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Research article

Feedback linearization controller design for a geared transmission system considering asymmetric backlash and friction

Shima Mirshahzadeh, Hamed Khodadadi*

Department of Electrical Engineering, Khomeinishahr Branch, Islamic Azad University, Isfahan, Iran

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Abstract

This study is focused on the mathematical modeling, analysis, and controlling of a Geared Transmission System (GTS). Although GTS is used widely in mechanical equipment and mechatronics, the existence of nonlinearities such as backlash and friction made some challenges for its position control. In this paper's proposed method for GTS, the stribeck friction nonlinearity is estimated based on the motor velocity by a linear model. By this approach, the GTS model is reduced to a two-piecewise function. In addition, the asymmetric non-differentiable dead zone is approximated by a differentiable function. Additionally, using the differentiable function for the approximation leads to a reduction in the number of the piecewise function and therefore the number of switching in GTS output. Afterward, a feedback linearization controller is designed for the introduced model, and its stability and tracking of the reference trajectory are investigated. Simulation results indicate the designed controller on the proposed model has a good performance compared to the other models and complete tracking is realized without any steady-state error. Furthermore, due to the structure of the proposed model, position tracking is performed at the lowest time and by the minimum switching number.

Keywords: Geared transmission system, Feedback linearization control, Backlash, Mechatronics

1- Introduction

Gear transmissions are extensively used in robotic mechanisms and mechatronics to extract low speeds and large forces from commercial servo actuators [1]. Gear transmission servo systems are composed of a motor, gearbox, and other control elements [2]. Controllable parameters in the GTS include angular position, load speed, and engine speed. Up to now, various models have been proposed for power transmission in the GTS [3, 4]. Backlash is a clearance or lost motion in a mechanism caused by gaps between the parts. It can be defined as the maximum distance or angle through which any part of a mechanical system may be moved in one direction without applying appreciable force or motion to the next part in a mechanical sequence [5, 6]. It is not only inevitable in mechanical transmission but also one of the most important nonlinear factors that affect the dynamic performance and steady precision, and even stability of the controlled plants, if they are not well, handled [1, 7]. Friction is another nonlinear characteristic of the GTS which is appeared due to the presence of connections. The behavior of the used friction model is such that at low speeds, the friction force decreases with increasing speed, and at high speeds, the friction force increases with increasing speed [8].

The presence of two nonlinear and unavoidable factors of backlash and friction in the GTS make an oscillatory state in the system behavior. On the other hand, the backlash nonlinearity is described in the form of a dead zone, which is presented by a non-derivative function. This model will reduce the dynamic performance of the system and cause a steady-state error [9, 10]. The presence of a non-differentiable backlash factor in the existing models will create a limit cycle in the closed-loop response of the system. Not compensating for the backlash and collision between gears will cause severe shocks between mechanical parts in the mechatronic systems [10, 11]. Therefore, it is necessary to provide a complete model that can consider the effect of both friction and backlash on the gear system. In addition, the presented model should be in such a way that provides the designing a suitable controller that firstly guarantees the stability of the system and secondly places the load angular position on the desired input precisely.

The literature survey shows that several studies have been published on the modeling of backlash and friction in gear transmission servo systems. In [12], considering the nonlinear property of asymmetric backlash in the gear motor servo system, a new asymmetric and differentiable model has been introduced. besides, adaptive controllers [9], backstepping [10, 11], and state feedback linearization methods [12] have been designed on the introduced model. In [13], the mentioned system has been examined by considering the friction model and separating the dynamic model of the system based on the backlash and connection states related to the backlash. More complete friction models, including Stribeck and viscous models, have been evaluated in [14]. In [15], the linear two-mass system is considered with the assumption of a nonlinear backlash model, and then the angular velocity of the system is controlled using the PI loop shaping control approach. In [16], the system model was selected similarly to [15], albeit the effect of friction and backlash was not considered. In this paper, the system is controlled by PI and PD controllers firstly, and then the state feedback linearization method is used to control the system more precisely. Using the suboptimal convex optimization approach and Lyapunov theory, the sufficient conditions and closed loop response of designing state feedback stabilization controller for systems with input backlash are stated in [17, 18]. The analytical expression of backlash characteristics with separate parameters using switching and approximate functions is proposed in [18]. In [19], a continuous dynamical model is presented to describe a group of hysteresis curves similar to backlash. Then, a solution is presented in the form of a linear function into the input signal, which facilitates the process of controller design. The presented approach by [20] for hysteresis is transforming into a time-varying system considering the uncertainty and controlling utilizing an adaptive method. Similarly, considering the uncertain parameters in the dead zone model, adaptive control [21, 22], optimal model predictive control [23], and sliding mode control [24] are used for parametric uncertainty caused by backlash nonlinearity.

