Sliding Mode Control based On Fractional order Calculus for DC-DC Converters

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Abstract. The aim of this paper is to design a Fractional Order Sliding Mode Controllers (FOSMC) for a class of DC-DC converters such as boost and buck converters. Firstly, the control law is designed with respect to the properties of fractional calculus, the design yields an equivalent control term with an addition of discontinuous (attractive) control law. Secondly, the mathematical proof of the stability condition and convergence of the proposed fractional order sliding surface is presented. Finally the effectiveness and robustness of the proposed approaches compared with classical SMCs are demonstrated by simulation results with different cases.

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1. Introduction

Since the middle of the 20th century, DC-DC converters have gained an increasing place in industrial applications, especially in the field of power electronics. These converters are electronic circuits which convert a voltage from one level to a higher or lower one. Among these converters we have the Buck and Boost converter.

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Many works are reported for the regulation of the output voltage of these converters in the closed loop, such as the Proportional Integral (PI) and hysteretic control....etc.

Besides, the Sliding Mode Control (SMC) for example was largely proved its efficiency through the reported theoretical studies [1], [2], [3], [4]. The first step of SMC design is to select a sliding surface that models the desired closed-loop performance in state variable space. The second step is to design the equivalent and a hitting control law such as the system state trajectories forced toward the sliding surface and slides along it to the desired attitude.

Many works based on sliding mode are reported in integer order control of DC-DC converters [5], [6], [7], [8] before the apparition of the fractional order sliding mode control by [9], [10] and [11]; these fractional controllers are based on the fractional order PI, PD and PID sliding surfaces.

Motivated by the above discussion this paper designs a Fractional Order Sliding Mode Controller (FOSMC) for buck and boost converters in which the sliding surface proposed for the buck converter is a generalization of the classical PD sliding surface $S = \dot{e} + \lambda e$, where the integer order derivative of error will be of fractional order $(D_t^{(\alpha-1)}\dot{e}, \; 0 \prec \alpha \prec 1)$ and the term $(\lambda.e)$ will be $(\lambda.e^P, 0<P<1)$ (see [12]).

With these two added parameters (α and P) we can say that, the performance of the system can be improved.

For boost converter, the proposed fractional order sliding surface exploits the advantages of the fractional integrator that are fast convergence and precision, in which the integer order sliding surface S= λ.e [8] will be $S = \lambda D_t^{-\alpha}e, \; 0 \prec \alpha \prec 1$.

The rest of this article is organized as follows. Basic Definitions of Fractional Calculus in section II. DC-DC converters in section III. The fractional order sliding mode controller design in section IV. And finally the simulation results and conclusion are given in Sections V and VI, respectively.

2. Basic Definitions of Fractional Calculus

The fractional differo-integral operators denoted by $_aD_t^\alpha f(t)$ (where a and t are the bounds of the operation) are a generalization of integration and differentiation of the operators of a non integer order. In the literature we find different definitions of fractional differo-integral, but the commonly used are:

The Riemann-Liouville (RL) definition:

$$_aD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \tag{1}$$

The Caputo's definition:

$$_aD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \tag{2}$$

Where m-1$<\alpha<m$ and $\Gamma(.)$ is the well-known Euler's gamma function, and its definition is:

$$\Gamma(x) = \int_0^\infty e^{-t} t^{(x-1)} dt, \; x > 0 \tag{3}$$

On the other hand, Grunwald-Letnikov (GL) reformulated the definition of the fractional order differ-integral as follows:

$$_aD_t^\alpha f(t) = \underset{h \to 0}{\lim} \frac{1}{h^\alpha} \sum_{k=0}^{(t-a)/h} (-1)^k \binom{\alpha}{k} f(t-kh) \tag{4}$$

Because the numerical simulation of a fractional differential equation is not simple as that of an ordinary differential equation [21], [22], so the Laplace transform method is often used as being a tool for the resolution of the problems arising in engineering [13], [14].

In the following, we give the Laplace transforms of the fractional order derivative given previously.

