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Research article

Static analysis of guyed tower regarding cable sagging by using finite difference model

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Abstract

The static analysis of guyed towers is very complex because of high degree of non-linearity they exhibit due to their slender and flexibility. In the static analysis of cable systems, due to the effects of nonlinearity of loading and large functions, the interference of loads and displacement have no much value. Because of having no analytical solution for these systems, numerical methods have been applied. Finite difference method (FDM) is one of the known methods which are directly related to nonlinear relative equations and, the cable system is defined using calculating of them and deformation equations . In this research, the non-linear analysis of cables under three dimension static loads is analyses using FDM. This model can be analyzed three dimension response of cable using sequential method. In this model, long and inclined pre-tensioned cables have been studied in which only axial stiffness of cable is considered while bending and torsion stiffness are assumed to be negligible.

Keywords: Static analysis, Guyed cable, Cable sagging, Finite difference method.

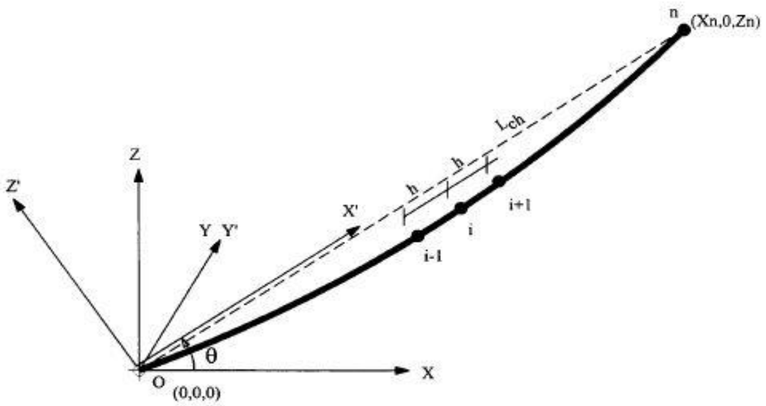
1- Introduction

A cable structure is defined as a structure in which a cable or a cable system is used as an element to withstand the initial load. They are simple in assembling, light in weight and safe in maintenance. In these structures, pre-tensioned cables are used to provide stability of system. Most of these cables are under combination of three dimension forces due to their supporting action [1].Structure cables are important in the modern structural engineering which increase tower stability and support the flexible rods at the top of the mast that are supposed to wind forces. It is necessary for a structure engineer to know how to deal with these forces. Many existing structures are vulnerable to loads therefore their resistance to these loads should be increased [2]. Regarding growing importance of cables structures application, understanding their properties are very important, so there is a need for more information on cable behavior [3]. Recently, cable three dimensional structures have been designed with high tension stress tolerance.

Sometimes the cables are under different loadings. In the cable supported structures, wind aerodynamic forces [4] are important. In the most of cable structures, cable hardness is low compared to other structure components. Since the frequency is proportional to the square of stiffness to mass ratio, natural frequencies of cable structure are less than other structures therefore the system dynamic response is determined if the natural frequencies are approximately three fold of force frequency, In the normal state, the cable structures have lower frequency (less than 4 Hez) while guyed structures have high natural frequency. Dynamic responses of cable structures provided based on created loads in different times and dynamic tensions applied on statistic pretensions lead to problems in the structural members which lead to high pressure or fatigue. When the power frequency is closed to natural frequency, the dynamic stresses close to resonance conditions are very high [5]. An efficient way to solve nonlinear equations is to use a finite difference method for static analysis.

2- Static equilibrium equations

Today, modeling is very important step for cognition and analysis of designers to meet their needs and design based on facilities to accelerate production [6].Consider a flat sa, single span and inclined under influence of a three dimensional static load applied to its self-weight deflected shape. Fig. 1 shows a cable static model. As shown in this figure, the left of cable (O) is considered as origin of coordinate system and the cable is initially placed in the z-x plane [7].



