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Stress Intensity Factor at the Hole-Edge Crack Tips in a Finite Plate

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Abstract

In the current research work, the problem of fracture mechanics in a plate with a central hole under tensile loading is studied. The stress intensity factors are calculated for a finite plate containing two symmetrical hole-edge cracks. The problem is solved by two different methods, namely the finite element method and the FRANC software analysis. At first the finite element method is used and by writing a program in MATLAB software the stress intensity factors at the crack tips are calculated. The same problem is then reanalyzed with the Franc software and the results are compared. The effects of various factors such as the hole diameter, crack length and crack angle have been investigated on stress intensity factors. The results show that for small crack lengths, the effect of cracks length is more than that of the hole diameter on variation of normalized stress intensity factors, while it is the opposite for large crack lengths, the effect of hole diameter is more than that of the cracks length on variation of normalized stress intensity factors.

Keywords: Stress intensity factor, hole-edge cracks, finite element, Franc software.

1- Introduction

The presence of cracks in a structure can lead to its failure at stresses below the yield strength. A special situation that is often of practical interest is a crack growing from a stress raiser, such as a hole or notch. Experimental, numerical and analytical methods are used to investigate fracture mechanics problems. Yan [1] has discussed a numerical analysis of cracks emanating from an elliptical hole in a plate using the boundary element method. Cirello et. al. [2] have calculated the stress intensity factor (SIF) by numerical simulation and experimental measurements in perforated plates. Chakherlou et. al. [3] have investigated and calculated the effect of bolt clamping force on the fracture strength and the SIF of a plate containing a fastener hole with edge cracks. Zhao [4] has calculated the SIF in an infinite plate containing multiple hole-edge cracks. Torshizian and Molazem [5] by using the finite element method (FEM) have studied the SIFs in a cracked gear for different states of loading cases. Torshizian and Kargarnovin [6] have considered the mixed-mode fracture mechanics analysis of an arbitrarily oriented crack in a two dimensional functionally graded material using plane elasticity theory. Evans et. al. [7] have devised a method for calculating the geometric correction of the SIF and calculated four different configurations of the SIFs using the innovative formula. Torshizian [8] has studied mode III fracture in a functionally graded materials plate with an internal crack and determined the SIFs at crack tips using the analytical method and finite element method. Torshizian and Andarzjoo [9] have considered the mixed-mode fracture mechanics analysis in a plate containing two symmetrical hole-edge cracks, which is bonded with two dissimilar planes. Dowling [10] has proposed an approximate equation to determine the stress intensity factor for an infinite plate containing two symmetrical hole-edge cracks.

The almost of previous works have considered the hole-edge crack in infinite plate. In this study, the SIFs are calculated in a finite plate containing two symmetrical hole-edge cracks, as shown in Figure 1. The effects of various factors such as the hole diameter, crack length and crack angle on the SIF have been investigated and the author offer an approximate solution for SIFs at the crack tips in a finite plate, containing two symmetrical hole-edge cracks. To determine the SIF the finite element method is used. A program is coded in MATLAB to obtain the displacement of surrounding nodes of the crack. Based on the displacement field stress intensity factors at the crack tips are calculated. The same problem is investigated by FRANC program and the results are compared with the previous values.

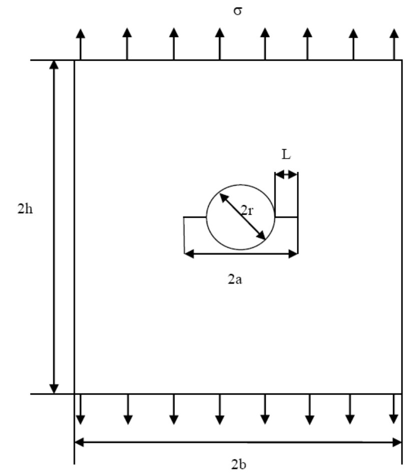

Fig. 1 The finite plate containing two symmetrical hole-edge cracks.

