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A Non-radial Rough DEA Model

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Abstract. There are situations that Decision Making Units (DMU's) have uncertain information and their inputs and outputs cannot alter redially. To this end, this paper combines the rough set theorem (RST) and Data Envelopment Analysis (DEA) and proposes a non-redial Rough-DEA (RDEA) model so called additive rough-DEA model and illustrates the proposed model by a numerical example.

Keywords: Rough data envelopment analysis (RDEA), Data envelopment analysis (DEA), Uncertainty.

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1. Introduction

Data envelopment analysis (DEA), was first put forward by Charnes et. al. (CCR model) in 1978 [1], it is performance measurement technique which can be used for evaluating the relative efficiency of Decision Making Units (DMU's) in organizations. One of research of DEA is Rough-DEA that researches on Rough-DEA (RDEA) are still very restricted. Therefore, the research on combining DEA with Rough set theory is an attractive study field. Pawlak introduced a theory of Rough sets in 1982[4]. Since then rough-set theory has been developed very rapidly and has resulted in a number of applications. In order to provide an axiomatic theory to describe rough events, Liu [3], established the thrust theory which is branch of mathematics that studies the behavior of rough events. Liu [3], defined a rough variable ξ by a measurable function from a rough space $(\mu, \Delta, \Lambda, \pi)$ to the set of real numbers, that is for any Borel set B of R, $\{\lambda; \xi(\lambda) \in B\} \in \Lambda$ hold-Based on

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the thrust theory, Liu, A., et al., [2,3], studied some rough programming models with variables as parameters. The remainder of this paper is organized as follows.

Section 2 wants to bring some concepts about measurable space, Rough space and trust measure. Section 3 presents the Additive Rough DEA (ARDEA) model; and section 4 provides an application example to illustrate the efficiency of the ARDEA model. Finally, concluding remarks are outlined in section 5.

2. Background

This section recalls several concepts and definitions of rough variable theory to characterize the rough uncertainty.

DEFINITION 2.1 *Let μ be a nonempty sets, a collection Λ is called a σ-algebra of subsets of μ if is satisfied in the following conditions [3]:*

a) $\mu \in \Lambda$;
b) If $A \in \Lambda$, then $A^c \in \Lambda$;
c) If $A_i \in \Lambda$, for $i = 1, \ldots, n$ then $\cup_{i=1}^{\infty} A_i \in A$.

DEFINITION 2.2 *Let μ be a nonempty set, and Λ a σ-algebra of subsets of μ, then (μ, Λ) is called a measurable space and the elements in Λ are called measurable sets furthermore, if π is measure defined on (μ, Λ), then triplet (π, Λ, π) is called measure space [3].*

DEFINITION 2.3 *Let μ be a nonempty set, Λ a σ-algebra of subsets of μ, Δ an element in Λ, and π a set function on Λ, then $(\mu, \Delta, \Lambda, \pi)$ is called rough space, further more, the triplet (μ, Λ, π) is a measure space [3].*

DEFINITION 2.4 *Uncertain variables: An uncertain variable is a measurable function from an uncertainty space (μ, Λ, π) to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi \in B\} = \{\gamma \in \mu(\xi(\gamma) \in B)\}$ is an event [3].*

Example 2.5 Random variables, fuzzy variables and hybrid variable are instance of uncertain variable.

DEFINITION 2.6 *Uncertain vector: An n-dimensional uncertain vector is a measurable function from an uncertainly space (μ, A, π) to the set of n-dimensional real vectors, i.e., for any Borel set B of R^n, the $\{\xi \in B\} = \{\gamma \in \mu | \xi(\gamma) \in B\}$ is an event [3].*

DEFINITION 2.7
Let $(\mu, \Delta, \Lambda, \pi)$ be a rough space then the upper trust of event $A \in \Lambda$ is defined by $\bar{T}r\{A\} = \frac{\pi\{A\}}{\pi\{\mu\}}$ [3].

The lower trust of the event A is defined by $\underline{T}r\{A\} = \frac{\pi\{A \cap \Delta\}}{\pi\{\Delta\}}$ and the trust of the event A is defined by $Tr\{A\} = \frac{1}{2}(\bar{T}r\{A\} + \underline{T}r\{A\})$.

