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A Simplified LagrangianMultiplier Approach for Fixed Head Short-Term Hydrothermal Scheduling

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Abstract. This paper presents a simplified lagrangian multiplier based algorithm to solve the fixed head hydrothermal scheduling problem. In fixed head hydrothermal scheduling problem, water discharge rate is modeled as quadratic function of hydropower generation and fuel cost is modeled as quadratic function of thermal power generation. The power output of each hydro unit varies with the rate of water discharged through the turbines. It is assumed that hydro plants alone are not sufficient to supply all the load demands during the scheduling horizon. In hydro scheduling, the specified total volume of water should be optimally discharged throughout the scheduling period. A novel mathematical approach has been developed to determine the optimal hydro and thermal power generation so as to minimize the fuel cost of thermal units. The performance of the proposed method is demonstrated with three test systems. The test results reveal that the developed method provides optimal solution which satisfies the various system constraints of fixed head hydrothermal scheduling problem.

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1. Introduction

The hydrothermal scheduling plays an important role in the operation planning of an interconnected power plant. The short-term hydrothermal scheduling problem is one of the most important daily activities for a utility company. Short-term hydrothermal scheduling

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involves the hour-by-hour scheduling of all generating units on a system to achieve minimum generation cost. The objective of fixed head hydrothermal scheduling is to minimize the fuel cost of thermal generating units by determining the optimal hydro and thermal generation schedule subject to satisfying water availability constraint and load demand in each interval of the scheduling horizon. The head of the reservoir is assumed to be constant for the hydro plants having reservoirs with large capacity. The hydraulic constraint imposed in the problem formulation is that the total volume of water discharged in the scheduling horizon must be exactly equal to the defined volume. The operational cost of hydroelectric units is insignificant because the source of hydro power is the natural water resources.

Numerous mathematical methods for the optimal resource allocation problems have been developed in the literature [6, 7]. Different mathematical methods for hydrothermal scheduling problem have been reported in the literature. In [12] , an iterative technique based on computing LU factors of the Jacobian with partial pivoting for solving hydrothermal scheduling problem has been developed. A Lagrangian multiplier method for optimal scheduling of fixed head hydro and thermal plants is reported in [5]. This method linearizes the coordination equation and solves for the water availability constraint separately from unit generations. Dynamic programming approach [10] has been implemented for the solution of hydrothermal scheduling problem. The disadvantages of dynamic programming method are computational and dimensional requirements grow drastically with increase in system size and planning horizon. Lagrangian relaxation techniques have been proposed in the literature to solve hydrothermal coordination problem [11, 4]. A Lagrangian relaxation technique solves the dual problem of the original hydrothermal coordination problem. However the perturbation procedures are required to obtain primal feasible solution. These perturbation procedures may deteriorate the optimality of the solution obtained.

Recently, as an alternative to the conventional mathematical approaches, the heuristic optimization techniques such as simulated annealing [1], genetic algorithm [9], artificial immune algorithm [2] have been used to solve fixed head hydrothermal scheduling problem. Heuristic methods use stochastic techniques and include randomness in moving from one solution to the next solution. Due to the random search nature of the algorithm, these methods provide feasible optimal solution for the optimization problems. Neural network based approach has been developed for scheduling thermal plants in coordination with fixed head hydro units [3]. This method provides near-optimal solution for hydro thermal scheduling problem. An integrated technique of predator-prey optimization and Powell's method for optimal operation of hydrothermal system has been reported in [8]. In this hybrid method, predator-prey optimization is used as a base level search in the global search space and Powell's method as a local search technique.

In this paper, a new deterministic method based on Lagrangian multiplier and Newton method is developed for the solution of short-term fixed head hydrothermal scheduling problem. The scheduling period is divided into a number of subintervals each having a constant load demand. The proposed approach has been validated by applying it to three test systems.