As it is obvious, in several models that have been proposed to express the effect of nonlinear elements of the GTS system, only the effect of backlash has been taken into account and friction has either been neglected or a very simple model such as the viscous model has been considered. In this paper, the effect of the presence both of backlash and friction factors on the GTS model is investigated. Furthermore, considering the four rules model of the literature for the GTS, this research tries to reduce the number of switching and rules of the model. For this purpose, by introducing the definition of asymmetric differentiable backlash, the condition of the gear connection will be removed. Moreover, by converting Stribeck's friction model into two linear equations, the dynamics of the proposed system are reduced into two equations. In the following, the feedback linearization controller is designed for the proposed model and the tracking accuracy of the reference input is evaluated.

After the introduction, the description of the GTS and its dynamical model by considering friction and backlash in the form of four rules is performed in the second section. In the third section, the proposed model of the gear transmission servo systems is presented by assuming two rules. The fourth section is dedicated to designing a suitable controller to track the desired angular position. In the fifth part, the simulation results of the designed controller on the proposed model will be presented and compared with other studies. Finally, the conclusion is discussed in the last section.

2- GTS modelling

A servomotor consists of a simple electric motor that is placed next to some other electronic elements and all parts are provided in a single package. The purpose of this system is to control the angle, speed, or acceleration of the servomotor [8]. The mentioned servomotor is coupled with a gearbox which is shown in Fig. 1. A dynamic model for the gear servomotor system is obtained as (1)

$$\begin{aligned} J_m \ddot{\theta}_m + c_m \dot{\theta}_m &= \tau - Dead[\theta] \\ J_l \ddot{\theta}_l + c_l \dot{\theta}_l &= N_0 Dead[\theta] \end{aligned} \quad (1)$$

where, J_m, q_m and c_m represent the moment of inertia, position, and coefficient of viscous friction on the engine side and J_l, q_l and c_l represent the moment of inertia, position, and coefficient of viscous friction on the load side. K_g is the reduction factor, τ is the control torque, $q = q_m - K_g q_l$ is the relative displacement, and $Dead[\theta]$ is the transmission torque due to backslash [12].

2-1 The definition of backlash

One of the nonlinear characteristics of the gearbox is backlash [25]. Backlash is used to create better lubrication and also to compensate for errors like distortion, deformation, thermal expansion, etc. backlash can be described as (2).

$$Dead(\theta) = \begin{cases} k_r(\theta - \alpha_r) & \text{if } \theta \geq \alpha_r \\ 0 & \text{if } -\alpha_l \leq \theta \leq \alpha_r \\ k_l(\theta + \alpha_l) & \text{if } \theta \leq -\alpha_l \end{cases} \quad (2)$$

where k_l and k_r describe stiffness coefficients and α_r and α_l indicate the separation points.

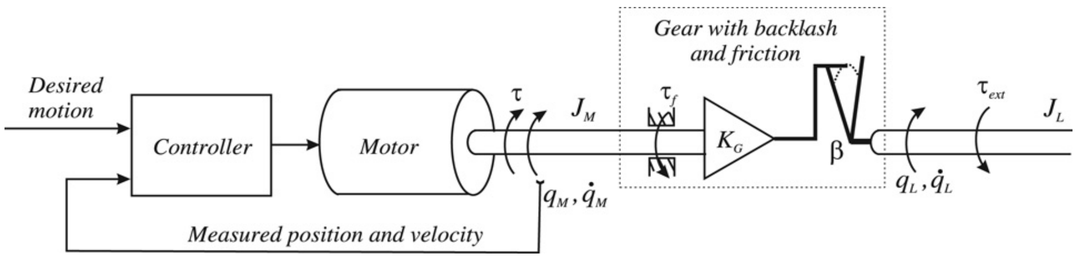

Fig. 1 Mechanical system with backlash and friction [8]

Although this definition of dead zone torque is widely used due to its high compatibility with real systems, it is considered as one of the nonlinear and nondifferentiable factors of these systems. In the real world of engineering, gearbox parameters $(\alpha_r, \alpha_l, k_r, k_l, N_0)$ are among the technical specifications of the product [12]. Fig. 2 shows the backlash behavior of the system in terms of input and output angles.