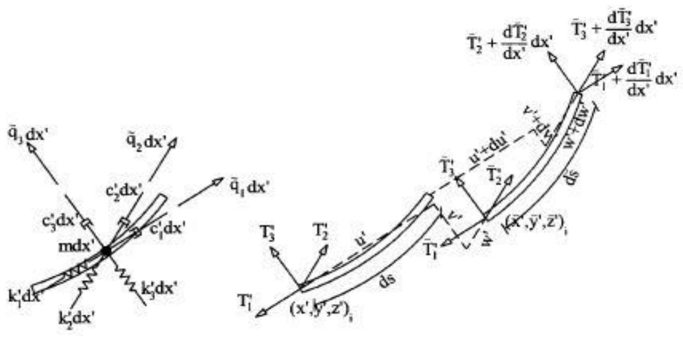


Fig. 1 the model of 3D cable for static analysis

In each chord length unit, the weigh $(q_d)T$, elastic modulus *(E)*, the same cross-section *(A)*, chord length *(L_q)*, inclination angle *(θ)* due to *X* axis are considered. At any point of cable, deformed geometrical shape is presented by Cartesian coordinate space vector (x,y,z) or $(x_{1,}x_2,x_3)$ using index markings.3-D uniform loading and concentrated loading vectors

$\tilde{q} = \{\tilde{q}_X , \tilde{q}_Y , \tilde{q}_Z\}$ and $\tilde{P} = \{\tilde{P}_X \ \tilde{P}_Y \ \tilde{P}_Z\}$, respectively) are applied to self- weight deformed cable in a varying chord length. Due to 3-D loading, the internal cable tension $T = \{T_x T_y T_z\}$ in any point (x,y,z) of cable is different from one point to another along the cable.

2-1- The main hypotheses

To simplify the derivation of cable equilibrium equation, the followings points are assumed:

(1) elastic cable materials with finite strain (Lagrangian non-linear strain)
(2) long and pre tensioned cables with common axial stiffness (negligible torsional and bending stiffness)
(3) Along the x axis, cable tension varies (x is considered as an independent variable).

2-2- static equilibrium equations

Static analysis is very important to design of a structure and using a static analysis, it can be sure about resistance of the structure [7]. Consider the equilibrium conditions for infinite element of length (d_s) of self-weight deformed cable, in the (x-y) plane. Its force equilibrium in x vector is [8]:

$$\left(\frac{\partial T'_{x'}}{\partial x'}\right)dx' + (\tilde{q}_{x'})dx' = 0 \quad i.e \frac{\partial T'_{x'}}{\partial x'} = -\tilde{q}_{x'} \quad (1)$$

By defining the x', y', z' directions as 1, 2 and 3 index notation, equation 1 generalizes for 3-D local directions:

$$\frac{\partial T'_i}{\partial x'} = -\tilde{q}_{i'} \quad (2)$$

Using vector algebra expressions, the following equation is obtained for 3-D cable tension component:

$$T'_i = n'_i T = \frac{\partial x'_i}{\partial s}.T \quad i = 1,2,3 \quad (3)$$

Using convention of equation 3, we can reduce the number of unknown tension component as follow:

$$T'_2 = T'_1.\frac{\partial y'}{\partial x'} \quad (4\text{-}1)$$

$$T'_2 = T'_1.\frac{\partial y'}{\partial x'} \quad (4\text{-}2)$$

Using equations 4, the force equilibrium in y' is as follow:

$$\begin{aligned} &\frac{\partial}{\partial x'}\left(T'_1.\frac{\partial y'}{\partial x'}\right) = -\tilde{q}'_2 \\ (5) \quad &T'_1.\frac{\partial^2 y'}{\partial x'^2} + \frac{\partial T'_1}{\partial x'}.\frac{\partial y'}{\partial x'} = -\tilde{q}'_2 \end{aligned}$$