2. Problem Formulation

The basic problem is to find the real power generation of committed hydro and thermal generating units in the system as a function of time over a finite time period from 1 to T. The goal is to minimize the total fuel cost required for the thermal generation in the scheduling period.

$$\text{Minimize}\, \varphi = \sum_{k=1}^{T}\sum_{i=1}^{NT} t_k F_{ik}(P_{Tik}) \tag{1}$$

where $F_{ik}(P_{Tik})$ is cost function of each thermal generating unit in interval k, which is expressed by $F_{ik}(P_{Tik}) = a_i P_{Tik}^2 + b_i P_{Tik} + c_i$

T- number of periods for dividing the scheduling time horizon

NT- number of thermal generators

t_k - time in interval k

P_{Tik} - power generation level of ith thermal generating unit in interval k

a_i, b_i and c_i are the fuel cost coefficients of ith thermal generating unit

Subject to

(i) Power balance constraint

$$C_k = \sum_{i=1}^{NT} P_{Tik} + \sum_{i=1}^{NH} P_{Hik} - P_{Dk} = 0 \quad \text{for } k = 1, 2, \dots, T \tag{2}$$

where P_{Hik} - power generation level of ith hydro generating unit in interval k

P_{Dk} - Total generation demand in interval k.

NH - number of hydro generators

(ii) Water availability constraint

$$W_i = \sum_{k=1}^{T} t_k q_{ik}(P_{Hik}) = V_i - S_i \text{ for } i= 1, 2, \dots, NH \tag{3}$$

where q_{ik} is the discharge rate of hydro unit i during the kth interval, which is expressed by $q_{ik}(P_{Hik}) = \alpha_i P_{Hik}^2 + \beta_i P_{Hik} + \delta_i$.

V_i is the pre-specified volume of water available for hydro unit i during the scheduling period. S_i is the total spillage discharge of ith hydro unit during the scheduling period and the spillage discharge is not used for power generation. α_i, β_i, and δ_i are the discharge coefficients of ith hydro unit with

$$P_{Ti}^{min} \le P_{Tik} \le P_{Ti}^{max} \tag{4}$$

$$P_{Hi}^{min} \le P_{Hik} \le P_{Hi}^{max} \tag{5}$$

where P_{Ti}^{min}, P_{Ti}^{max} are the minimum and maximum generation limits of ith thermal unit and P_{Hi}^{min}, P_{Hi}^{max} are the minimum and maximum generation limits of ith hydro unit.

3. ProposedMethodology

The lagrangian equation for hydrothermal scheduling is

$$L(P,\lambda,\gamma) = \varphi - \sum_{k=1}^{T} \lambda_k C_k + \sum_{i=1}^{NH} \gamma_i (W_i - (V_i - S_i)) \tag{6}$$

where λ_k is the Lagrangian multiplier for power balance constraint in kth interval and γ_i is the lagrangian multiplier for hydro unit i. Substituting eqns. (1),(2) and (3) in eqn (6) gives

$$L(P,\lambda,\gamma) = \sum_{k=1}^{T}\sum_{i=1}^{NT} t_k F_{ik}(P_{Tik}) - \sum_{k=1}^{T} \lambda_k \left(\sum_{i=1}^{NT} P_{Tik} + \sum_{i=1}^{NH} P_{Hik} - P_{Dk}\right) + \sum_{i=1}^{NH} \gamma_i \left(\sum_{k=1}^{T} t_k q_{ik}(P_{Hik}) - (V_i - S_i)\right) \tag{7}$$

The minimum value is obtained by partially differentiating the Equation (7) with respect to P_{Tik}, P_{Hik}, λ_k, γ_i and equating to zero

i.e
$$\frac{\partial L}{\partial P_{Tik}} = 0 \tag{8}$$

i.e
$$t_k \frac{\partial F_{ik}(P_{Tik})}{\partial P_{Tik}} - \lambda_k \frac{\partial C_k}{\partial P_{Tik}} = 0 \tag{9}$$