The Laplace transform of (RL) definition is as follows [13], [15]:

$$L\left\{_0D_t^\alpha f(t); s\right\} = s^\alpha F(s) - \sum_{k=0}^{(m-1)} s^k \left[_0D_t^{(\alpha-k-1)} f(t)\right]_{t=0} \tag{5}$$

The Laplace transform of Caputo's definition is given by [15]:

$$L\left\{_0D_t^\alpha f(t); s\right\} = s^\alpha F(s) - \sum_{k=0}^{(m-1)} s^{(\alpha-k-1)} f^k(0) \tag{6}$$

Where $s = jw$ denotes the Laplace operator. For zero initial conditions, the Laplace transform of fractional derivative of Riemann-Liouville, Caputo and Grunwald-Letnikov reduced to (7) [15], [16].

$$L\left(_0D_t^\alpha f(t)\right) = s^\alpha F(s) \tag{7}$$

In this paper the fractional order element s^α is approximated with Oustaloup's filter. The Oustaloup's filter [17] is based on the approximation of a function of the form:

$$G(s) = s^\alpha, \ \alpha \in R^+ \tag{8}$$

By a rational function:

$$\hat{G}(s) = K' \prod_{k=-N'}^{N'} \frac{s + w_k'}{s + w_k} \tag{9}$$

Where the parameters of this function (zeros, poles, and gain) can be determined by the following formulas:

$$
\begin{aligned}
w'_k &= w_b \cdot \left(\frac{w_h}{w_b} \right)^{(k+N'+0.5(1-\alpha))/(2N'+1)} \\
w'_k &= w_b \cdot \left(\frac{w_h}{w_b} \right)^{(k+N'+0.5(1+\alpha))/(2N'+1)} \\
K &= w_h^\alpha
\end{aligned}
\tag{10}
$$

$(2N'+1)$ is the order of the filter, w_b and w_h are respectively the Low and High transient-frequencies.

The following properties of Caputo's definition are used in this paper:

Fractional order derivative of fractional integration of a function $f(t)$ [15]:

$$
D_t^\alpha \left(D_t^{-\alpha} f(t) \right) = f(t)
\tag{11}
$$

Fractional integration of fractional order derivative of a function $f(t)$ [15]:

$$
D_t^{-\alpha} \left(D_t^\alpha f(t) \right) = f(t) - f(0) \quad \text{for} \quad 0 < \alpha < 1
\tag{12}
$$

3. DC-DC Converters

The DC/DC converters are electronic circuits allow to generate a continuous and variable source of tension from a continuous and fixed source of tension.

In general case these converters consist of a switch (S_w) with control input u between 0 and 1, a fast diode D and R, L, C components.

For the controller design, it is necessary to give the mathematical model of these converters, which can be obtained by applying the two Kirchhoff's laws (current and voltage).

3.1 Boost Converter

The boost converter as given in figure 1, it steps up the input voltage to produce a higher output voltage.
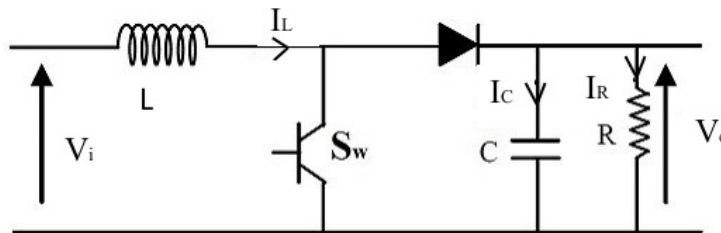

Figure 1. DC-DC Boost converter schematics

The state space equations when the power switch (S_w) is in *ON* state are represented by:

$$
\begin{aligned}
\dot{I}_L &= \frac{1}{L} \cdot (V_i) \\
\dot{V}_o &= \frac{1}{C} \cdot \left(\frac{V_o}{R} \right)
\end{aligned}
\tag{13}
$$

And, when the power switch (S_w) is in *OFF* state:

$$\dot{I}_L = \frac{1}{L} \cdot (V_i - V_o)$$
$$\dot{V}_o = \frac{1}{C} \cdot (I_L - \frac{V_o}{R})$$

(14)

The selection of the output voltage (V_o) and inductor current (I_L) as state variables of the system, such as:

$$x_1 = I_L$$
$$x_2 = V_o$$

(15)

Leads to state space model describing the system as the following:

$$\dot{x}_1 = \frac{(V_i - x_2)}{L} + \frac{x_2}{L} \cdot u$$
$$\dot{x}_2 = \frac{1}{C}(x_1 - \frac{x_2}{R}) - \frac{x_1}{C} \cdot u$$

(16)

Where the two equations below are considered for simplification of calculations:
$$f_1(x) = \frac{(V_i - x_2)}{L}, \quad b_1(x) = \frac{(x_2)}{L}$$
$$f_2(x) = \frac{1}{C}(x_1 - \frac{x_2}{R}), \quad b_2(x) = \frac{-(x_1)}{C}$$