2- Finite element modeling

Based on the division of the cracked plate to small components with relatively simple forms, the FEM is able to model the stress field and displacement in the area surrounding the crack. Therefore, this method is widely used in the context of linear fracture mechanics. In this section, by employing the weak form formulation on equilibrium equations, appropriate finite element matrices are obtained. Then by using a singular isoparametric eight-node element that is able to model discontinuities of the stress field at the crack tip, weak form equilibrium equations have become equivalent to a system of algebraic equations. The solution proposed by this system of algebraic equations makes it possible to calculate the stress intensity factor at the crack tip. In the fracture problem, to which only an in-plane external mechanical loads are applied and in the absence of body forces, the strong form of the equilibrium equation can be written as:

$$[L]^T \{\sigma_{ij}\} = 0 \tag{1}$$

Also displacement-strain and stress-strain relations are as:

$$\{\varepsilon_{ij}\} = [L]\{u_i\} \ , \ \{\sigma_{ij}\} = [D_{ij}]\{\varepsilon_{ij}\} \tag{2}$$

where σ_{ij} and ε_{ij} are the stress and strain components, respectively and u_i s are the displacement components. For an isotropic material, for plain stress, the displacement, strain and stress vector are defined as:

$$\{u\}=\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \quad , \quad \{\varepsilon\}=\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}, \quad \{\sigma\}=\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} \qquad (3)$$

And the operator of $[L]$ and elastic matrix $[D_{ij}]$ are defined as follows:

$$[L]=\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \quad , [D]=\frac{E}{1-2\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \qquad (4)$$

Considering the equivalence of internal and external works in conjunction with the variational formulation of the boundary value problem, one can obtain:

$$\int_V \delta\{\varepsilon\}^T \{\sigma\} dV - \int_S \delta\{u\}^T \{f^t\} dS = 0 \qquad (5)$$

where $\{f^t\}$ represents function of any force applied on crack surfaces. By substituting Eq. (3) and (4) into Eq. (5) the weak form of the equilibrium equation can be written as follows:

$$\int_V ([L]\delta\{u\})^T [D]([L]\{u\}) dV - \int_S \delta\{u\}^T \{f^t\} dS = 0 \qquad (6)$$

Now, the domain can be discretized using eight-node quadrilateral elements. For the problem under consideration, the nodal displacement vector on each element is defined as:

$$\{U\}_e = \{u_{x1} \;\; u_{y1} \;\; \dots \;\; u_{x8} \;\; u_{y8}\}^T \qquad (7)$$

In the finite element analysis, the trial function for the displacement field is written in terms of the shape function. The finite element approximation of the trial solution on each element can be expressed as follows:

$$\{u\} = [N]_e \{U\}_e \qquad (8)$$

where the elements of shape functions can be written as follows:

$$[N]_e = \begin{bmatrix} N_1 & 0 & N_2 & 0 & \dots & \dots & N_8 & 0 \\ 0 & N_1 & 0 & N_2 & \dots & \dots & 0 & N_8 \end{bmatrix} \qquad (9)$$

By inserting Eq. (8) into Eq. (6) one can obtain:

$$\int_V ([B]_e \delta\{U\}_e)^T [D]_e ([B]_e \{U\}_e) dV - \int_S ([N]_e \delta\{U\}_e)^T \{f^t\} dS = 0 \qquad (10)$$

where, the strain-displacement matrix $[B]_e$ is defined as:

$$[B]_e = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \dots & \frac{\partial N_8}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & \dots & 0 & \frac{\partial N_8}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \dots & \frac{\partial N_8}{\partial y} & \frac{\partial N_8}{\partial x} \end{bmatrix} \qquad (11)$$

Equation (10) can be summarized as follows:

$$[K]_e \{U\}_e = \{F\}_e \qquad (12)$$

where in this case $\{U\}_e$ is the displacement vector element node, $[K]_e$ is the element stiffness matrix and $\{F\}_e$ is the force vector element which are defined as follows:

$$[K]_e = \int_V [B]_e^T [D]_e [B]_e \, dV \qquad (13)$$

$$\{F\}_e = \int_S [N]_e^T \{f^t\} dS \qquad (14)$$

By superposition of stiffness matrices, force vectors and node displacement vectors of all the elements, the relationship between the total stiffness matrix, total external force vector and total displacement vector can be written as follows:

$$[K]\{U\} = \{F\} \quad (15)$$

Quadrilateral eight-node elements will be used for singular elements, which takes care of singularity condition at the crack tips. It is shown that in an eight-node isoparametric element if the mid-side node along the edge neighboring the crack is placed at the quarter distance to the crack tip, the singularity characteristic of the linear elastic fracture mechanic can be obtained. Positions of mid-side nodes along the edges in singular elements around the crack tip are shown in Fig. 2.