Equation (9) gives
$$t_k(2a_i P_{Tik} + b_i) = \lambda_k \tag{10}$$

i.e
$$\frac{\partial L}{\partial P_{Hik}} = 0 \tag{11}$$

therefore
$$-\lambda_k + \gamma_i \frac{\partial}{\partial P_{Hik}}(W_i - (V_i - S_i)) = 0 \tag{12}$$

Equation (12) gives
$$t_k \gamma_i [2\alpha_i P_{Hik} + \beta_i] = \lambda_k \tag{13}$$

i.e
$$\frac{\partial L}{\partial \lambda_k} = 0 \tag{14}$$

therefore $C_k = 0$ (15)

Equation (15) gives
$$\sum_{i=1}^{NT} P_{Tik} + \sum_{i=1}^{NH} P_{Hik} = P_{Dk} \tag{16}$$

i.e
$$\frac{\partial L}{\partial \gamma_i} = 0 \tag{17}$$

therefore
$$W_i - (V_i - S_i) = 0 \tag{18}$$

Equation (18) gives
$$\sum_{k=1}^{T} t_k [\alpha_i P_{Hik}^2 + \beta_i P_{Hik} + \delta_i] = V_i - S_i \tag{19}$$

From the Equation (10), we get
$$P_{Tik} = \frac{\lambda_k - b_i}{2a_i} = \lambda_k \frac{1}{2a_i} - \frac{b_i}{2a_i}$$
$$P_{Tik} = \lambda_k A_i - B_i \text{ for i = 1, 2, … , NT, k= 1, 2, … , T} \tag{20}$$
$$\text{where } A_i = \frac{1}{2a_i};\ B_i = \frac{b_i}{2a_i}$$

From the Equation (13), we get
$$P_{Hik} = \frac{\lambda_k - \beta_i \gamma_i}{2\alpha_i \gamma_i} = \frac{\lambda_k}{2\alpha_i \gamma_i} - \frac{\beta_i}{2\alpha_i}$$
$$P_{Hik} = \frac{\lambda_k}{\gamma_i} C_i - D_i \text{ for i = 1, 2, … , NH, k= 1, 2, … , T} \tag{21}$$

where $C_i = \frac{1}{2\alpha_i};\ D_i = \frac{\beta_i}{2\alpha_i}$

Substituting Equations (20) and (21) in Equation (16) gives
$$\sum_{i=1}^{NT} (\lambda_k A_i - B_i) + \sum_{i=1}^{NH} \left(\frac{\lambda_k C_i}{\gamma_i} - D_i \right) = P_{Dk} \text{ for k= 1, 2, … , T}$$

therefore
$$\lambda_k = \frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}} \quad \text{for k= 1, 2, … , T} \tag{22}$$

SubstitutingEquation (22) in Equation (21) gives

$$P_{Hik} = \frac{C_i}{\gamma_i}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right] - D_i \tag{23}$$

for i =1, 2, … , NH, for k= 1, 2, … , T

Substituting (23) in (19), we get

$$\sum_{k=1}^{T} t_k (\alpha_i \left(\frac{C_i}{\gamma_i}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right] - D_i\right)^2 + \beta_i \left(\frac{C_i}{\gamma_i}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right] - D_i\right) + \delta_i) = V_i - S_i \tag{24}$$

for i =1, 2, … , NH

By expanding the Equation (24)

$$\sum_{k=1}^{T} t_k (\alpha_i \left(\frac{C_i^2}{\gamma_i^2}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right]^2 + D_i^2 - \frac{2C_iD_i}{\gamma_i}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right]\right) + \frac{\beta_i C_i}{\gamma_i}\left[\frac{P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i}{\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH} \frac{C_i}{\gamma_i}}\right] - \beta_i D_i + \delta_i) = V_i - S_i \tag{25}$$

