1$	$J_2 = J_1$	$J_2 \prec J_1$

In table 2 all previous methods give $E_1 < E_2$ which is an intuition contradiction but AMM gives $E_2 < E_1$

5. Conclusions

In this paper, we have presented a simple ranking method for triangular (trapezoidal) fuzzy numbers , which associates an intuitive geometrical representation. It does not imply much computational effort and does not require a priori knowledge of the set of all alternatives. Comparative examples illustrate the advantage of the new approach.

References

1. S. Abbasbandy, B. Asady, Ranking of fuzzy numbers by sign distance, Volume 176, Issue 16, 2006, 2405 – 2416.
2. S. Abbasbandy, B. Asady, Anew approach for defuzzification, Fuzzy Sets Syst. 128 (2002) 131-132.

3. M. Adabitabar Firozja, B. Agheli, M. Hosseinzadeh, Ranking function of two LR-fuzzy numbers, Journal of Intelligent and Fuzzy Systems, Volume 26, Number 3, 2014, 1137-1142.

4. Oumayma Bahri, Nahla Ben Amor, Talbi El-Ghazali, New Pareto Approach for Ranking Triangular Fuzzy Numbers, Communications in Computer and Information Science Volume 443, 2014, 264-273.

5. J. F. Baldwin and N. C. F. Guild, Comparison of fuzzy numbers on the same decision space, Fuzzy Sets and Systems 2 (1979) 213-233.

6. G. Bortolan and R. Degani, A review of some methods for ranking fuzzy numbers, Fuzzy Sets and Syst. 15 (1985) 1-19.

7. S. Bass, H. Kwakernaak, Rating and ranking of multiple-aspect alternatives using fuzzy sets, Automatica, 13 (1977) 47-58.

8. G. Bortolan and R. Degani, A review of some methods for ranking fuzzy numbers, Fuzzy Sets and systems 15, 1-19, (1985).

9. S-J. Chen and C-L. Hwang, Fuzzy Multipe Attrioute Decision Making, Springer-Verlag, Berlin,(1992).

10. F. Choobineh and H. Li, An index for ordering fuzzy numbers, Fuzzy Sets and Systems 54, 287-297,(1993). (1985).

11. S. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems 17 (1985) 113-129.

12. T. Chu and C. Tsao, Ranking fuzzy numbers with an area between the centroid point and orginal point, Comput. Math. Appl. 43 (2002) 11-117.

13. L. H. Chen, H. W. Lu, An approximaite approach for ranking fuzzy numbers based on left and right dominance, Comput. Math. Appl. 41 (2001) 1589-1602.

14. C. H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Syst. 95 (1998) 307-317.

15. D. Dubios, H. Prade, Oprations on fuzzy numbers, J. System Sci. 9 (1978) 613-626.

16. O. Dias, Ranking altternatives using fuzzy numbers: A computational approach, Fuzzy Sets and systems 56, 247-252, (1993).

17. M. Delago, A. Vila, W. Voxman, On a cononical representation of fuzzy number,Fuzzy Sets Syst. 93 (1998). 125-135.

18. P. Fortemps and M. Roybens, Ranking and defuzzification methods based on area compensation, Fuzzy Sets and Systems 82,319-330, (1996).

19. Ching-Fen Fuh, Rong Jea, Jin-Shieh Su, Fuzzy system reliability analysis based on level (λ , 1) interval-valued fuzzy numbers, Information Sciences Volume 272(2014) 185–197.

20. R. Jain, Decision-making in the presence of fuzzy variables, IEEE Trans. Systems Man and Cybernet. SMC-6, 698-703,(1976).

21. R. Jain, A procedure for multi-aspect decision making using fuzzy sets, Internat. J. Systems Sci. 8, 1-7, (1978). defuzzification, Fuzzy sets and Systems 111 (2000) 351-356.

22. K. M. Lee, Cho and H. Lee-kWANG, Ranking fuzzy values with satisfaction function, Fuzzy Sets and Systems 64(1994) 295-311, .

23. M. Ma, A. Kandel and M. Friedman, A new approach for defuzzification, Fuzzy sets and Systems 111 (2000) 351-356.

24. M. Modarres, S. S. Nezhad, Ranking fuzzy numbers by preference ratio, Fuzzy Sets Syst. 118 (2001).

25. P. A. Raj and D. N. Kumar, Ranking alternatives with fuzzy weights using maximizing set and minimizing set, Fuzzy Sets and Systems 105(1999) 365-375.

26. Nazirah Ramli, Daud Mohamad, Ranking Fuzzy Numbers based on Sokal and Sneath Index with Hurwicz Criterion, Malaysian Journal of Mathematical Sciences 8(1)(2014) 117-137 .

27. I. Requena, M. Delgado and J. I. Verdegay, Automatic ranking of fuzzy numbers with the criterion of decision maker learnt by an artificial neural network, Fuzzy Sets and Systems 64(1994) 1-19.

28. Chitrasen Samantra, Saurav Datta, Siba Shankar Mahapatra, Risk assessment in IT outsourcing using fuzzy decision-making approach: An Indian perspective, Expert Systems with Applications, Volume 41, Issue 8(2014) 4010–4022.

29. S. Seikkala, On the fuzzy initial value problem, Fuzzy Sets and Systems 24 (1987) 319-330.

30. X. Wang, E. E. Kerre, Reasonable properties for the ordering of fuzzy quantities (I), Fuzzy Sets and Syst. 118 (2001) 375-385.

31. J. Yao and K. Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems 116 (2000) 275-288.

32. R. R. Yager, A procedure for ordering fuzzy subests of the unit interval, Information Sciences 24 (1981) 143-161.

33. Vincent F. Yu, Luu Quoc Dat, An improved ranking method for fuzzy numbers with integral values, Applied Soft Computing, Volume 14, Part C, January 2014, 603–608.

34. H. J. Zimmermann, Fuzzy Sets Theory and its Application, Kluwer Academic Press, Dordrecht, 1991