3.2 Buck Converter

The buck converter as shown in figure 2, is a voltage step down and current step up converter.
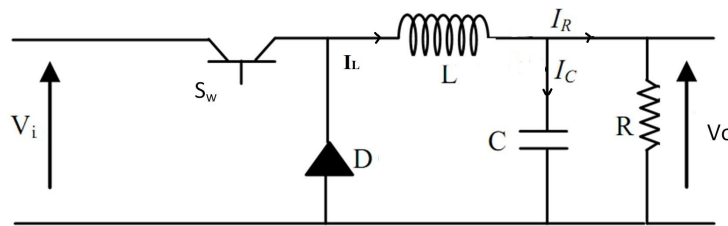

Figure 2. DC-DC Buck converter schematics

When the ideal switch (S_w) is in *ON* state, the dynamics of the inductor current $I_L(t)$ and the output voltage $V_o(t)$ are given by:

$$\dot{I}_L = \frac{1}{L} \cdot (V_i - V_o)$$
$$\dot{V}_o = \frac{1}{C} \cdot (I_L - \frac{V_o}{R})$$

(17)

And when the switch (S_w) *is in OFF state* :

$$\dot{I}_L = \frac{1}{L} \cdot (V_o)$$
$$\dot{V}_o = \frac{1}{C} \cdot (I_L - \frac{V_o}{R})$$

(18)

Selecting the output voltage (V_o) and its derivative (dV_o/dt) as system state variables, that is:

$$
\begin{aligned}
x_1 &= V_o \\
x_2 &= \dot{V}_o
\end{aligned}
\tag{19}
$$

Leads to the state space model describing the system, derived as:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{x_1}{LC} - \frac{x_2}{RC} + \frac{V_i}{LC}u
\end{aligned}
\tag{20}
$$

For the controller design we set the following simplification:
$f_1(x) = -\frac{x_1}{LC} - \frac{x_2}{RC}$ and $b_1(x) = \frac{V_i}{LC}$

4.   Fractional order Sliding Mode Controller (FOSMC) Design

The sliding mode control strategy is divided into three steps such as:

(1) **Selection of sliding surface**: it is usually designed to full fill the desired control objectives.
(2) **Calculation of the controller** $u(t)$ which is given by the following equation :

$$
u(t) = u_{eq}(t) + u_n(t)
\tag{21}
$$

Where u_{eq} is the equivalent control law that is derived by setting $\dot{S}(t) = 0$, u_n is called the discontinuous (or attractive) control low.

(3) **Stability analysis**: For the stability analysis, the candidate Lyapunov function given below is considered:

$$
V = \frac{1}{2}S^2
\tag{22}
$$

We say the system is stable when we have:

$$
\dot{V} = S\dot{S} \leq 0
\tag{23}
$$

4.1   *FOSMC for Boost Converter*

For the boost converter presented by equation (16), we propose the following sliding surface based on fractional order integrator as follows:

$$
S(t) = \lambda D_t^{-\alpha}e(t)
\tag{24}
$$

Where $e(t) = x_1(t) - x_{1d}(t)$, λ is positive constant, and $0 \prec \alpha \prec 1$

Remark: it is clear that selecting $\alpha = 0$ the classical sliding surfaces $S(t) = \lambda e(t)$ can be recovered [7].

The desired current (x_1) is obtained from the outer voltage loop as followings [7]:

$$x_{1d} = \frac{V_o^2}{R.V_i} \tag{25}$$

Differentiating both sides of (24) to the order unity yields the equality in (26):

$$\begin{aligned}
\dot{S}(t) &= \lambda D_t^{-\alpha} \dot{e}(t) \\
&= \lambda D_t^{-\alpha}(\dot{x}_1(t) - \dot{x}_{1d}(t)) \\
&= \lambda D_t^{-\alpha}(f_1(x) + b_1(x).u - \dot{x}_{1d}(t))
\end{aligned} \tag{26}$$

By setting $\dot{S}(t) = 0$ the equivalent control is obtained, and it has the owing formula:

$$u_{eq}(t) = \frac{-1}{b_1(x)}(f_1(x) - \dot{x}_{1d}(t)) \tag{27}$$

Then, the global control is given by:

$$u(t) = \frac{-1}{b_1(x)}(f_1(x) - \dot{x}_{1d}(t) + K.D_t^{\alpha}(sgn(S))) \tag{28}$$