for i =1, 2, … , NH

Simplifying the Equation (25)

$$\frac{\alpha_i C_i^2}{\gamma_i^2}\sum_{k=1}^{T}\left[P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i\right]^2 + \left[\frac{\beta_i C_i}{\gamma_i} - \frac{2C_iD_i\alpha_i}{\gamma_i}\right]\left[\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH}\frac{C_i}{\gamma_i}\right]\sum_{k=1}^{T}\left[P_{Dk} + \sum_{i=1}^{NT} B_i + \sum_{i=1}^{NH} D_i\right] + \left[\sum_{i=1}^{NT} A_i + \sum_{i=1}^{NH}\frac{C_i}{\gamma_i}\right]^2\left[24(\alpha_i D_i^2 - \beta_i D_i + \delta_i) - (V_i - S_i)\right] = 0 \tag{26}$$

for i =1, 2, … , NH

The Equation (26) is solved by Newton's method.The solution of these equations gives the values of $\gamma_1, \gamma_2, \dots, \gamma_n$. The detailed procedure for solving the simultaneous equations with two variables γ_1 and γ_2 by Newton's method is given below:

Let $f(\gamma_1, \gamma_2) = 0$, and $g(\gamma_1, \gamma_2) = 0$ (27)

Take γ_1^0andγ_2^0 be the initial approximate solution for the above equation. The actual solution is given by $(\gamma_1^0 + h)$ and $(\gamma_2^0 + m)$,where h is an incremental value of γ_1 and m is an incremental value of γ_2 .

Therefore, $f\,(\gamma_1^0 + h,\ \gamma_2^0 + m) = 0$ (28)

$$g(\gamma_1^0 + h,\ \gamma_2^0 + m) = 0 \quad (29)$$

Expanding equations (28) and (29) by Taylor series,

$$f\,(\gamma_1^0,\ \gamma_2^0) + h\frac{\partial}{\partial\gamma_1}[f\,(\gamma_1^0,\ \gamma_2^0)] + m\frac{\partial}{\partial\gamma_2}[f\,(\gamma_1^0,\ \gamma_2^0)] = 0 \quad (30)$$

$$g\,(\gamma_1^0,\ \gamma_2^0) + h\frac{\partial}{\partial\gamma_1}[g\,(\gamma_1^0,\ \gamma_2^0)] + m\frac{\partial}{\partial\gamma_2}[g\,(\gamma_1^0,\ \gamma_2^0)] = 0 \quad (31)$$

Equations (30) and (31) can be written as

$$f_0 + h(f_{\gamma 1})_0 + m(f_{\gamma 2})_0 = 0 \quad (32)$$

$$g_0 + h(g_{\gamma 1})_0 + m(g_{\gamma 2})_0 = 0 \quad (33)$$

Equations (32) and (33) are solved for h and m using determinants,

i.e $h = \frac{-D_{\gamma 1}}{D}$ and $m = \frac{-D_{\gamma 2}}{D}$ (34)

where $D = \begin{vmatrix} (f_{\gamma 1})_0 & (f_{\gamma 2})_0 \\ (g_{\gamma 1})_0 & (g_{\gamma 2})_0 \end{vmatrix}$; $D_{\gamma 1} = \begin{vmatrix} f_0 & (f_{\gamma 2})_0 \\ g_0 & (g_{\gamma 2})_0 \end{vmatrix}$; $D_{\gamma 2} = \begin{vmatrix} (f_{\gamma 1})_0 & f_0 \\ (g_{\gamma 1})_0 & g_0 \end{vmatrix}$

By using the incremental values, the new values of γ_1 and γ_2 are obtained by

$$\gamma_1^{new} = \gamma_1^o + h\ ;\ \gamma_2^{new} = \gamma_2^o + m \quad (35)$$

Now γ_1^{new} and γ_2^{new} are taken as initial values and above process is repeated till the convergence criterion is satisfied.