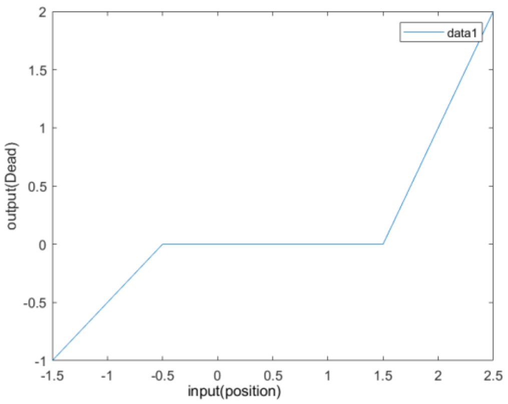

Fig. 2 Backlash behavior based on input angle [15]

2-2 Friction

Another nonlinear specification of the gearbox is friction, which exists in any system whose components are in contact with each other [25]. So far, various models have been employed to describe friction. For example, in the proposed model of [12] derived in Eq. 1, the friction model is considered as viscous, which is one of the simplest of these models. In order to make the GTS model more practical, it is necessary to apply more realistic models such as Stribeck to describe friction [26]. The behavior of the Stribeck model is as follows. At low speeds with increasing speed, friction force decreases, and at high speeds with increasing speed, friction increases. Eq. 3 describes the mentioned behavior [25].

$$\tau_f = \begin{cases} \tau \quad for \;\; \dot{q} = 0 \; and \; |\dot{q}| < F_s \\ (F_c + (F_s + F_c)e^{|\dot{q}|/\dot{q}_m})sign(\dot{q}) \\ +F_v \dot{q} \quad for \; otherwise \end{cases} \tag{3}$$

where F_c, F_s and F_v indicate the coulomb, static, and sticky friction coefficients, respectively. Moreover, $\dot{q}$ is the angular velocity in rotational motion, $\dot{q}_s$ is Stribeck velocity, τ describes the control torque, and exp present the exponential function [8].

2-3 Linearized Friction Model

A linear approximation has been made to achieve a simpler friction model. Suppose the mechanical system moves in the speed range of $[0, \dot{q}_{Msw}]$. It should be considered that a linear approximation for an exponential curve is drawn using two lines: where d_{1+} passes through the point of $(\tau_f(0), 0)$ tangent to the curve and d_{2+} passes through the point of $[\dot{q}_{M\max}, \tau_f(\dot{q}_{M\max})]$ tangent to the curve. These two lines meet at speed $\dot{q}_{Msw}$. The line d_{1+} in the interval $[0, \dot{q}_{Msw}]$ and the line d_{2+} in the interval $[\dot{q}_{Msw}, \dot{q}_{M\max}]$ have been used as curve linearizers [8]. Therefore, the exponential friction model can be linearized by applying two lines in the defined speed domain with limited error. Fig. 3 shows the behavior of Stribeck friction in non-linear and linearized modes. The equations related to d_{1+}, d_{2+} using Taylor's expansion are as Eqs. 4 and 5.

$$d_{1+} : F_{L1f+}(\dot{q}_m) = a_1 + b_1 \dot{q}_m \quad \text{for} \; 0 < \dot{q}_m \le \dot{q}_{Msw} \tag{4}$$

$$d_{2+} : F_{L2f+}(\dot{q}_m) = a_2 + b_2 \dot{q}_m \quad \text{for} \; \dot{q}_{msw} < \dot{q}_m \le \dot{q}_{\max} \tag{5}$$