Where:

$$sgn(S) = \begin{cases} -1 & if & S \prec 0 \\ 0 & if & S = 0 \\ 1 & if & S \succ 0 \end{cases} \tag{29}$$

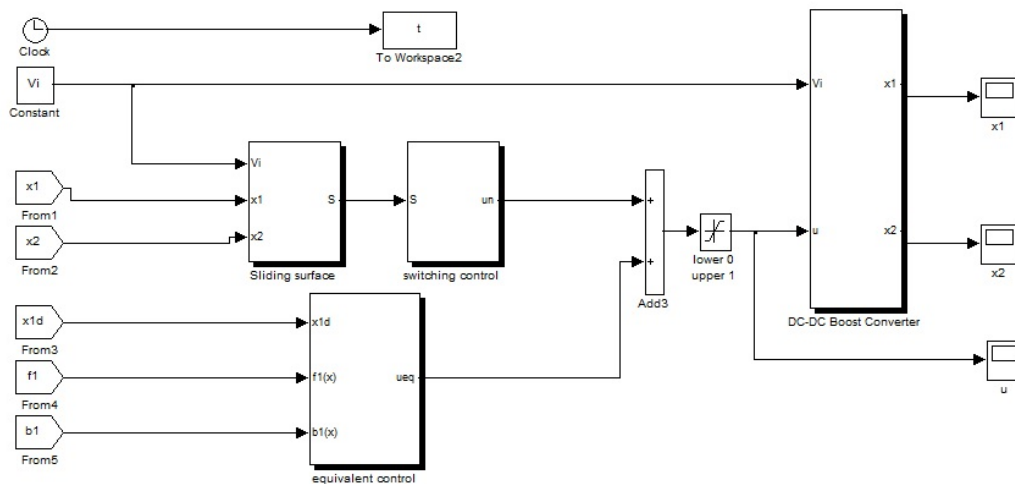The following figure shows the functional diagram of proposed FOSMC for DC-DC Boost converter.

Figure 3. Functional diagram of proposed FOSMC for boost converter

For the stability analysis, substituting of (28) in (26) results:

$$\dot{S}(t) = -\lambda.K.(sgn(S(t))) - \lambda.K.(sgn(S(0))) \tag{30}$$

For initial condition $x_1(0) = 0$, the sliding surface S at $t=0$ is 0, then equation (30) can be rewritten as the following:

$$\dot{S}(t) = -\lambda.K.(sgn(S(t))) \tag{31}$$

Using equation (23):

$$\begin{aligned} S\dot{S} &= -\lambda.K.S.(sgn(S)) \\ &= \lambda.K.\,|S| \leq 0 \end{aligned} \tag{32}$$

As conclusion, the proposed sliding surface can satisfy the stability condition; on the other hand the *sgn* function can causes the chattering phenomenon; and to avoid this problem we replace the *sgn* function by a saturation (*sat*) one; and the control signal $u(t)$ will be:

$$u(t) = \frac{-1}{b_1(x)}(f_1(x) - \dot{x}_{1d}(t) + K.D_t^\alpha(sat(S))) \tag{33}$$

where:

$$\mathrm{sat}(\phi) = \begin{cases} \phi & if \ |\phi| < 1 \\ sgn(\phi) & if \ |\phi| \geq 1 \end{cases} \tag{34}$$

4.2   *FOSMC for Buck Converter*

For the Buck converter presented by (20), firstly we define the following fractional order sliding surface using Caputo's definition as:

$$S = D_t^{(\alpha-1)}\dot{e} + \lambda e^P \tag{35}$$

This proposed sliding surface is somewhat similar to [12], it has the potential to improve the control performance, because extra real parameters α and P are involved.
    Where $e = x_1 - x_{1d}$, λ is positive constant, and $0 < \alpha$, $P < 1$.
Remark: It is clear that selecting $\alpha = P = 1$, the classical sliding surface $S = \dot{e} + \lambda e$ can be recovered.

Differentiating both sides of (35) to the order unity yields the equality in (36);

$$\dot{S} = D_t^{(\alpha-1)}(\ddot{x}_1 - \ddot{x}_{1d}) + \lambda P(e^{(P-1)}\dot{e}) \tag{36}$$

The expression of the equivalent control is easily derived by setting $\dot{S} = 0$ as:

$$u_{eq} = \frac{-1}{b_1(x)}\left[f_1(x) - \ddot{x}_{1d} + \lambda.P.D_t^{(1-\alpha)}(e^{(P-1)}.\dot{e})\right] \tag{37}$$

The switching or attractive control u_n is generally equal to $(-K\text{sign}(S))$. But a large control gain K often causes the chattering effect. In order to tackle this problem, several method of SMC with reaching law, were designed [18], [19]. In this paper we chose the one of [20] given by the following equation of integer order:

$$u_n = -K\,sign(S) - \rho S \qquad (38)$$

Because the controller is of fractional order, this structure will be modified (see below) to satisfy the stability condition.