The computational procedure for implementing the proposed method for the solution of fixed head hydrothermal scheduling problemis given in the following steps:

Step 1: Calculate the initial generation of thermal, hydro plants and initial $\lambda_k$from the following equations

$P_{Tik} = \frac{P_{Dk}}{NT + NH}$ for $i = 1, 2, \ldots, NT,\ k = 1, 2, \ldots, T$

$P_{Hik} = \frac{P_{Dk}}{NT + NH}$ for $i = 1, 2, \ldots, NH,\ k = 1, 2, \ldots, T$

$\lambda_k = 2a_i P_{Tik} + b_i$ for $i = 1, 2, \ldots, NT,\ k = 1, 2, \ldots, T$

Step 2: Determine the initial values of γ_i^0 using Equation (13)

i.e $\gamma_i^0 = \frac{\lambda_k}{2\alpha_i P_{Hik} + \beta_i}$ for $i = 1, 2,\ \ldots,\ NH$

Step 3: Substitute the values of γ_i^0 in Equation (26).

Step 4: Calculate the incremental values of $\gamma_i$usingEquation (34).

Step 5: Calculate thenew$\gamma_i$values using Equation(35).

Step 6: Substitute the new $\gamma_i$values in Equation(26) and then go to Step 4. This iterative procedureis continued till the difference between two consecutive iterations is less than specified tolerance.

Step 7: Calculate $\lambda_k$valuesfrom the Equation (22)bysubstitutingtheoptimal values of γ_i.

Step 8: Determine the optimal generation schedule of thermal and hydro plants using eqns. (20) & (21).

4. Numerical Examples and Results

A mathematical approach developed in this paper has been implemented on three fixed head hydrothermal scheduling problems. The program is developed using MATLAB 7. The system data and results obtained through the proposed approach are given below

Table 1 Optimal generation schedule for Example 1

Interval k	Demand P_{Dk}	Water discharge q_{1k}	Hydro generation P_{H1k}	Thermal generation P_{T1k}
1	455	101.9329	234.2742	220.7258
2	425	101.0896	231.8794	193.1206
3	415	100.8105	231.0812	183.9188
4	407	100.5879	230.4426	176.5574
5	400	100.3937	229.8838	170.1162
6	420	100.9500	231.4803	188.5197
7	487	102.8422	236.8285	250.1715
8	604	106.2529	246.1679	357.8321
9	665	108.0848	251.0372	413.9628
10	675	108.3886	251.8354	423.1646
11	695	108.9992	253.4319	441.5681
12	705	109.3059	254.2301	450.7699
13	580	105.5423	244.2522	335.7478
14	605	106.2827	246.2477	358.7523
15	616	106.6104	247.1258	368.8742
16	653	107.7215	250.0793	402.9207
17	721	109.7988	255.5073	465.4927
18	740	110.3874	257.0240	482.9760
19	700	109.1524	253.8310	446.1690
20	678	108.4799	252.0749	425.9251
21	630	107.0292	248.2433	381.7567
22	585	105.6899	244.6513	340.3487
23	540	104.3705	241.0592	298.9408
24	503	103.3007	238.1057	264.8943

Problem- 1

Minimize $F_1(P_{T1}) = \sum_{k=1}^{24} 0.001991\, P_{T1k}^2 + 9.606\, P_{T1k} + 373.7$ dollars

Subject to satisfying power demand in each interval and total water discharge during the entire scheduling horizon should be equal to the given water availability.

Total volume of water available for discharge (24 hours) V = 2559.6 M cubic ft

Water discharge $q_{1k}(P_{H1k}) = 0.0007749\, P_{H1k}^2 - 0.009079\, P_{H1k} + 61.53$ M cubic ft. per hour ;Total spillage S= 25.596 M cubic ft/24 hours

The optimal generation schedule obtained through the proposed method is given in Table 1. The solution converged in fifth iteration. The optimal value of γ = 29.61852312 and total amount of water utilized for hydro power generation exactly satisfied water availability constraint. The total fuel cost of thermal power generation is $ 92097.74.