After determining the total displacement vector from Eq. (15) the mode I and mode II stress intensity factors can be determined by the displacement of nodes near the crack tip (Fig. 2), which will be calculated as follows [11]:

$$K_I = \frac{2\mu}{k+1}\sqrt{\frac{2\pi}{L}}\left[-3u_{yC} + 4(u_{yB} - u_{yD}) - (u_{yA} - u_{yE})\right] \quad (16)$$

$$K_{II} = \frac{2\mu}{k+1}\sqrt{\frac{2\pi}{L}}\left[-3u_{xC} + 4(u_{xB} - u_{xD}) - (u_{xA} - u_{xE})\right] \quad (17)$$

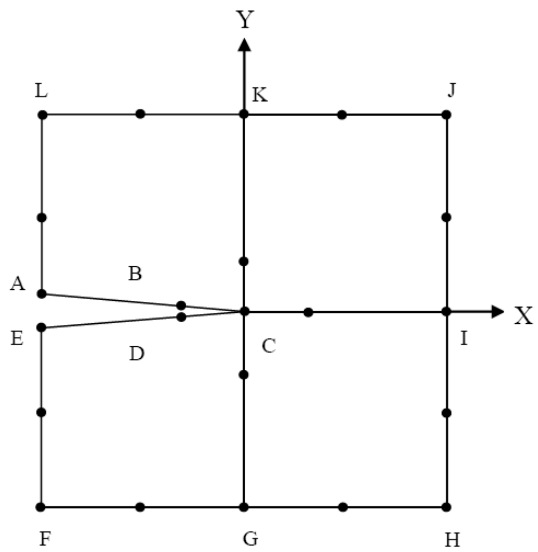

Fig. 2 Positions of mid-side nodes along the edges in singular elements around the crack tip

3- FRANC software

The FRANC is a highly interactive program for simulation of crack growth in layered structures. The program is an extension of the FRANC to make possible the representation of layered structures such as lap joints or bonded repairs. The FRANC uses standard eight or six nodded serendipity elements with quadratic shape functions. These elements perform well for elastic analysis and have the advantage that the stress singularity at the crack tip can be incorporated in the solution by moving the side nodes to the quarter-point locations.

The FRANC can model quasi-static crack propagation and crack propagation due to fatigue loading. The crack will propagate in the direction predicted using any of the three propagation theories implemented in the FRANC. The FRANC program, however, does not have the ability to produce the part geometry and networking. Piece is modeled by other programs and networking event. For this reason a piece of software components for network modeling is introduced as CASCA. In the CASCA program, no analysis has been prepared solely for completing the FRANC program. The CASCA program is a simple mesh generating program. Although strictly speaking, it is not part of the FRANC program, it is distributed with the FRANC, and can be used to generate initial meshes for The FRANC simulations.

4- Calculation of stress intensity factor and verification

In this study, a square plate containing two symmetrical hole-edge cracks has been studied. The side length of the square plate is considered 100 mm with 1 mm thickness. It is assumed that uniformly distributed tension stress of 1 MPa is applied on the typical geometry and loading as shown in Fig. 1. For the square plate with this size, 720 quadrilateral eight-node elements are used. As it can be seen in Fig. 3, very fine mesh near the crack tips is considered. In addition, around each crack tip, four singular elements are used.