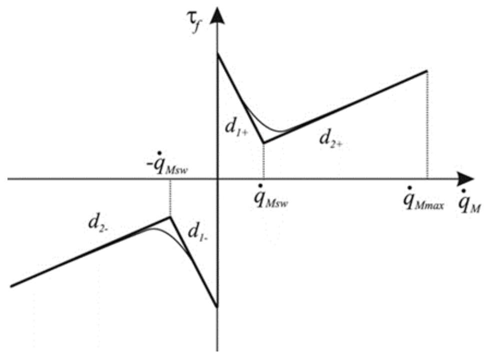

Fig. 3 Modeling Stribeck friction by nonlinear and linearized curves [8]

The point $\dot{q}_{Msw}$ can be obtained by Eq. 6:

$$\dot{q}_{Msw} = \frac{a_1 - a_2}{b_2 - b_1} \tag{6}$$

Therefore, the linearized friction model will be as follows [8]:

$$F_{l1f+}(\dot{q}_m) = \begin{cases} a_1 + b_1 \dot{q}_m & if \;\; \dot{q}_m \in (0, \dot{q}_{Msw}) \\ a_2 + b_2 \dot{q}_m & if \;\; \dot{q}_m \in (\dot{q}_{Msw}, \dot{q}_{M\max}) \\ -a_1 + b_1 \dot{q}_m & if \;\; \dot{q}_m \in (-\dot{q}_{Msw}, 0) \\ -a_2 + b_2 \dot{q}_m & if \;\; \dot{q}_m \in (-\dot{q}_{Msw}, -\dot{q}_{M\max}) \end{cases} \tag{7}$$

2-4 The dynamical model of GTS regarding backlash and friction

By using the linearized model of friction and regarding the contact and backlash modes, the complete model of the GTS system considering backlash and friction can be provided as a hybrid system Eqs. 8-11 [8].

if (BM) and $(|\dot{q}_m| \le \dot{q}_{Msw})$

$$\begin{cases} J_M \ddot{q}_m + b_1 = \tau - a_1 sign(\dot{q}_m) \\ J_L \ddot{q}_l = -\tau_{ext} \end{cases} \tag{8}$$

if (BM) and $(|\ddot{q}_m| \succ \dot{q}_{Msw})$

$$\begin{cases} J_M \ddot{q}_m + b_2 \dot{q}_m = \tau - a_2 sign(\dot{q}_m) \\ J_L \ddot{q}_l = -\tau_{ext} \end{cases} v \tag{9}$$

if (CM) and $(|\dot{q}_m| \le \dot{q}_{Msw})$

$$\begin{cases} (J_M + J_L / \ddot{q}_m) + b_1 \dot{q}_m = \tau - \tau_{ext} / k_G - a_1 sign(\dot{q}_m) \\ \dot{q}_l = \dot{q}_m / k_G \end{cases} \tag{10}$$

if (CM) and $(|\dot{q}_m| \succ \dot{q}_{Msw})$

$$\begin{cases} (J_M + J_L / k_G^2)\ddot{q}_m + b_2 \dot{q}_m = \tau \\ \qquad - \tau_{ext} / k_G - a_2 sign(\dot{q}_m) \\ \dot{q}_l = \dot{q}_m / k_G \end{cases} \tag{11}$$

$$\begin{cases} (J_M + J_L / \ddot{q}_m) + b_1 \dot{q}_m = \tau - \tau_{ext} / k_G - a_1 sign(\dot{q}_m) \\ \dot{q}_l = \dot{q}_m / k_G \end{cases}$$

The obtained model is hybrid and has four rules. Accordingly, by changing the speed of the system, switching occurs between different rules.

3- Proposed model for GTS considering differentiable approximation for deadzone

Generally, finding an appropriate model for describing the relation between system components has great importance [27-29]. As mentioned, although the GTS system model proposed in Eq. 8-11 is a complete model due to simultaneous consideration of friction and backlash, converting the system into a four-rule hybrid model and switching between different modes will reduce the accuracy of the system. Moreover, the behavior of the system is assumed to be completely separate in the two modes of contact and backlash in this model, which will make several challenges in the controller design process.