Similarly, the force equilibrium in

z' is as follow

$$\begin{aligned} &\frac{\partial}{\partial x'}\left(T'_1.\frac{\partial z'}{\partial x'}\right) = -\tilde{q}'_3 \\ &T'_1.\frac{\partial^2 z'}{\partial x'^2} + \frac{\partial T'_1}{\partial x'}.\frac{\partial z'}{\partial x'} = -\tilde{q}'_3 \end{aligned} \quad (6)$$

Finally, after rearranging, three differential equations for the cable equilibrium are provided as:

$$\frac{\partial T'_1}{\partial x'} = -\tilde{q}'_1 \quad (7\text{-}1)$$

$$T_1'\left(\frac{\partial^2 y'}{\partial x'^2}\right)=\left(\tilde{q}_1'.\frac{\partial y'}{\partial x'}-\tilde{q}_2'\right) \quad (7\text{-}2)$$

$$T_1'\left(\frac{\partial^2 z'}{\partial x'^2}\right)=\left(\tilde{q}_1'.\frac{\partial z'}{\partial x'}-\tilde{q}_3'\right) \quad (7\text{-}3)$$

where, T_1', T_2', T_3' are strain components in the x', y', z' directions.

$$T_1=+T_1'.Cos(\theta)-T_3'.Sin(\theta) \quad (8\text{-}1)$$

$$T_2=+T_2' \quad (8\text{-}2)$$

$$T_3=+T_1'.Sin(\theta)+T_3'.Cos(\theta) \quad (8\text{-}3)$$

$\tilde{q}_x$, $\tilde{q}_y$, $\tilde{q}_z$ are applied equivalent static force per unit chord length in x', y', z' direction.

$$\tilde{q}_1'=+\left(\tilde{q}_1+\frac{\tilde{P}_1}{h}\right).cos(\theta)+\left(\tilde{q}_d+\tilde{q}_3+\frac{\tilde{P}_3}{h}\right).Sin(\theta) \quad (9\text{-}1)$$

$$\tilde{q}_2'=+\left(\tilde{q}_2+\frac{\tilde{P}_2}{h}\right) \quad (9\text{-}2)$$

$$\tilde{q}_3'=-\left(\tilde{q}_1+\frac{\tilde{P}_1}{h}\right).Sin(\theta)+\left(\tilde{q}_d+\tilde{q}_3+\frac{\tilde{P}_3}{h}\right).Cos(\theta) \quad (9\text{-}3)$$

$\tilde{p}_x$, $\tilde{p}_y$, $\tilde{p}_z$ are point loads in x, y,z directions which are active in special points and h is cable segment lenth:

$$h=\left(\frac{L_{CH}}{n-1}\right) \quad (9\text{-}4)$$

in which, (n-1) is number of cable divisions. Thus the nonlinear static analysis of cable reduces the problem of finding the unknown values of the tension components (T_1') and (y', z') coordinates in a point using given vector (x') and for a certain loading vector $\tilde{q}_x$, $\tilde{q}_y$, $\tilde{q}_z$ .This is achieved by using S_0 as the only cable invariant to perform the required iterations to find correct tension of cable at the next loading. Then the calculation of other unknowns can be made based on these determined values.

2-3- Partial non-linear differential equations of equilibrium using finite difference method

An effective method to solve different nonlinear partial equilibrium equations is to use the finite difference model for the spatial discretization of the system equations. Using this method, and consider equations (7) and after rearrenging, we have [9]:

$$\left(\frac{\partial^2 y'}{\partial x'^2}\right)=\frac{\tilde{q}_1}{T_1'}.\left(\frac{\partial y'}{\partial x'}\right)-\frac{\tilde{q}_2}{T_1'} \quad (10)$$