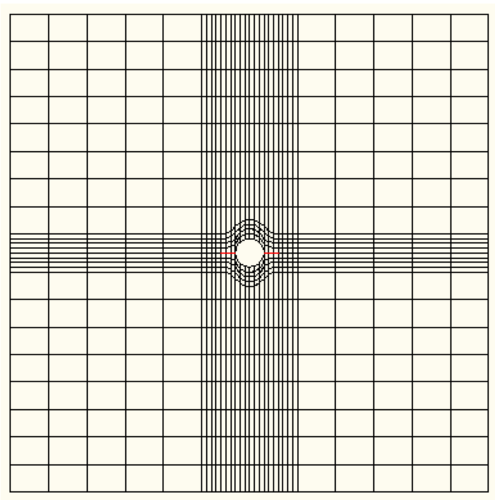

Fig. 3 Mesh with center hole plate in finite element method

Thus, Figs. 4 and Fig. 5 show a view of meshing the same plate with a central hole in the FRANC program and Fig. 6 shows the stress field around the crack.

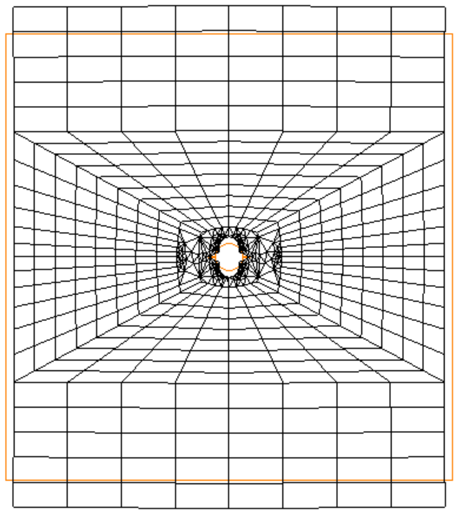

Fig. 4 The mesh plate before opening the crack

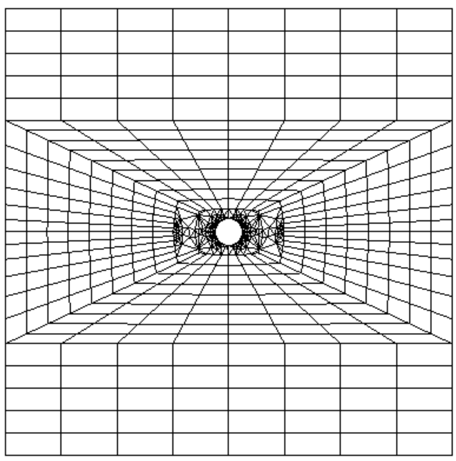

Fig. 5 The mesh plate after opening the crack

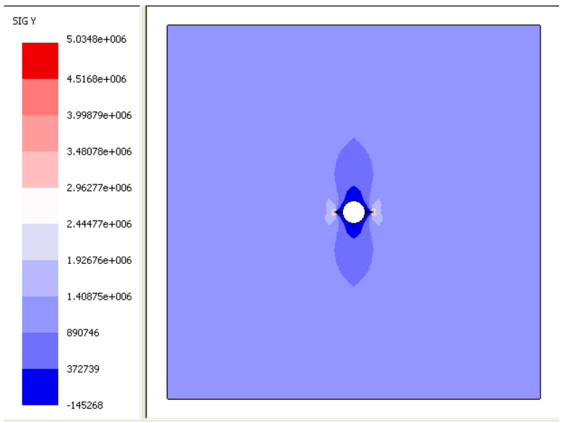

Fig. 6 Stress field around the hole and crack

Dowling [10] proposed the following approximate equation to determine the stress intensity factor for an infinite plate containing two symmetrical hole-edge cracks:

$$K_I = 0.5(3-d)\left[1+1.243(1-d)^3\right]\sigma\sqrt{\pi l} \quad (18)$$

where l and r are the crack length and hole radius respectively, and d is defined as $d = l/(l+r)$.

Consider a square plate with a hole diameter of 6 mm and various crack lengths from 1 to 20 mm. Stress intensity factors at crack tips are calculated by FEM and FRANC. The result obtained in the present study are shown in Fig. 7 and compared with Ref. [10].