Then, the global fractional order sliding mode controller u will be:

$$u = \frac{-1}{b(x)}\left[f(x) - \ddot{x}_{1d} + \lambda.P.D_t^{(1-\alpha)}(e^{(P-1)}.\dot{e})... \atop +D_t^{(1-\alpha)}(Ksgn(S) + \rho.S)\right] \qquad (39)$$

The following figure summarizes the developed controller for DC-DC buck converter.
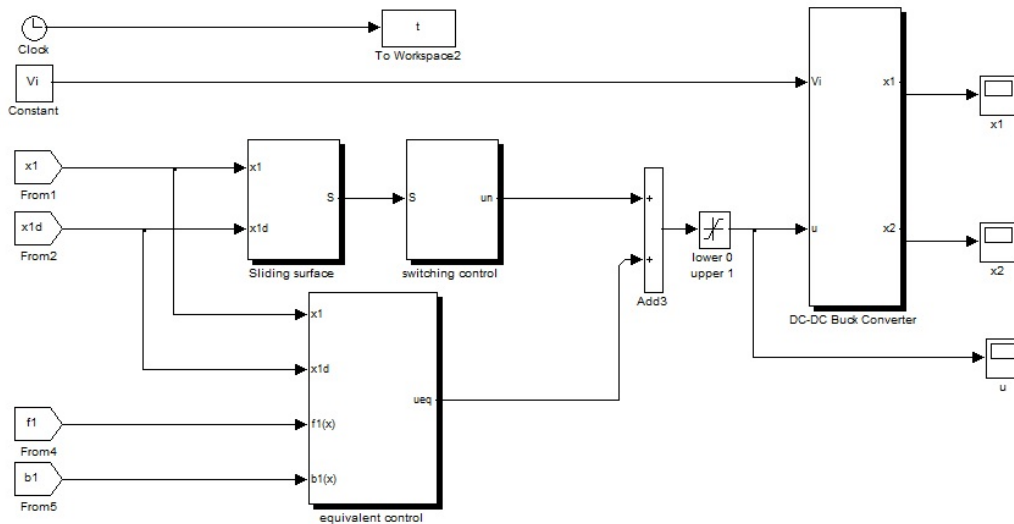

Figure 4. Functional diagram of proposed FOSMC for buck converter

Substituting (39) into (36), and taking into account the properties in (11) and (12), results in:

$$S\dot{S} = S\left[-\lambda.P.e^{(P-1)}(0).\dot{e}(0) - (Ksgn(S) + \rho.S) \atop -(K.sgn(S(0)) + \rho.S(0))\right] \qquad (40)$$

If one assume that:
$-\lambda.P.e^{(P-1)}(0).\dot{e}(0) - (K.sgn(S(0)) + \rho.S(0)) = 0$ then simply:

$$S\dot{S} = S\left[-(Ksgn(S) + \rho.S)\right] \atop = -K\,|S| - \rho.S^2 \leq 0 \qquad (41)$$

Otherwise, if:

$$\lambda.P.e^{(P-1)}(0).\dot{e}(0) + (K.sgn(S(0)) + \rho.S(0)) \leq \zeta \tag{42}$$

This lets us have:

$$\begin{aligned} S\dot{S} &= S\left[-(Ksgn(S) + \rho.S) - \zeta\right] \\ &= -(K + \zeta)\left|S\right| - \rho.S^2 \leq 0 \end{aligned} \tag{43}$$

In summary; the proposed fractional order sliding surface can guarantee the stability condition.

5. Simulation Results

5.1 *For Boost Converter*

The simulation of the functional diagram in figure 3 is carried out using the "Matlab/Simulink" tools with different cases. Where the following specifications are used.

Parameters of the converter: L=10 mH, C=100 μF.

Parameters of the controller: λ=10, K=20, α= variable.

First case:

R=30 Ω, V_o=40 volt, Vi=12 volt.

Second case:

R: variable from 50 to 30 Ω.

Third case:

Vi: variable from 12 to 20 volt.

Fourth case:

Vo: variable from 40 to 25 volt.