The proposed model of this study to solve the mentioned challenges is employing the backlash function as (2) and replacing it with switching between backlash and contact modes. Besides, to overcome the problems caused by the non-differentiability of the introduced model, Eq. 2 is approximated with a soft symmetric backlash nonlinear model proposed in [10] and leads to (12).

$$T_s(q) = \frac{1}{\rho}\ln\frac{1+e^{\rho k_r(q-\alpha_r)}}{1+e^{-\rho\kappa_l(q+\alpha_l)}} \tag{12}$$

In this equation, $\alpha_l, \alpha_r, k_l, k_r$ are the parameters defined in Eq. 2, and ρ is a positive adjustable parameter called a soft degree. By combining Eqs. 12 and 8-11, the proposed model of this study for the GTS system is described as two rules (13-14) considering backlash and friction.

$$\text{if } \left|\dot{q}_m\right| \le \dot{q}_{Msw}$$

$$\begin{Bmatrix} J_M \ddot{q}_m + b_2 \dot{q}_m = \tau - a_2 sign(\dot{q}_m) \\ J_L \ddot{q}_l = -\tau_{ext} \end{Bmatrix} \tag{13}$$

$$\text{if } \left|\dot{q}_m\right| > \dot{q}_{Msw}$$

$$(J_M + |sign(T_s)|.(J_L / k_G^2))\ddot{q}_m = \tau - a_2 sign(\dot{q}_m) - b_2 q \dot{q}_m \tag{14}$$
$$\ddot{q}_l = (-c_l / j_l)\dot{q}_l + (k_g / j_l)T_s$$

In this model, the four rules are reduced to two rules, and the modes of contact and backlash are considered simultaneously. In backlash mode, parameter $T_s(q)$ chooses the middle numerical parameter $k_l(q+\alpha_l) \le T_s(q) \le k_r(q-\alpha_r)$ as the coefficient (J_l / k_g^2). Therefore, $B = J_m + T_s(J_l / k_g^2)$ appears in the dynamics of the system. Besides, due to $T_s = 0$ in the contact mode, $B = J_m$. It should be noted that since the parameter B is the coefficient of $\ddot{q}_m$, it is not allowed to be zero. Accordingly, B is considered as Eq. 15.

$$B = J_m + |sign(T_s)|.(J_l / k_g^2) \tag{15}$$

To convert the system model into the state space form, the state vector is defined as (16).

$$X = (x_1, x_2, x_3, x_4)^T = (q_l, \dot{q}_l, q_m, \dot{q}_m)^T \tag{16}$$

The state space equation of the nonlinear system will be defined as (17-18). Where τ is the system input, $T_s(q)$ the approximation of the differentiable-inadmissible model, and $q = x_3 - k_g x_1$ indicates the difference angle between the motor and the load.

$$if \;\; -\dot{q}_{Msw} \le x_4 \le \dot{q}_{Msw}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ (-c_l/J_l)x_2 + (-k_g/J_l)T_s \\ x_4 \\ (-b_l/B)x_4 - (-a_1/B)sign(x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\tau \tag{17}$$

$$if \;\; x_4 \ge \dot{q}_{Msw} \;\; or \;\; x_4 \le -\dot{q}_{Msw}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ (-c_l/J_l)x_2 + (-k_g/J_l)T_s \\ x_4 \\ (-b_2/B)x_4 - (-a_2/B)sign(x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\tau \tag{18}$$

4- Designing a state feedback linearization controller

In addition to conventional control methods such as PID, which is utilized to control robots and mechatronic systems [30, 31], the sliding mode control method [32] and state feedback linearization are two control approaches that are very suitable and applicable to nonlinear systems.

In the feedback linearization method, by applying an appropriate map and choosing a suitable control signal, the nonlinear dynamical system will be linearized; afterward, by utilizing conventional linear methods system can be controlled. In fact, the main purpose of this method is to eliminate the nonlinear parts and choose a suitable linear controller for stabilization. A nonlinear system can be assumed as (19):

$$\dot{x} = f(x) + g(x)u \tag{19}$$

The dynamics of the discussed system include two rules, which are based on the $x_4(t)$ signal (motor speed), switching will be performed. After checking the necessary conditions for applying the feedback linearization method for each of the rules, it is shown that each of the subsystems can be linearized. Therefore, a map of $Z = T(x)$ can be found that by applying the control rule as $u = \alpha(x) + \beta(x)v$ makes the closed-loop system linear.

4-1- Evaluating the relative degree and internal dynamics of the system

First, the relative degree of the system and the presence of internal dynamics are evaluated. Since the relative degree of the system is 4 and equal to the number of system dynamics, this system does not have internal dynamics.