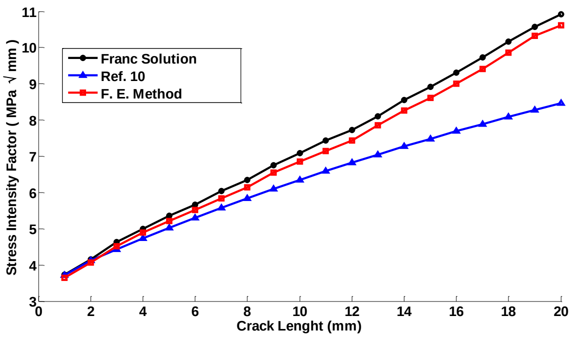

Fig. 7 Variations of SIFs at crack tip for various crack length with holes diameters 6 mm

In Fig. 7 a very good agreement is observed for small crack length. But by increasing the crack length the difference increases between the results of present study and those of Ref. [10]. This is because Eq. (18) is defined for an infinite plate. Then the results obtained from Eq. (18) are committed error for large crack that located in a finite plate. Then, it seems that Eq. (18) needs a correction factor when the plate is finite.

The author offers an approximate solution for the SIFs at the crack tips as:

$$K_I = \left[1-0.16c+3.13c^2-2.11c^3\right]\times 0.5(3-d)\left[1+1.243(1-d)^3\right]\sigma\sqrt{\pi l} \quad (19)$$

where c is defined as $c=(l+r)/b$ and b is the plate width.

For example variations of the SIFs at the crack tip for a square plate with a hole diameter of 6 mm are listed in Table 1.

Table 1 Variations of the SIFs ($MPa\sqrt{mm}$) at the crack tip for variation of crack length

Crack length (L) mm	Present study			Reference [10]
	Franc solution	F.E. method	Propose d Eq.(19)	
2	4.197	4.061	4.188	4.134
4	4.992	4.892	4.882	4.726
6	5.697	5.524	5.618	5.298
8	6.392	6.151	6.389	5.841
10	7.101	6.853	7.191	6.347
12	7.822	7.434	8.027	6.821
14	8.574	8.265	8.898	7.266
16	9.358	9.019	9.804	7.687
18	10.188	9.865	10.743	8.086

5- Numerical example and discussion

Consider a square plate containing two symmetrical hole-edge cracks with a length of 100 mm and thickness of 1 mm under uniformly tension stress 1 MPa (Fig. 1). The effect of different factors such as hole diameter, crack length and crack angle on the stress intensity factors are investigated and shown in the following figures. As shown in these figures, the stress intensity factors are normalized by dividing into $\sigma\sqrt{\pi a}$ where a is the sum of hole radius and crack length ($a=l+r$).

The variation of the normalized SIFs for various crack lengths with different values of hole diameter are presented in Fig. (8). In Fig. 8, it is assumed that the crack length varies from 1 to 20 mm for the hole diameters of 6 and 10 mm.

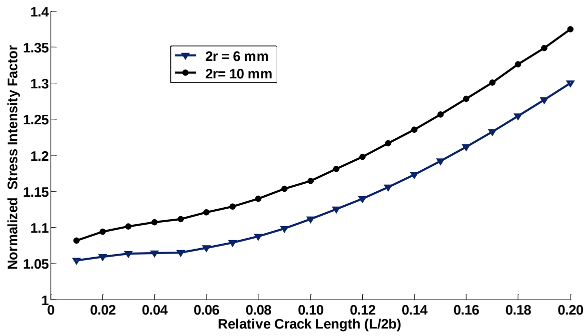

Fig. 8 Variations of the normalized SIFs at the crack tip for various crack lengths with hole diameters of 6 and 10 mm

Fig. 9 shows the normalized SIFs for various crack lengths with different values of the hole diameter. It is assumed that the crack length varies from 1 to 10 mm for hole diameters of 20 and 30 mm.

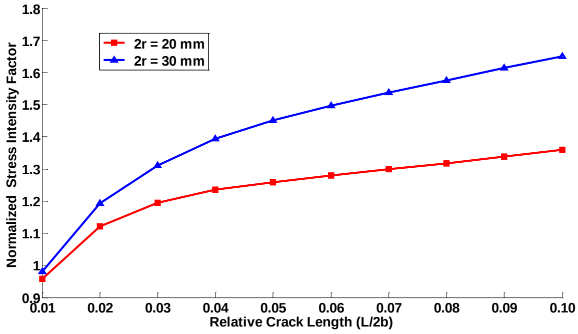

Fig. 9 Variations of the normalized SIFs at the crack tip for various crack lengths with hole diameters of 20 and 30 mm

These curves show that for the same crack lengths the normalized SIFs increase with increasing the hole diameter.