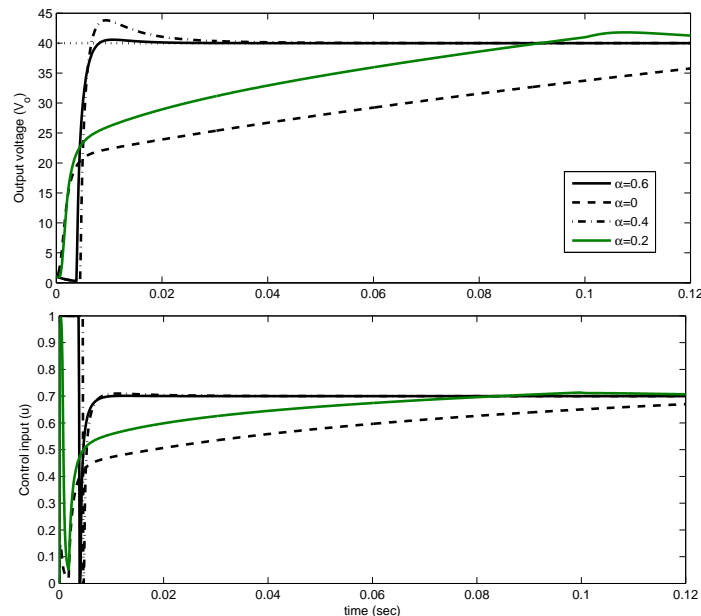The obtained results are given in the following figures.

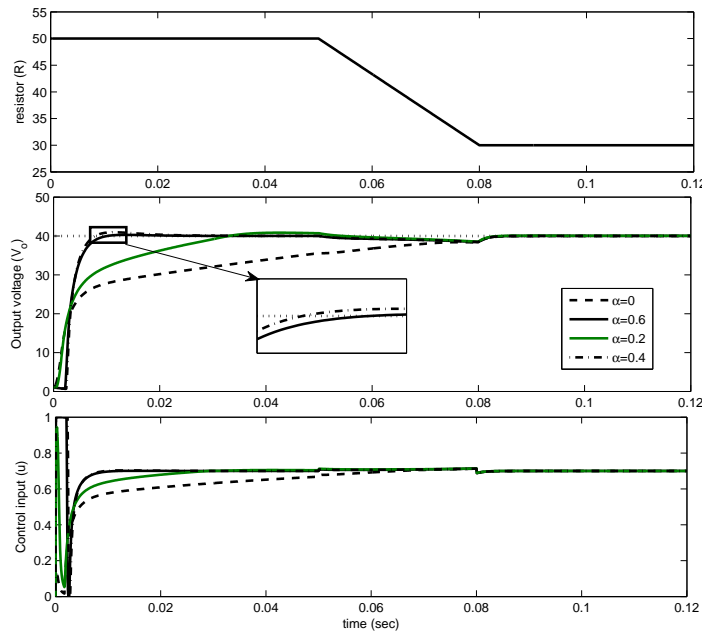Figure 5. Simulation results of the first case for boost converter

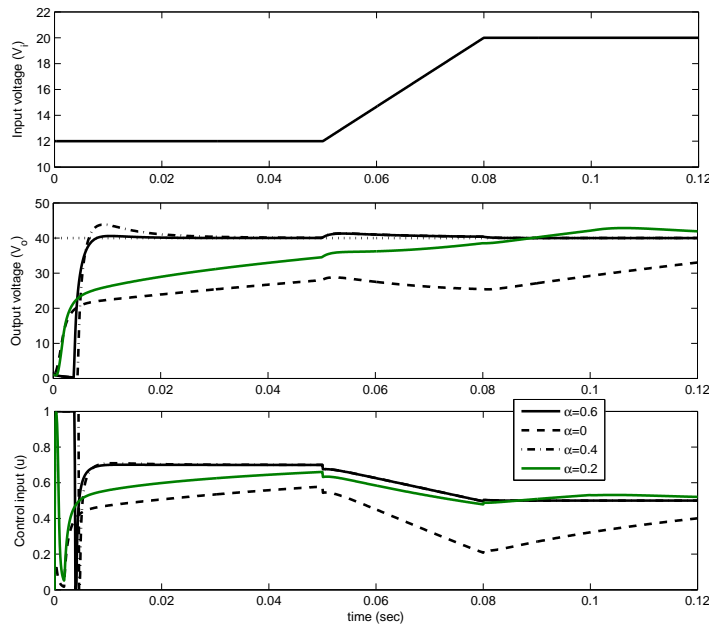Figure 6. Simulation results of the second case for boost converter

Figure 7. Simulation results of the third case for boost converter

From the simulation results given by figures 5-8, it can be seen that the proposed approach of FOSMC gives the best results compared to the conventional one of integer order, and that is due to the integrator effect that it can accelerate the settling time around $S=0$.

