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Solving a Step Fixed Charge Transportation Problem by a Spanning Tree-Based Memetic Algorithm

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Abstract. In this paper, we consider the step fixed-charge transportation problem (FCTP) in which a step fixed cost, sometimes called a setup cost, is incurred if another related variable assumes a nonzero value. In order to solve the problem, two metaheuristic, a spanning tree-based genetic algorithm (GA) and a spanning tree-based memetic algorithm (MA), are developed for this NP-hard problem. For comparing GA and MA, twenty eight problems with different specifics have been generated at random and then the quality of the proposed algorithms was evaluated using the relative percentage deviation (RPD) method. Finally, based on RPD method, we investigate the impact of increasing the problem size on the performance of our proposed algorithms.

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1. Introduction

The transportation problem is one of the most important areas of supply chain design that offers great potential to reduce costs. The first formulation and discussion of a classical transportation problem (TP) as an optimization problem was introduced by Hitchcock [13] with the objective of minimizing the total costs in order to transport homogeneous products from several origins to several destinations. In several decades, a vast variety of

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deterministic and/or nondeterministic models for TP have been developed.

A basic assumption in any transportation problem is that the cost is directly proportional to the number of units transported, while, in most real-world applications, a fixed cost for distributing of products in each route is also considered. Many practical supply chain distribution problems with fixed charge can be formulated as fixed charge transportation problems (FCTP).

Balinski [6] first formulated the FCTP and presented an approximate algorithm to solve it. Hirsch and Dantzig [12] proved that the FCTP is NP-hard. This problem is formulated as a mixed integer network programming problem and solved by some exact algorithms, such as branch and bound and cutting plane; however, these algorithms are usually inefficient and computationally expensive, especially for large-sized instances. Therefore, in the last two decades, several heuristics and metaheuristics have been presented to solve FCTPs (see, for example, heuristics [4–8]; tabu search [24]; simulated annealing [5]; genetic algorithm (GA) [11–16]; artificial immune and genetic algorithm [19]; simplex-based simulated annealing [26]; minimum cost flow-based genetic algorithm [25]).

Step fixed charge transportation problem (SFCTP) is an extended version of the FCTP and is introduced by Kowalski and Lev [16]. The SFCTP has received little attention in the transportation problem literature. To the best of our knowledge, two heuristics proposed by Kowalski and Lev [16] Altassan, et al., [3] and an artificial immune algorithm by El-Sherbiny [9] have been presented to solve SFCTPs.

In this paper, we consider the step fixed charge transportation problem (SFCTP). Up until now, no one has considered neither GA nor MA for any kind of SFCTPs. So, we presented GA and MA for solving the SFCTP for the first time.

The rest of the paper is organized as follows. In Section 2, the SFCTP model is described, while in Sections 3 the solution approach is discussed. The experimental design and comparisons are presented in Section 4. Finally, the conclusion and future work are reported in Section 5.

2. Mathematical Model and Descriptions

SFCTP can be stated as a transportation problem in which there are *m* suppliers and *n* customers. Each of the *m* suppliers can ship to any of the *n* customers at a shipping cost per unit c_{ij} plus a fixed cost k_{ij}, assumed for opening this route. Each supplier *i*=1,2,…,m has S_i units of supply, and each customer *j*=1,2,…,n has a demand of D_j units. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized. This problem can be formulated as follows:

Min $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} k_{ij}\, y_{ij}$

s.t

$$\sum_{i=1}^{m} x_{ij} = S_i \mathrm{i} = 1,2,\ldots,\mathrm{m},$$

$$\sum_{i=1}^{n} x_{ij} = D_j \qquad j = 1,2,\ldots,n,$$

$$x_{ij} \geq 0, \qquad \forall\, i,j,$$

Where

$$y_{ij} = \begin{cases} 1 & x_{ij} > 0, \\ 0 & Otherwise \end{cases}$$

The fixed cost k_{ij} for route (i, j) is related to the transported units through its route. This consists of a fixed cost$\tilde{k}_{ij,1}$ for opening the route (i, j) and an additional cost $k_{ij,2}$ when the transported units exceeds a certain amount A_{ij} . Therefore, $k_{ij} = b_{ij,1}\ k_{ij,1} + b_{ij,2}\ k_{ij,2}$, where

$$b_{ij,1} = \begin{cases} 1 & x_{ij} > 0, \\ 0 & Otherwise \end{cases}, \qquad b_{ij,2} = \begin{cases} 1 & x_{ij} > A_{ij}, \\ 0 & Otherwise \end{cases},$$

and $k_{ij,1}$, $k_{ij,2}$, k_{ij}, $A_{ij} \geq 0$.