Fig. 10 and 11 show variations of the normalized SIFs for various a for different hole diameters where a is the sum of the hole radius and crack length ($a = l + r$).

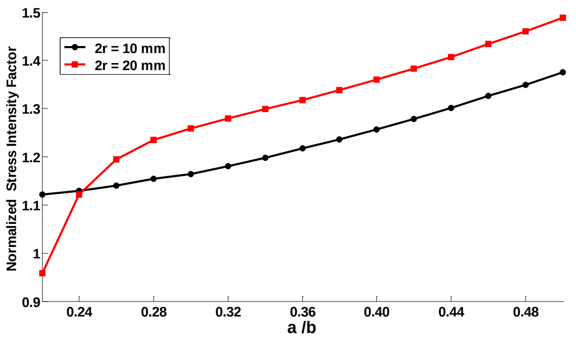

Fig. 10 Variations of the normalized SIFs for various a for hole diameters of 10 and 20mm

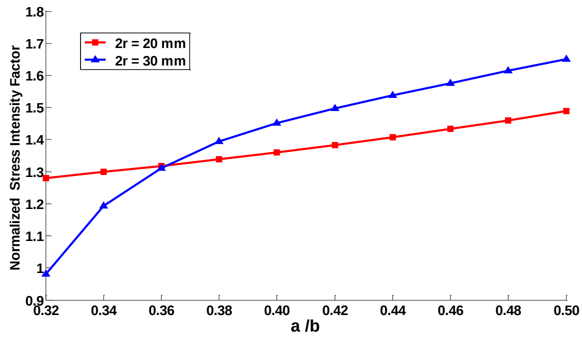

Fig. 11 Variations of the normalized SIFs for various a and different hole diameters of 20 and 30mm

Fig. 12 shows variations of the normalized SIFs in the mode I and mode II for various crack angle with a hole diameter of 10 mm and a crack length of 5 mm.

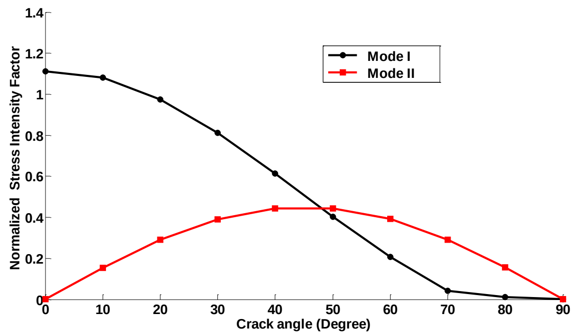

Fig. 12 Variations of the normalized SIFs in mode I and mode II for various crack angles

6- Conclusion

The fracture behavior of a plate with central hole under tensile loading has been studied. The stress intensity factors are calculated in a finite plate containing two symmetrical hole-edge cracks. To determine the SIF the finite element method is used. The same problem is investigated by FRANC program and the results are compared with the previous values. Several different examples are solved and effects of hole diameter, crack length and crack angle on the SIF have been investigated.

The results show that for different hole diameters, the normalized SIF increases with increasing the crack length. Also, for the same crack length the normalized SIFs increase with the increase of the hole diameter.

For a certain amount of crack length ($a = l + r$) for small crack lengths the effect of cracks length on variation of normalized SIFs is more than the hole diameter but for large crack lengths the effect of hole diameter on variation of normalized SIFs is more than the cracks length.

For any arbitrary crack orientation θ, when the crack length is not perpendicular to the direction of an external load, a mixed mode condition occurs. In this case by increasing θ the normalized SIFs for the mode I decrease. Also by increasing θ, the normalized SIFs for the mode II, first increase and then decrease and the maximum SIFs at crack tips occur at $\theta = 45^\circ$.

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