5.2 *For Buck Converter*

For the simulation of the functional diagram of figure 4, the used parameters are the following:
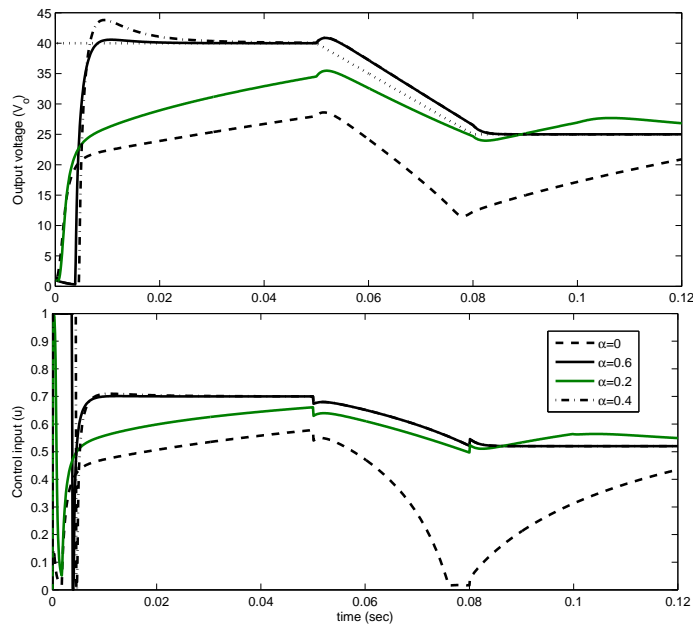
Buck converter: $L=10\ mH$, $C=100\ \mu F$

Figure 8. Simulation results of the fourth case for boost converter

Controller: λ=7, P=0.25, α=0.8, K=35, ρ= 60.

The following cases are discussed:

First case:

V_{in}=24 volt, Vo= 12 volt and R=50 Ω.

Second case:

R: variable from 50 to 30 Ω.

Third case:

V_i: variable from 15 to 20 volt.

Fourth case**:**

Vo: variable from 12 to 20 volt.

The simulation results are shown in the following figures:

From figures 9-12, it is clear that the proposed FOSMC gives the best control specification such as small settling time, and precision in trajectory tracking, compared with the classical one of integer order, because of the two added extra parameters α and P.

6.   Conclusion

In this paper a proposed fractional order sliding mode control techniques are investigated for the DC-DC buck and boost converters. The obtained simulations results with different cases confirm the effectiveness and robustness of these control techniques, which give best results in term of small settling time and precision compared with classical SMC based on integer order sliding surfaces.

Ongoing research will involve an optimization of the proposed controller, using the intelligence of particle swarms. Also we will try to implement the optimized controller on FPGA or DSP circuit.
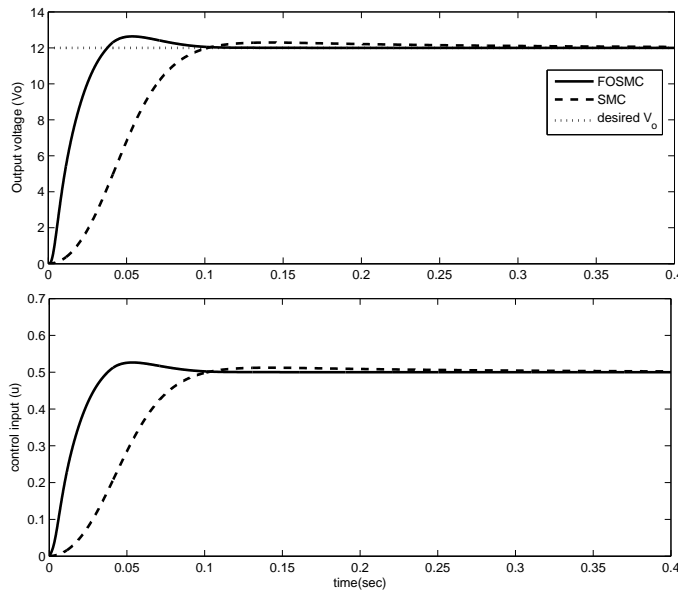
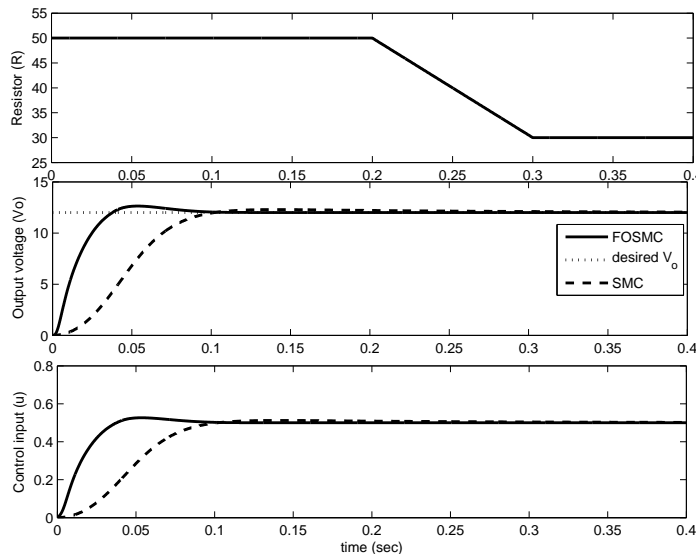Figure 9. Simulation results of the first case for buck converter