4-2- Converting nonlinear system to linear system under mapping

Consider the map of $h(x) = x_l$ which is satisfied in the below equations for the system described by Eq. 17-18.

$$\begin{aligned} &\nabla z_1 . ad_{f^i} g = 0 \quad \text{i=0,1,2} \\ &\nabla z_1 . ad_{f^3} g \neq 0 \end{aligned} \tag{20}$$

The other variables of the system are as follows:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} L_{f^0} z_1 \\ L_{f^1} z_1 \\ L_{f^2} z_1 \\ L_{f^3} z_1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ (-c_l/J_l)x_2 + (-k_g/J_l)T_s \\ \dfrac{-K_g^2 x_2 (k_r s_1 + k_l s_2) + c_l^2 x_2}{J_l} \\ \qquad - \dfrac{c_l K_g T_s + K_g (k_r s_1 + k_l s_2) x_4}{J_l} \end{bmatrix} \tag{21}$$

where

$$s_1 = \frac{e^{\rho k_r (q-\alpha_r)}}{1+e^{\rho k_r (q-\alpha_r)}} \qquad s_2 = \frac{e^{-\rho k_l (q+\alpha_l)}}{1+e^{-\rho k_l (q+\alpha_l)}}$$

$$s_3 = \frac{s_1}{1+e^{\rho k_r (q-\alpha_r)}} \qquad s_4 = \frac{s_2}{1+e^{-\rho k_l (q+\alpha_l)}} \tag{22}$$

For each system, the feedback linearization controller is designed separately in the form of $u = \alpha(x) + \beta(x)$. We derive:

$$\alpha = \frac{L_{f^4} h}{L_g L_{f^2} h} \qquad \beta = \frac{1}{L_g L_{f^2} h} v \tag{23}$$

As a result, for system (17), the control signal is $u = \alpha_1(x) + \beta_1(x)v$, in which

$$\alpha_1(x)=\frac{\rho.kg.x_2^2(k_r^2s_3-k_l^2s_4)}{j_l}+\frac{\rho.kg^2.x_4(-k_r^2s_3+k_l^2s_4)x_2}{j_l}+\frac{kg^2.x_2.c_l(k_rs_1-k_ls_2)}{j_l^2}+\frac{-kg^2(k_rs_1+k_ls_2)(-c_lx_2+kgT_s)}{j_l^2}+\frac{-c_l^3x_2+c_l^2kgT_s}{j_l^3}+\frac{\rho*kg^2*x_2*x_4(-k_r^2s_3+k_l^2s_4)}{j_l}-\frac{\rho*kg*x_4(k_r^2s_3-k_l^2s_4)}{j_l}-\frac{(k_rs_1+k_ls_2)*k_g*(b_1x_4+a_1sign(x_4))}{j_l.B} \tag{24}$$

$$\beta_1=\frac{J_1.B}{\left(k_rs_1+k_ls_2\right)K_g} \tag{25}$$

In a similar way, the control signal for for system (18) is $u=\alpha_2(x)+\beta_2(x)v$, in which

$$\alpha_2=\frac{\rho.kg.x_2^2(k_r^2s_3-k_1^2s_4)}{j_l}+\frac{\rho.kg^2.x_4(-k_r^2s_3+k_1^2s_4)x_2}{j_l}+\frac{kg^2.x_2.c_l(k_rs_1-k_ls_2)}{j_l^2}-\frac{kg^2(k_rs_1+k_ls_2)(-c_lx_2+kgT_s)}{j_l^2}+\frac{-c_l^3x_2+c_l^2kgT_s}{j_l^3}+\frac{\rho.kg^2.x_2.x_4(-k_r^2s_3+k_l^2s_4)}{j_l}+\frac{\rho.kg.x_4(k_r^2s_3-k_l^2s_4)}{j_l}-\frac{(k_rs_1+k_ls_2).k_g.(b_2x_4+a_2sign(x_4))}{j_l.B} \tag{26}$$

$$\beta_2=\frac{J_1.B}{\left(k_rs_1+k_ls_2\right)K_g} \tag{27}$$

By using the map (21) and obtained control signals, the equations of the system in z coordinates are obtained as follows:

$$\begin{aligned}&\dot{z}=AZ+bv\\&y=cZ\\&\dot{z}=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}.\begin{pmatrix}z_1\\z_2\\z_3\\z_4\end{pmatrix}+\begin{pmatrix}0\\0\\0\\1\end{pmatrix}v\\&y=z_1\end{aligned} \tag{28}$$

4-3- State feedback linearization controller design

The last stage of the designing process is the appropriate selection of the control signal in such a way that the closed-loop system becomes stable. For this purpose, the control signal is considered as (29):

$$v=-k.z+M.r\left(t\right) \tag{29}$$

In this equation, $r(t)$ is the reference input which is considered a step function.