Note that k_{ij} has two steps. It could have multiple steps, depending on the problem structure. Without loss of generality, we assume that

$$\sum_{i=1}^{n} S_i = \sum_{j=1}^{m} D_j\, S_i, D_j \geq 0.$$

Where x_{ij} is the unknown quantity to be transported on the route (i, j) that from plant *i* to consumer *j*, c_{ij} is the shipping cost per unit from plant *i* to consumer *j*. k_{ij} is the fixed cost associated with route (*i*, *j*). In this paper, we assume a balanced transportation problem, because the unbalanced transportation problem can be converted to a balanced transportation problem by introducing a dummy supplier or a dummy consumer. Despite its similarity to a standard TP problem, SFCTP is significantly harder to solve because of the discontinuity in the objective function *Z* introduced by the fixed costs.

3. Solution Approach

3.1. Representation and Initialization

Most of the metaheuristics use a random procedure to generate an initial set of solutions. The initialization of a solution is performed from randomly generated *m* + *n* –2 digits in range [1, *m* + *n*]. Figs. 1 and 2 illustrate a transportation graph and it's spanning tree.

	D_1	D_2	D_3	D_4	D_5	a_i
O_1		6	5			11
O_2	2			16	1	19
O_3	12		5			17
O_4					7	7
b_j	14	6	10	16	8	

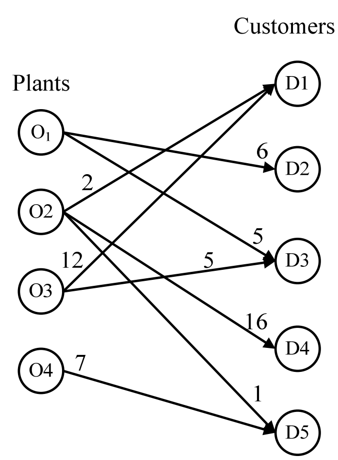

Fig. 1. Illustration of basis on the transportation tableau and the transportation graph.

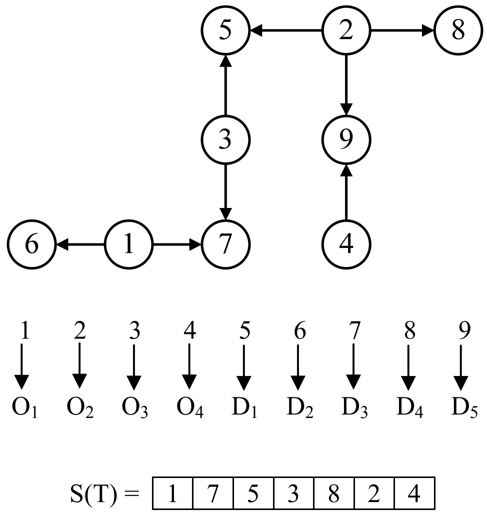

Fig. 2. A Spanning tree and its solution representation.

When generating the solution, there will be a possibility that it cannot be adapted into the transportation network graph. For this purpose, the feasible solution generation procedure proposed by Hajiaghaei-Keshteli et al. [13] is used. The feasibility criterion is as follows:

$$\sum_{i=1}^{m}(L_i+1)=\sum_{i=m+1}^{m+n}(L_i+1) \tag{1}$$

Where L_i is the appearance number of node i in solution S(T). The criterion can be showed by equation 2:

$$\sum_{i=1}^{m}L_i+m=\sum_{i=m+1}^{m+n}L_i+n \tag{2}$$

Considering the length of solution, the following equation is obtained:

$$\sum_{i=1}^{m}L_i+\sum_{i=m+1}^{m+n}L_i=m+n-2 \tag{3}$$

So we can easily show the feasibility criteria from (2) and (3) as follows:

$$\sum_{i=1}^{m}L_i=n-1 \tag{4}$$

And

$$\sum_{i=m+1}^{m+n}L_i=m-1 \tag{5}$$

A solution has $m+n-2$ digits. Considering the feasibility criteria (equations (4) and (5)), we randomly generate a string with $n-1$digits from set O, and another with $m-1$digits from set D. To design a feasible solution, the two produced strings are combined together at random, as depicted in Fig. 3. After generating a feasible solution, the transportation network graph can be determined by using the following decoding procedure:

Procedure: Convert solution S(T) to the transportation tree
Input: Transportation network graph and solution S(T)
Output: A transportation tree

Step 1: Let S(T) be the original solution and let S'(T) be the set of all the nodes that are not part ofS(T) and designed as eligible for consideration.
Step 2: Repeat the following process – (2.1) – (2.5) – until no digits are left in S(T).
2.1 Let *i* be the lowest numbered eligible node in S'(T). Let j be the leftmost digit of S(T).
2.2 If *i* and j are not in the same set O or D, add the edge (*i*,*j*) to tree T. Otherwise, select the next digit k from S(T) that is not included in the same set with *i*, exchange *j* with *k*, and add the edge (*i*,*k*) to the tree *T*.
2.3 Remove *j* (or *k*) from S(T) and *i* from S'(T). If *j* (or *k*) does not occur anywhere in the remaining part of S(T), put it into S'(T).
2.4 Assign the available amount of units to $x_{ij} = \min\{a_i, b_j\}$ (or $x_{ik} = \min\{a_i, b_k\}$) to the edge (*i*,*j*) or (*i*,*k*)) where $i \in O$ and $j, k \in D$.
2.5 Update availability $a_i = a_i - x_{ij}$ and $b_j = b_j - x_{ij}$ (or $b_k = b_k - x_{ik}$).
Step 3: If no digits remain in S(T) then there are exactly two nodes, *i* and *j*, still eligible in S'(T) for consideration. Add edge (*i, j*) to tree *T* and form a tree with $m + n - 1$ edges.
Step 4: If there are no available units to assign, then stop. Otherwise, there are *y* plants with $a > 0$ units, and z costumers with $b > 0$ demands yet. One of these states occurs:

I. If $y = 1$ and $z = 1$, Add the edge between the plant and the customer to the tree and assign the available amount to the edge.

II. If $y > 1$ and $z = 1$, Add the edge between the plants and the customer to the tree and assign the available amount to the edge.

III. If $y = 1$ and $z > 1$, Add the edge between the plant and the customers to the tree and assign the available amount to the edge.

IV. If $y > 1$ and $z > 1$, Consider them as a new transportation model with *y* plants and *z* customers, then generate solution, and Repeat step 1 to 4.

If a cycle exists; remove the edge that is assigned zero flow. A new spanning tree is formed with $m + n - 1$ edges.

m=3 and *n=5*→ $\sum_{i=1}^{3} L_i = 4$ and $\sum_{i=4}^{8} L_i = 2$

Random n-1 digits from set O	1	3	2	3		
Random m-1 digits from set D	7	6				
feasible solution	1	7	3	2	6	3

Fig. 3. Illustration of generating feasible solution.

All conditions that may occur in designing transportation tree in step 4 are considered, while in the previous procedure, some states are not involved. Therefore it does not produce any transportation tree in some situation. To more explanation and clarify this difference, we give an example and illustrate it in Fig. 4.

S(T) = | 1 | 7 | 5 | 3 | 8 | 2 | 4 |

	D_1	D_2	D_3	D_4	D_5	a_i	
O_1		11	0			11	0
O_2	0			16		19	3
O_3	4		10			17	3
O_4				0	7	7	0
b_j	4	16	10	16	8		
	0	5	0	0	1		

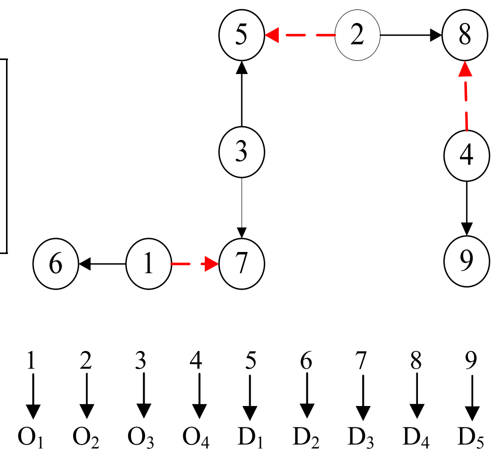

Fig 4. Illustration of the previous procedure proposed by Jo et al. [15] to designing transportation.