Problem- 2

Minimize $F_1(P_{T1}) = \sum_{k=1}^{24} 0.01\, P_{T1k}^2 + 3.0\, P_{T1k} + 15$ dollars

Subject to satisfying power demand in each interval and total water discharge during the

entire scheduling horizon should be equal to the given water availability constraints.
Total volume of water available for Hydro plant 1 (24 hours) V_1 = 25 M cubic ft
Total volume of water available for Hydro plant 1 (24 hours) V_2 = 35 M cubic ft

Table 2 Optimal generation schedule for Example 2

Interval k	Demand P_{Dk}	Water discharge		Hydro generation		Thermal generation P_{T1k}
		q_{1k}	q_{2k}	P_{H1k}	P_{H2k}	
1	30	0.7714	0.9543	18.4792	9.1003	2.4205
2	33	0.8105	1.0279	19.7041	10.2892	3.0067
3	35	0.8367	1.0772	20.5207	11.0817	3.3976
4	38	0.8760	1.1513	21.7456	12.2706	3.9838
5	40	0.9023	1.2009	22.5622	13.0631	4.3746
6	45	0.9684	1.3253	24.6038	15.0446	5.3517
7	50	1.0349	1.4505	26.6453	17.0260	6.3287
8	59	1.1556	1.6780	30.3201	20.5925	8.0874
9	61	1.1826	1.7288	31.1367	21.3851	8.4782
10	58	1.1421	1.6526	29.9118	20.1962	7.8920
11	56	1.1152	1.6019	29.0951	19.4037	7.5012
12	57	1.1286	1.6272	29.5034	19.7999	7.6966
13	60	1.1691	1.7034	30.7284	20.9888	8.2828
14	61	1.1826	1.7288	31.1367	21.3851	8.4782
15	65	1.2368	1.8310	32.7699	22.9702	9.2599
16	68	1.2776	1.9079	33.9948	24.1591	9.8461
17	71	1.3186	1.9851	35.2197	25.3479	10.4324
18	62	1.1961	1.7543	31.5450	21.7814	8.6737
19	55	1.1018	1.5766	28.6868	19.0074	7.3058
20	50	1.0349	1.4505	26.6453	17.0260	6.3287
21	43	0.9419	1.2754	23.7872	14.2520	4.9609
22	33	0.8105	1.0279	19.7041	10.2892	3.0067
23	31	0.7845	0.9788	18.8875	9.4966	2.6159
24	30	0.7714	0.9543	18.4792	9.1003	2.4205

Water discharge equations

$$q_{1k}(P_{H1k}) = 0.00005\ P_{H1k}^2 + 0.03\,P_{H1k} + 0.2 \quad \text{M cubic ft. per hour}$$

$$q_{2k}(P_{H2k}) = 0.0001\ P_{H2k}^2 + 0.06\,P_{H2k} + 0.4 \quad \text{M cubic ft. per hour}$$

Total spillage
S_1= 0.25 M cubic ft/24 hours, S_2= 0.35 M cubic ft/24 hours

The optimal generation schedule of hydro and thermal plants obtained through the proposed method is given in Table 2. The solution converged at fifth iteration. The optimal values of γ_1 = 95.717727 and γ_2 = 49.311017. The total fuel cost of thermal power generation is $ 821.32.