M is the gain of the forward path which is selected to have a zero steady-state error, according to the reference input signal based on Eq. 30.

The coefficients k are obtained based on the pole placement method that all the poles of the closed-loop system are placed on -3 as (31).

$$M=(-1)/(c^TA_c^{(-1)}b)=81 \tag{30}$$

$$k=[k_1,k_2,k_3,k_4]^T=[81,108,54,12]^T \tag{31}$$

5- Simulation results

In order to evaluate the performance of the designed controller on the proposed GTS model, considering the effect of backlash and friction, the simulation results are evaluated in this section. The numerical values of the system parameters are selected based on Table 1. Firstly, the simulation is performed on the proposed system model without applying the controller and presented in Fig. 4. The reference input which is defined as the difference between the angle of the load and the motor shaft is set to 2. Obviously, it is desired that the output remains at this value. In this case, the output (load angle) is fluctuating and did not follow the desired input.

Table 1: Numerical value of the system parameters

parameter	description	unit	Value
J_m	Motor Inertia	$Kg.m^2$	0.01
b_1	Friction coefficient	---	-2.2
a_1	Friction coefficient	---	1.5
a_2	Friction coefficient	---	0.5567
b_2	Friction coefficient	---	0.1
K_g	Gear ratio	---	5
J_l	Load Inertia	$Kg.m^2$	0.5
α_r	Angle breaking point	rad	0.001
α_l	Angle breaking point	rad	0.002
$\dot{q}_{Msw}$	Motor velocity breaking point	---	$\frac{a_1-a_2}{b_1-b_2}$
c_l	Load friction coefficient	Nm/rad	0.12
c_m	Motor friction coefficient	Nm/rad	0.1
ρ	Gear soft degree	---	20
k_r	Stifness coeficient	Nm/rad	0.3
k_l	Stifness coeficient	Nm/rad	0.2

In the next simulation, the reference input is selected to be 2 and the output should reach this value. The initial conditions of the GTS system are selected as $x_0 = [x_{10}, x_{20}, x_{30}, x_{40}]^T = [2, 0.5, 10, 1]^T$ and the smoothness coefficient of the gearbox is assumed to be 20. The value of the load angle by applying the feedback linearization controller is shown in Fig. 5.

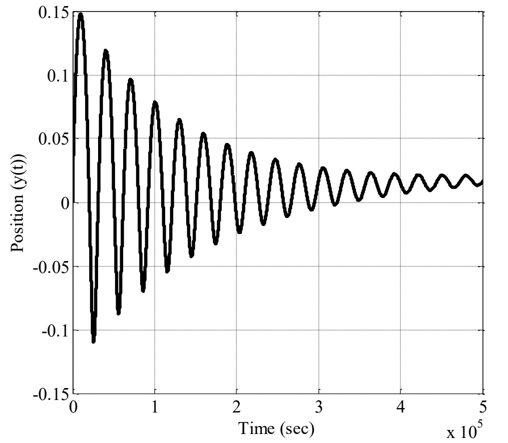

Fig. 4 Position of the load side without any control signal

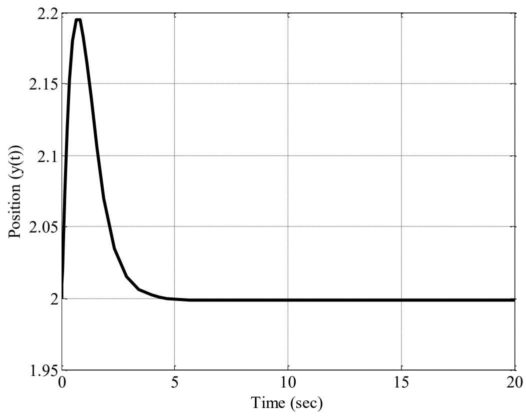

Fig. 5 Position of the load side by feedback linearization controller

As it is obvious, the oscillating behavior of the system is eliminated and the output follows the desired input without error. Additionally, the control torque applied to the system is shown in Fig. 6. The simulation results show that the feedback linearization controller on the proposed GTS model has suitable performance considering the nonlinear effects of backlash and friction. Furthermore, the control objectives are meted in the desired time and without steady-state error.

