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Cost Malmquist Productivity Index in Supply Chain

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Abstract. The index is excellent by the Malmquist index as extended to productivity measurement. The index developed here is defined in terms of input cost rather than input quantity distance functions in supply chain. Therefore, we propose productivity change is decomposed into overall efficiency and cost technical change. These decompositions provide a clearer situation of the root sources of supply chain productivity change, so that illustrated here in a sample of supply chain; so that results are computed using non-parametric mathematical programming.

Keywords: DEA, Productivity change, Malmquist index, Supply chain.

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1. Introduction

In recent years the analyses of productivity growth is one of the major sources of economic development and particularly supply chain management. On the other hand, performance evaluation is an important issue for supply chain where can be measured both in terms of customers level of satisfaction and the costs incurred, where supply chain comprising a key factor of corporate success [2]. Combining these multiple aspects, SCM can be defined as a systemic and strategic coordination of planning and managing production transportation and distribution until it reaches the end user [6] and [9]. The ability to integrate best supply chain performance practices is one way of defining productivity growth (major sources of economic development). Therefore, productivity measurement is an important research topic of supply chain management. Economists have traditionally focused on

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technological change for operations managed by a supply chain. This perspective is consistent with the competitive, independent company [10]. However, studies on productivity measurement of supply chain performance are still quite limited, particularly cost Malmquist productivity Index. A very useful method for productivity measurement in Data Envelopment Analysis (DEA) is the Malmquist productivity index [1, 3–5, 7] and [8] proposed a global Malmquist productivity index. In this paper, our research effort has focused on the investigation of the causes of productivity change and on its decomposition. DEA non-parametric mathematical programming is used to compute productivity and so this method is based on the cost Malmquist productivity index of period. The reminder of our work is organized as follows. Section 2 our method is proposed. In section 3, numerical examples are presented to demonstrate the method. In the last section conclusions is given.

2. Methodology

Suppose there are n supply chains to be evaluated in light of P inputs and Q outputs. For period t, let us define an input vector $x_j^t = (x_{1j}^t, ..., x_{Pj}^t)$ and an output vector $y_j^t = (y_{1j}^t, ..., y_{Qj}^t)$ and an intermediate product vector $i_j^t = (i_{1j}^t, ..., i_{Kj}^t)$. Denote by x_{pj}^t, i_{kj}^t and y_{qj}^t as well as , x_{pj}^{t+1}, i_{kj}^{t+1} and y_{qj}^{t+1} the inputs, intermediate product and outputs of supply chain (SC_j) at time periods t and *t+1*, respectively, where $p = 1, 2, ..., P; k = 1, 2, ..., K; q = 1, 2, ..., Q; j = 1, 2, ..., n$. [7], Fig.1.

When input price of supplier, $c_j^t = (c_{1j}^t, ..., c_{Pj}^t)$ and input (intermediate product) price of manufacture $w_j^t = (w_{1j}^t, ..., w_{Kj}^t)$ are available, we propose the following models to find optimal value of $Cost^t(y^t, c^t)$ i.e. cost of producing of period.

$$\begin{aligned}
&Cost^t(y^t, c^t) = \min(c_{pd}^t + w_{kd}^t i_{kd}^t)\\
&\text{s.t}\\
&\sum_{j=1}^{n} \lambda_j \times c_{pj}^t x_{pj}^t \leqslant c_{pd}^t \times x_{pd}^t, p = 1, 2, ..., P\\
&\sum_{j=1}^{n} \lambda_j \times i^{kj} \geqslant i_{kd}^t, k = 1, 2, ..., K\\
&\sum_{j=1}^{n} \eta_j \times w_{kj}^t i_{kj}^t \leqslant w_{kd}^t \times i_{kd}^t, k = 1, 2, ..., K\\
&\sum_{j=1}^{n} \eta_j \times y_{qj}^t \geqslant y_{qd}^t, q = 1, 2, ..., Q\\
&\lambda_j \geqslant 0, \eta_j \geqslant 0, i_{kd}^t \geqslant 0, x_{pd}^t \geqslant 0, j = 1, 2, ..., n, \quad k = 1, 2, ..., K, \quad p = 1, 2, ..., P.
\end{aligned} \tag{1}$$