Problem- 3

Minimize $F_1(P_{T1})+ F_2(P_{T2})$ where

$$F_1(P_{T1}) = \sum_{k=1}^{24} 0.0025\ P_{T1k}^2 + 3.2\,P_{T1k} + 25\,\text{dollars}$$

$$F_2(P_{T2}) = \sum_{k=1}^{24} 0.0008\ P_{T2k}^2 + 3.4\,P_{T2k} + 30\,\text{dollars}$$

Subject to satisfying power demand in each interval and total water discharge during the entire scheduling horizon should be equal to the given water availability constraints.
Total volume of water available for Hydro plant 1 (24 hours) V_1 = 2500 M cubic ft
Total volume of water available for Hydro plant 1 (24 hours) V_2 = 2100 M cubic ft

Table 3 Optimal generation schedule for Example 3

Interval k	Demand P_{Dk}	Water discharge		Hydro generation		Thermal generation	
		q_{1k}	q_{2k}	P_{H1k}	P_{H2k}	P_{T1k}	P_{T2k}
1	400	64.8580	21.3191	182.0811	32.6776	75.2100	110.0313
2	300	57.6834	9.5683	163.2297	13.9900	60.0679	62.7123
3	250	54.1537	3.7873	153.8040	4.6462	52.4969	39.0529
4	250	54.1537	3.7873	153.8040	4.6462	52.4969	39.0529
5	250	54.1537	3.7873	153.8040	4.6462	52.4969	39.0529
6	300	57.6834	9.5683	163.2297	13.9900	60.0679	62.7123
7	450	68.5029	27.2888	191.5068	42.0214	82.7810	133.6907
8	900	103.0338	83.8446	276.3381	126.1156	150.9203	346.6260
9	1230	130.3323	128.5549	338.5477	187.7847	200.8891	502.7785
10	1250	132.0405	131.3527	342.3180	191.5222	203.9175	512.2423
11	1350	140.6736	145.4921	361.1694	210.2098	219.0596	559.5612
12	1400	145.0477	152.6562	370.5951	219.5536	226.6306	583.2207
13	1200	127.7816	124.3772	332.8923	182.1784	196.3465	488.5828
14	1250	132.0405	131.3527	342.3180	191.5222	203.9175	512.2423
15	1250	132.0405	131.3527	342.3180	191.5222	203.9175	512.2423
16	1270	133.7549	134.1604	346.0883	195.2597	206.9459	521.7061
17	1350	140.6736	145.4921	361.1694	210.2098	219.0596	559.5612
18	1470	151.2359	162.7914	383.7911	232.6349	237.2301	616.3439
19	1330	138.9347	142.6441	357.3991	206.4723	216.0312	550.0974
20	1250	132.0405	131.3527	342.3180	191.5222	203.9175	512.2423
21	1170	125.2446	120.2221	327.2369	176.5721	191.8039	474.3871
22	1050	115.2350	103.8280	304.6152	154.1470	173.6334	417.6044
23	900	103.0338	83.8446	276.3381	126.1156	150.9203	346.6260
24	600	79.6677	45.5750	219.7839	70.0528	105.4941	204.6692

Water discharge equations

$$q_{1k}(P_{H1k}) = 0.000216\, P_{H1k}^2 + 0.306\, P_{H1k} + 1.98 \text{ M cubic ft. per hour}$$

$$q_{2k}(P_{H2k}) = 0.00036\, P_{H2k}^2 + 0.612\, P_{H2k} + 0.936 \text{ M cubic ft. per hour}$$

Total spillage

S_1= 25 M cubic ft/24 hours, S_2= 21 M cubic ft/24 hours

The optimal generation schedule of example 3 is given in Table 3. The solution converged at fifth iteration. The optimal values of γ_1 = 9.2967 and γ_2 = 5.6269. The total fuel cost of thermal power generation is $ 47985.76.

5. Conclusion

A novel mathematical approach for the solution of fixed head hydrothermal scheduling problem is presented in this paper. The proposed method is implemented with test systems. The practical constraints of fixed head hydrothermal scheduling such as water availability constraints, waterspillage,power balance constraint in each interval and generation limits of hydro and thermal units are taken into account in the problem formulation. Numerical results show that the proposed method has the ability to determine the global optimal solution and the method requires lesser number of iterations for the convergence.

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