Figure 10. Simulation results of the second case for buck converter

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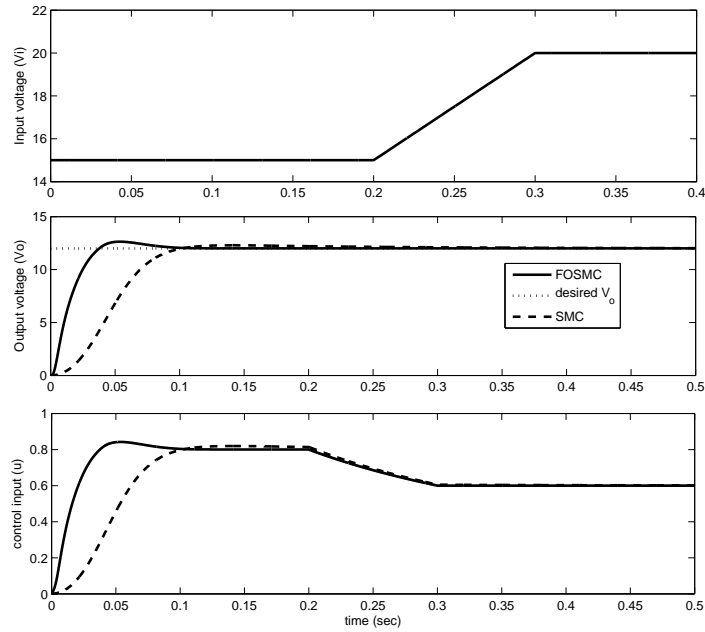
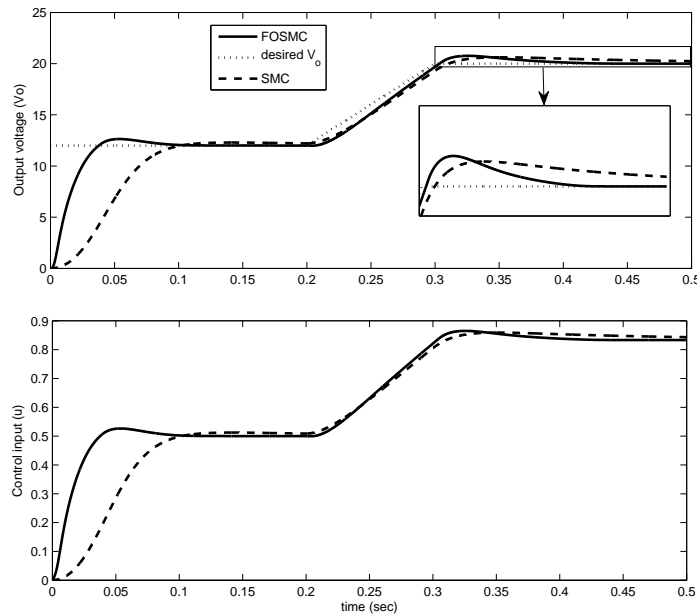Figure 11. Simulation results of the third case for buck converter

Figure 12. Simulation results of the fourth case for buck converter

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