Using finite difference first and second order approximations for the spatial derivatives, it is possible to write equation 10 in the well-known matrix to solve the n-dimension simulated for all internal nodes as follows:

$$\left\{[A]+\left(\frac{\hat{\tilde{q}}_1\hat{h}}{2\hat{T}_1}\right).[B]\right\}\times\{\hat{u}\}=\{\hat{f}\} \quad (11)$$

in which

$$[A]=\begin{bmatrix} +1 & & & & & \\ -1 & +2 & -1 & & & \\ & & \ddots & & & \\ & & & -1 & +2 & -1 \\ & & & & & +1 \end{bmatrix} \quad (12\text{-}1)$$

$$[B]=\begin{bmatrix} 0 & & & & & \\ -1 & 0 & +1 & & & \\ & & \ddots & & & \\ & & & -1 & 0 & +1 \\ & & & & & 0 \end{bmatrix} \quad (12\text{-}2)$$

$$for Y' \text{ - } direction \ \{u'\} = \begin{bmatrix} y'_1 \\ y'_2 \\ \\ \\ y'_{n-1} \\ y'_n \end{bmatrix} \quad (12\text{-}3)$$

$$for\ Z' \text{ - } direction \ \{u'\} = \begin{bmatrix} z'_1 \\ z'_2 \\ \\ \\ z'_{n-1} \\ z'_n \end{bmatrix}$$

$$for Y' \text{ - } direction \ \{f'\} = \begin{bmatrix} y'_1 \\ \left(\dfrac{\tilde{q}'_2 h^2}{T'_1}\right)_2 \\ \left(\dfrac{\tilde{q}'_2 h^2}{T'_1}\right)_{n-1} \\ y'_n \end{bmatrix} \quad (12\text{-}4)$$

$$for\ Z' \text{ - } direction \ \{f'\} = \begin{bmatrix} z'_1 \\ \left(\dfrac{\tilde{q}'_3 h^2}{T'_1}\right)_2 \\ \left(\dfrac{\tilde{q}'_3 h^2}{T'_1}\right)_{n-1} \\ z'_n \end{bmatrix}$$

2-4- Calculation method for solving equilibrium equation

2.4.1- Initial cable self-weight profile

The first step in complete nonlinear analysis is to determine the initial shape of the cable considering its self- weigh only, to initial cable profile approximated as its chord. This can be accomplished using the same method used in the following steps or the finite difference model combined with the sequential design. We have obtained (x,y,z) along with its fixed length. The fixed length of the cable, S_0, is considered as a mandatory parameter in cable analysis, so that only the fixed cable is used to determine the correct equilibrium state of the cable under load.

2-4-2- Cable response due to total static loading

In order to determine the response of the cable (for example, cable deformation function and the internal forces in terms of tensile components) due to 3-D loading, we completed the numerical solution of the nonlinear equilibrium differential equations using iteration schemes. The required computational method can be accomplished using the following steps: [10]

Step 1

Based on equation (7-1) in discrete form, for T'_1 it can be solved as follows:

$$F'_{i,i+1} = F'_{1,i} - \tilde{q}'_{1,i} \times \Delta x' |_i \quad (13)$$

where, $F'_{1,i} + F'_{1,i+1}$ are nodal forces in direction of x' for i th cable element.. At first, we assume that the $F'_{1,i}$ value at one cable end is equal to previous value and from then we continue calculation process until we reach to other end of the cable. The average amount of nodal forces for each element is taken to represent the discrete cable tension for that element, for example:

$$T'_{1,i} = \frac{1}{2}(F'_{1,i} + F'_{1,i+1}) \quad i = 1,2,...,n-1 \quad (14)$$

Step 2-1

Using finite difference method that is given in equations 11 and 13-2 and employing determined tension components, $T'_{1,i}$ determined previously for n-1 and i=1,2,…so componentization in (7-2) indicates the static balance of cable in unknown functions.

Step 2-2

Repeat the previous sub-step to solve discreet form of equation (3-7) which indicates the static balance of the cable along z' and is used for the unknown z' coordinate of cable.