DEFINITION 2.8 *Let ξ be a rough variable, and $\alpha \in (0,1]$, then $\xi_{\sup}(\alpha) = \sup\{r | Tr(\xi \geqslant r) \geqslant \alpha\}$ is called the α-optimistic value to ξ and $\xi_{\inf}(\alpha) = \inf\{r | Tr(\xi \leqslant r) \geqslant \alpha\}$ is called the α-pessimistic value to ξ [3].*

It is important to have focus that $\xi = ([a, b], [c, d])$ be a rough variable with $c \leqslant a \leqslant b \leqslant d$ then the α-optimistic value of ξ is

$$\xi_{\sup}(\alpha) = \begin{cases} (1-2\alpha)d + 2\alpha c & \text{if} \alpha \geqslant \frac{(d-b)}{2[d-c]} \\ 2(1-\alpha)d + (2\alpha-1)c & \text{if} \alpha \geqslant \frac{(2d-a-c)}{[2(d-c)]} \\ \frac{d(b-a)+b(d-c)-2\alpha(b-a)(d-c)}{(b-a)+(d-c)} & \text{otherwise} \end{cases}$$

and the α-pessimistic value of ξ is

$$\xi_{\inf}(\alpha) = \begin{cases} (1-2\alpha)c + 2\alpha d & \text{if} \alpha \leqslant \frac{(a-c)}{2(d-c)} \\ 2(1-\alpha)c + (2\alpha-1)d & \text{if} \alpha \geqslant \frac{b+d-2c}{2(d-c)} \\ \frac{c(b-a)+a(d-c)+2\alpha(b-a)(d-c)}{(b-a)+(d-c)} & \text{otherwise} \end{cases}$$

THEOREM 2.9 *Let $\xi_{\inf}(\alpha)$ and $\xi_{\sup}(\alpha)$ be the α-pessimistic and α-optimistic values of the rough variable ξ, respectively. Then we have [3]*

a) $Tr\{\xi \geqslant \xi_{\sup}(\alpha)\} \geqslant \alpha$ and $Tr\{\xi \leqslant \xi(\alpha)\} \geqslant \alpha$;
b) $\xi_{\inf}(\alpha)$ an increasing and left-continuous function of α;
c) $\xi_{\sup}(\alpha)$ an decreasing and left-continuous function of α;
d) If $0 < \alpha \leqslant 1$, then $\xi_{\inf}(\alpha) = \xi_{\sup}(\alpha)(1-\alpha)$, and $\xi_{\sup}(\alpha) = \xi_{\inf}(1-\alpha)$;
e) If $0 < \alpha \leqslant 0.5$, then $\xi_{\inf}(\alpha) \leqslant \xi_{\sup}(\alpha)$;
f) If $0.5 < \alpha \leqslant 0.5$, then $\xi_{\inf}(\alpha) \leqslant \xi_{\sup}(\alpha)$.

DEFINITION 2.10 *DMU_o is rough DEA efficient if its best possible upper bound efficiency $(\theta^*)^{\inf(\alpha)}$ is equal to one; otherwise, it is said to be rough DEA inefficient if $(\theta^*)^{\inf(\alpha)} < 1$ [2].*

3. The Additive Rough DEA

Let there are n DMUs with m inputs and s outputs, the input and output vectors of DMU$_j$ are rough vectors i.e.

$$\hat{x}_j = (\hat{x}_{ij}, \ldots, \hat{x}_{mj}) > 0 \qquad \text{and} \qquad \hat{y}_i = (\hat{y}_{ij}, \ldots, \hat{y}_{sj}) > 0 \qquad i = 1, \ldots, m,\ j = 1, \ldots, n.$$

According to the Additive DEA model (the following model), we can formulate a DEA model with rough.

$$\begin{array}{ll} \max & \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \\ \text{s.t.} & \sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = x_{io},\ i = 1, \ldots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+ = y_{ro},\ r = 1, \ldots, s \\ & \lambda_j \geqslant 0, \quad S_i^-, S_r^+ \geqslant 0, r = 1, \ldots, s,\ i = 1, \ldots, m,\ j = 1, \ldots, n. \end{array} \tag{1}$$

Generally, to deal with the rough uncertainty discussed above, Liu [3], proposed the rough expected value model (EVM) and α-optimistic value and α-pessimistic value operator of rough variable to transfer rough programming in to determinate one.

In this paper we suppose $0.5 < \alpha \leqslant 1$, according to the theorem, α-value of the rough variable are ξ are $\xi_{\sup}(\alpha)$ and $\xi_{\inf}(\alpha)$ and $\xi_{\inf}(\alpha) \geqslant \xi_{\sup}(\alpha)$, it is denoted by $\lfloor \xi_{\sup}(\alpha), \xi_{\inf}(\alpha) \rfloor$.