Using *t+1* instead of t for the above model, we get the other cost efficiency score for SC_j in time period *t+1* where it is defined as $Cost^{t+1}(y^{t+1}, c^{t+1})$. The first of the mixed period measures, which is defined as $Cost^t(y^{t+1}, c^t)$ (defines the minimum cost of producing a given output vector y^{t+1} given the input prices c^t and the technology of period t) is calculated as optimal value to the following linear programming problem:

$$\begin{aligned}
&Cost^t(y^{t+1},c^t)=\min(c^t_{pd}x^{t+1}_{pd}+w^t_{kd}i^{t+1}_{kd})\\
&\text{s.t}\\
&\sum_{j=1}^{n}\lambda_j\times c^t_{pj}x^t_{pj}\leqslant c^t_{pd}\times x^{t+1}_{pd},\quad p=1,2,...,P\\
&\sum_{j=1}^{n}\lambda_j\times i^t_{kj}\geqslant i^{t+1}_{kd},k=1,2,...,K\\
&\sum_{j=1}^{n}\eta_j\times w^t_{kj}i^t_{kj}\leqslant w^{t+1}_{kd}\times i^{t+1}_{kd},k=1,2,...,K\\
&\sum_{j=1}^{n}\eta_j\times y^t_{qj}\geqslant y^{t+1}_{qd},q=1,2,...,Q\\
&\lambda_j\geqslant 0,\eta_j\geqslant 0,i^{t+1}_{kd}\geqslant 0,x^{t+1}_{pd}\geqslant 0,j=1,2,...,n,\quad k=1,2,...,K,\quad p=1,2,...,P.
\end{aligned}\tag{2}$$

Using *t+1* instead and vice versa, this is defined as $Cost^{t+1}(y^t,c^{t+1})$.

$$\begin{aligned}
&Cost^{t+1}(y^t,c^{t+1})=\min(c^{t+1}_{pd}x^t_{pd}+w^{t+1}_{kd}i^t_{kd})\\
&\text{s.t}\\
&\sum_{j=1}^{n}\lambda_j\times c^{t+1}_{pj}x^{t+1}_{pj}\leqslant c^{t+1}_{pd}\times x^t_{pd},\quad p=1,2,...,P\\
&\sum_{j=1}^{n}\lambda_j\times i^{t+1}_{kj}\geqslant i^t_{kd},k=1,2,...,K\\
&\sum_{j=1}^{n}\eta_j\times w^{t+1}_{kj}i^{t+1}_{kj}\leqslant w^{t+1}_{kd}\times i^t_{kd},k=1,2,...,K\\
&\sum_{j=1}^{n}\eta_j\times y^{t+1}_{qj}\geqslant y^t_{qd},q=1,2,...,Q\\
&\lambda_j\geqslant 0,\eta_j\geqslant 0,i^t_{kd}\geqslant 0,x^t_{pd}\geqslant 0,j=1,2,...,n,\quad k=1,2,...,K,\quad p=1,2,...,P.
\end{aligned}\tag{3}$$

We propose cost Malmquist productivity index of period *t*, *t+1*

$$SC.CM=\left[\frac{(c^tx^{t+1}+w^ti^{t+1})/Cost^t(y^{t+1},c^t)}{(c^tx^t+w^ti^t)/Cost^t(y^t,c^t)}\times\frac{(c^{t+1}x^{t+1}+w^{t+1}i^{t+1})/Cost^{t+1}(y^{t+1},c^{t+1})}{(c^{t+1}x^t+w^{t+1}i^t)/Cost^{t+1}(y^t,c^{t+1})}\right]^{1/2}\tag{4}$$

Where $c^tx^t=\sum_{p=1}^{P}c^t_px^t_p$, p denotes the pth input of supplier and $w^ti^t=\sum_{k=1}^{K}w^t_ki^t_k$, k denotes the kth input of manufacturer. And also the cost ratios in (4) represent inflation factors. These factors were defined in terms of input quantities in the cost Malmquist productivity index. On the other hands the cost ratio $(c^tx^t+w^ti^t)/Cost^t(y^t,c^t)$ measures the extent to which the aggregate production cost in period t can be reduced while still securing the output vector y^t under the input price vector c^t [7]. The value lesser than 1 for index of $SC.CM$ is demonstrator of improvement in utilization, the value bigger than 1 equal to deuce of utilization and the value equal to 1 is demonstrator of stable utilization. To be continued, we show that index of $SC.CM$ can analyze to elements which made useful views toward main source of utilization in our authority. In this paper a decomposition of productivity change is introduced:

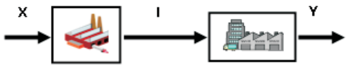

Figure 1. Two-stage process of supply chain, [6]

$$SC.CM = \underbrace{\frac{(c^{t+1}x^{t+1} + w^{t+1}i^{t+1})/Cost^{t+1}(y^{t+1}, c^{t+1})}{(c^{t}x^{t} + w^{t}i^{t})/Cost^{t}(y^{t}, c^{t})}}_{SC.CEC} \times$$
$$\underbrace{\left[\frac{(c^{t}x^{t+1} + w^{t}i^{t+1})/Cost^{t}(y^{t+1}, c^{t})}{(c^{t+1}x^{t+1} + w^{t+1}i^{t+1})/Cost^{t+1}(y^{t+1}, c^{t+1})} \times \frac{(c^{t}x^{t} + w^{t}i^{t})/Cost^{t}(y^{t}, c^{t})}{(c^{t+1}x^{t} + w^{t+1}i^{t})/Cost^{t+1}(y^{t}, c^{t+1})}\right]^{1/2}}_{SC.CTC} \quad (5)$$

Namely, the cost Malmquist productivity index can be decomposed into SC.CEC and SC.CTC, as follows:
SC.CEC: Supply chain Cost efficiency change
SC.CTC: Supply chain Cost technical change

Therefore, if $SC.CEC > 1$ indicates degrease of cost efficiency in the supply chaind from period t to $t+1$, while $SC.CECd = 1$ and $SC.CECd < 1$ respectively the status quo and deterioration in the cost efficiency.

3. Illustrative example

In this section, numerical example is given to illustrate the proposed method. Table 1 use two inputs to produce a intermediate product and Here, we consider to performance of the seven supply chain so that these chains include of the two members such as supplier and manufacturer. The suppliers of supply chains pieces consume to the factors input and it achieve to the output with selling of its products to profit manufacturer in this phase. In the following, manufacturer suffers to spend of expense for buy to products and primary materials of supplier and it produce the output as validity in producing chain.

Utilization index of Malmquist coat is shown in Table 2 so that second and third columns are demonstrator of efficiency of supply chain in t and $t+1$ times and fourth column is efficiency of supply chain cost in $t+1$ time but in regard to cost

Table 1. Table 1. List of 7 supply chain

				Period	(1)			
NO.	X_1	C_1	X_2	C_2	I_1	W_1	Y_1	Y_2
SC1	25	2	100	2.5	11	2.5	250	160
SC2	14	2.5	65	1.5	25	3	114	450
SC3	5	1.15	89	2	81	1.5	350	250
SC4	35	3	21	3	54	3	425	550
SC5	41	2	56	1.5	46	2	60	220
SC6	21	1.5	94	5	20	1.5	115	95
SC7	19	2.5	75	4	80	2.5	95	200
				Period	(2)			
NO.	X_1	C_1	X_2	C_2	I_1	W_1	Y_1	Y_2
SC1	35	3	120	2	21	3.5	350	260
SC2	24	1.5	55	2.5	25	4	134	400
SC3	15	2.5	89	2	75	2.5	330	220
SC4	15	1.5	35	2.5	64	3	425	580
SC5	41	2.5	66	3.5	40	3	160	220
SC6	11	1.5	84	6	50	2.5	105	195
SC7	29	3.5	85	3	75	4.5	95	250

time of t and also, fifth column is cost of supply chain in t time and in regard to cost of next year, in other words, fourth and fifth columns calculate after replacing time courses of t and *t+1*. The change of efficiency of cost in sixth column and its survey is cited in seventh column also. Finally, decrease and increase of efficiency of Malmquist in final column is shown in regard to $SC.CM$ value.

Table 2. Table 2. Cost Malmquist productivity index of supply chain

NO.	$Cost^t$ (y^t, c^t)	$Cost^{t+1}$ (y^{t+1}, c^{t+1})	$Cost^t$ (y^{t+1}, c^t)	$Cost^{t+1}$ (y^t, c^{t+1})	CEC	State	CM	Results
SC1	54.454	110	84.81	78.281	0.8843	C.E.I	1.0488	P.R
SC2	130	140	120	160	1.3917	C.E.D	1.1932	P.R
SC3	110	120	94.861	120	1.1817	C.E.D	1.0819	P.R
SC4	160	240	170	230	0.3990	C.E.I	0.8315	P.G
SC5	78.292	90.899	79.141	86.51	1.8845	C.E.D	1.1098	P.R
SC6	40.688	84.43	82.207	44.616	0.9289	C.E.I	0.6321	P.G
SC7	63.917	86.874	79.716	70.348	0.8217	C.E.I	0.8172	P.G

C.E.I= Cost efficiency increase; C.E.D= Cost efficiency decrease; P.R= Productivity regress; P.G= Productivity growth

4. Conclusion

In this paper Malmquist productivity index (MPI) for supply chain (SC) has been evaluated. A method for assessing Malmquist productivity index using cost efficiency also has been developed, so that this paper develops supply chain productivity index applicable when producers are cost minimize and input prices are known.

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