In order to have a better evaluation of the contribution of this study's proposed model, the performance of this study is compared with [8]. The friction model is selected similarly in both studies; however, the modes of backlash and contact are separated from each other in [8]. Therefore, there is a four-rule model in that several switching occurs between different situations.

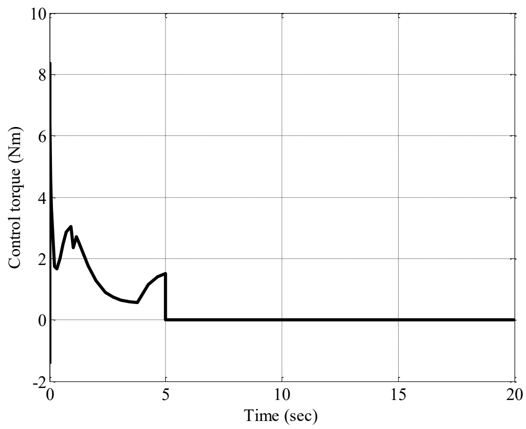

Fig. 6 Control torque

Fig. 7 shows the output response of the GTS system to the unit step reference input for the load angle in the range of $|\mathring{q}_m| > \mathring{q}_{Msw}$.

In the upper part of this figure, the load and motor angles are shown. The next part of this figure represents the motor speed. The modes of backlash, contact, and switching between these two states are given in the next part of Fig. 7. The last part of this figure shows the torque of the system as a control signal. The remarkable point is that the load angle does not reach its steady state due to the existence of many switchings between the two modes of contact and backlash, and the load angle is constantly fluctuating.

Meanwhile, in the proposed model, the behavior of the output of the GTS system is without oscillation by applying the feedback linearization controller. Employing the combination of the proposed model and controller, the system output will reach its desired value.

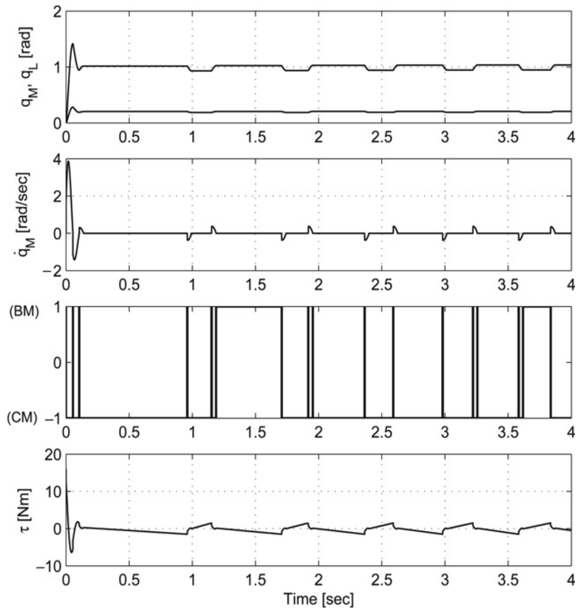

Fig. 7 Simulation results of GTS [8]

6- Conclusion

As mentioned previously, the modeling and controlling of the gear transmission servo system has two nonlinear challenges of backlash and friction, making several problems for their application in industries. Therefore, this paper tries to consider the effect of both factors simultaneously, by presenting a proposed model.

By considering the Stribeck model for friction and converting it to linear models and employing a symmetric differentiable function for backlash, the new model of the GTS system is proposed as two rules. The advantages of this model compared to the previous models are that it is more practical and also differentiable, which provides the possibility of designing different controllers.

In the following, the feedback linearization controller is designed for the proposed

GTS system model. The simulation results showed that the state feedback linearization controller can eliminate the system output oscillations and follow the desired input with zero steady-state error. Besides, compared to other models, the accuracy of the system increased due to the reduction in the number of switching.

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