3.1 *Transfer Rough Model into Deterministic Model*

Jiuping Xu et. al. [2], transferred the rough variables in model (1) into an interval programming under trust level α, therefore rough variables $\hat{x}_j = (\hat{x}_{ij} \dots \hat{x}_{mj})^t > 0$ and $\hat{y}_j = (\hat{y}_{ij} \dots \hat{x}_{mj})^t > 0$ can be transformed to $\left[x_j^{\sup(\alpha)}, x_j^{\inf(\alpha)}\right]$ and $\left[y_j^{\sup(\alpha)}, y_j^{\inf(\alpha)}\right]$, respectively. And with assumption $0.5 < \alpha \leqslant 1$, we have $\xi_{\inf}(\alpha) \geqslant \xi_{\sup}(\alpha)$. Now RDEA model (1) can be transformed in to following program:

$$
\begin{aligned}
\max\ & \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j [x_{ij}^{\sup(\alpha)}, x_{ij}^{\inf(\alpha)}] \leqslant [x_{io}^{\sup(\alpha)}, x_{io}^{\inf(\alpha)}], \quad i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j [y_{rj}^{\sup(\alpha)}, y_{rj}^{\inf(\alpha)}] \geqslant [y_{ro}^{\sup(\alpha)}, y_{ro}^{\inf(\alpha)}], \quad r = 1, \dots, s \\
& \lambda_j \geqslant 0, \quad S_i^-, S_r^+ \geqslant 0, \quad r = 1, \dots, s,\ i = 1, \dots, m.\ j = 1, \dots, n
\end{aligned}
\tag{2}
$$

In order to transform the interval programming (2) in to deterministic linear programming with α trust level we assume:

$$
\begin{aligned}
&\frac{\min}{\text{inputs}} \quad \mathrm{D}MU_o \longrightarrow \max \quad \text{output} \\
&\frac{\max}{\text{outputs}} \quad \mathrm{D}MU_j \longrightarrow \min \quad \text{output} \quad j = 1, \dots, n\ j \neq o.
\end{aligned}
$$

Now interval programming (2) can be transformed in to programming (3) as following

$$
\begin{aligned}
\max\ & \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j x_{ij}^{\inf(\alpha)} + \lambda_{jo} x_{jo}^{\sup(\alpha)} + S_i^- = x_{io}^{\sup(\alpha)}, \quad i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{\sup(\alpha)} + \lambda_{jo} y_{ro}^{\inf(\alpha)} - S_r^+ = y_{ro}^{\inf(\alpha)}, \quad r = 1, \dots, s \\
& \lambda_j \geqslant 0, \quad S_i^-, S_r^+ \geqslant 0, \quad r = 1, \dots, s,\ i = 1, \dots, m.\ j = 1, \dots, n
\end{aligned}
\tag{3}
$$

The variables in the model (3) are not rough, i.e. they are deterministic, and the model (3) is a linear program. In model (3) DMU_o is ARDEA-efficient if and only if $S^{-*} = 0$ and $S^{+*} = 0$.

4. Practical Application

In this section, we will evaluate the operation performance of the six DMUs with two inputs and one output (see Table 1) using the proposed ARDEA model.

Table 1. Input and output data of 6 DMUs

	X_1	X_2	Y
DMU_1	$([1650, 1775], [1545, 1867])$	$([520, 560], [450, 600])$	6790
DMU_2	$([2040, 2240], [1915, 2460])$	$([560, 600], [500, 650])$	8000
DMU_3	$([1980, 2080], [1700, 2180])$	$([620, 720], [560, 780])$	6550
DMU_4	$([1760, 1840], [1650, 1900])$	$([710, 730], [680, 760])$	5250
DMU_5	$([2120, 2210], [1920, 2300])$	$([565, 585], [470.620])$	8260
DMU_6	$([1940, 2010], [1880, 2100])$	$([620, 685], [560, 720])$	6280

4.1 *Performance Evaluation*

Using the transformation technique described in the previous section, we transform the rough variables in Table 1 into certain variables. Suppose the trust level $\alpha = 0.9$, the corresponding programming model's solutions of DMUs are summarized in table2:

Table 2. The result for ARDEA model

DMU	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6
ARDEA model	0.00	432.88	0.00	0.00	788.10	0.00

In Table 2, DMU_1, DMU_3, DMU_4, and DMU_6 are ARDEA-efficient, i.e. the summations of their slacks are zero.

5. Conclusions

In this paper, we developed an ARDEA model with rough parameters. This model can be used to evaluate the performance of DMUs when they want to change their inputs and outputs non-redially. In the process of solving the ARDEA model, we used the α-optimistic value and α-pessimistic value of rough variable to transfer the rough model into deterministic line programming. Finally, we illustrated the proposed method by an example.

References

[1] Charnes A., W. W. Cooper and E. Rhodes. Measurement the efficiency of decision making units, Eur.J.Oper.Res. **2**, (1978) 429-444.
[2] Jiuping Xu, Bin Li, Desheng Wu. Rough data envelopment analysis and its application to supply chain performance evaluation, (2009).
[3] Liu, B. D. Uncertain Theory: An Introduction to its Axiomatic Foundation. Springer, Berlin, (2004) .
[4] Pawlak, Z. Rough sets. International Journal of computer and information science, **11**, (1982) 341-356.