Step 3

Calculate strain components in y', z' axis using (4-1) and (4-2) for i[th] element, and define $\left(\frac{\partial y'}{\partial x'}\right), \left(\frac{\partial z'}{\partial x'}\right)$ as follow [11]:

$$\left(\frac{\partial y'}{\partial x'}\right)_i = \frac{(\Delta y')_i}{(\Delta x')_i} = \frac{y'_{i+1} - y'_i}{x'_{i+1} - x'_i} = \frac{y'_{i+1} - y'_i}{h} \quad (15\text{-}1)$$

$$\left(\frac{\partial z'}{\partial x'}\right)_i = \frac{(\Delta z')_i}{(\Delta x')_i} = \frac{z'_{i+1} - z'_i}{x'_{i+1} - x'_i} = \frac{z'_{i+1} - z'_i}{h} \quad (15\text{-}2)$$

Then calculate total amount of T_i' as:

$$T_i' = \sqrt{T_{1_i}'^2 + T_{2_i}'^2 + T_{3i}'^2} \quad (16)$$

Step 4-1

Compute the length of the deformed cable (S) using deformed geometrical shape of cable mentioned earlier, from the following equation:

$$S = \sum_{i=1}^{n=1} (\Delta S)_i \qquad i = 1, \ldots, n-1 \quad (17)$$

in which

$$\Delta S_i = \sqrt{(\Delta x')_i^2 + (\Delta y')_i^2 + (\Delta z')_i^2} \quad (18\text{-}1)$$

$$(\Delta x')_i = x'_{i+1} - x'_i = h, (\Delta y')_i = y'_{i+1} - y'_i, (\Delta z')_i = z'_{i+1} - z'_i \quad (18\text{-}2)$$

Step 4-2

Using the elastic modulus elasticity of the material of cable (E), and the cross-sectional area (A), along with the deformed length of the element (Δs), and the total tension (T_i), we can calculate the new fixed length from the following equation [12]:

$$S_a^* = \sum_{i=1}^{n-1} (\Delta S_0^*)_i \qquad i = 1, \ldots, n-1 \quad (19)$$

In which:

$$\Delta S_0^* = \frac{(\Delta S)_i}{1 + 2\left(\frac{T}{EA}\right)_i} \quad (20)$$

Step 5

Find the exact value of the tension at the end of the cable, $F'_{1,1}$, that keeps the cable in the cable in equilibrium under the existing loads, while maintaining a constant cable length a, which represents the net weight s_0. This can be accomplished using iterative scheme that require defining the upper and lower bounds on H'. Above mention calculation method leads to providing accurate solutions to solve the problem of a single cable under a three-dimensional static load. This can be accomplished by ensuring the balance of the forces and the adaptation of the deformation which is described by the differential equations in the cable system.

5-1- Pre-determined displacements applied in the end of cable

Displacement continuity at the lower and upper end of cable $(\tilde{u}, \tilde{v}, \tilde{w})_1$ is calculated using $(\tilde{u}, \tilde{v}, \tilde{w})_n$ at the lower and upper part of cable.it has effect on strain changes of cable (H). However, the only fixed parameter of the cable the fixed length (S_0), will be remain. This method has an important application for checking the non-linear interaction between cable and structure (for example, in guyed masts). In these masts, the movement at the end of the cable is related to the interaction between the cable and the supported structure. The interaction has a great effect on the internal forces of the supporting cables and the supported structure.

3- Conclusion

This research includes a new non-dimensional finite difference formulation that takes into account all the important parameters affecting the swing characteristics of restraining cables. The last hypotheses have been used to access the partial nonlinear differential equations for spatial displacement. Also, a normalized method has been used to provide the non-dimensional form of equations in an appropriate parametric research. Finally, parametric research has been done to investigate the effect of loading and cable parameters on cable response. According to the analysis, it has been concluded:

1- Cables with high initial tension (low bending ratio) show stiffness behavior in the displacements of lower parts (strain) and hence show low dynamic tension.
2- Due to the relatively light weight of the structure, the fluctuation and change of the mass of the cable does not have much effect on the displacement or tensile response of the cables.
3- Higher levels of in-plane loading, perpendicular to the cable chord and out-of-plane loading can significantly increase the response of the cable, while in-plane loading along the cable chord has much less effect.

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