In this example, one can easily conclude that it cannot be solved by the previous procedure presented by Jo et al. [15]. There are still two plants ($O_2$and O_3) and two customers (D_2 and D_5) remain. We solve this dilemma with step 4 in the presented procedure in Fig. 5.

	D_2	D_5	
O_2	3	0	3
O_3	2	1	3
	5	1	

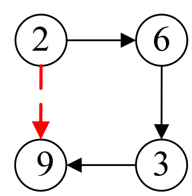

Fig 5. Solving the dilemma by step 4 in the presented procedure

Fig. 6 shows combined two Figs 4 and 5. Therefore we can reach to final transportation tree and transportation graph in Fig. 7 by removing the edges which are assigned zero flow.

	D_1	D_2	D_3	D_4	D_5	a_i
O_1		11	0			11
O_2	0	3		16	0	19
O_3	4	2	10		1	17
O_4				0	7	7
b_j	4	16	10	16	8	

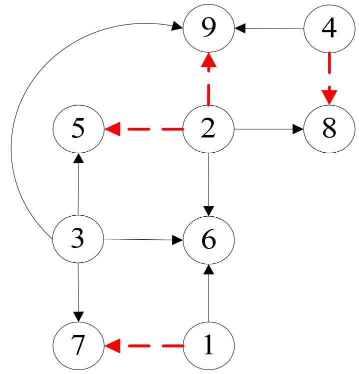

Fig. 6. Combined two Figs. 4 and 5.

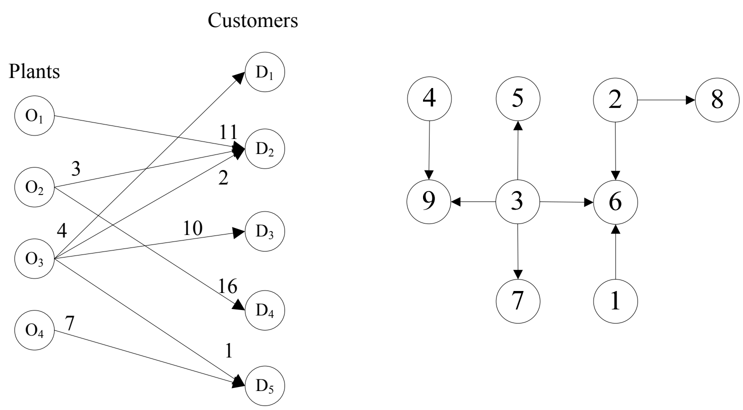

Fig. 7. Final transportation tree and transportation graph for the example.

3.2. The Proposed Memetic Algorithm

Biological genes in parents influence offspring. Additionally, MAs are inspired by Dawkin's concept of a meme [7], which represents a unit of cultural evolution that can exhibit local refinement. The same concept is the basis for the expansion of a GA. MA is a GA in which a separate local search procedure plays a significant role. In the proposed MA, the local search procedure is applied to each child to search for a better solution. From an optimization point of view, MAs are hybrid metaheuristics that combine the global and local search to perform exploration while the local search method performs exploitation.

Selection mechanism:

Analogous to natural selection, the more fit the parent is the more likely they are to have offspring. A simple way of carrying out this selection is via roulette wheel selection. The wheel has different width spaces so that the worst solution has the minimum wedge increasing up to the best solution with the maximum wedge. Since the objective function is the minimization of the total cost, better chromosomes are those results in a lower objective function. The higher fitness value means the better solution, so we define the following function to evaluate each fitness value:

$$\text{Fitness Value} = \frac{1}{\text{Objective Function}}$$

Using the roulette-wheel selection mechanism, the higher fitness value a solution has, the more chance it has to be selected.

Reproduction:
The best solution or Chromosomes with higher fitness values are more desirable than the other and should be considered in next generations. At a minimum, the best solution from the current population needs to be copied to the next generation thus ensuring the best score of the next generation is at least as good as the previous generation. Here elite is expressed as a percent, so the top pr% of the chromosomes is kept with the better fitness values. Hence they are copied to the next generation.

Crossover:
Crossover is the breeding of two parents to produce offsprings. The main purpose of this operator is to generate 'better' offspring, i.e. to create better sequences after combining the parents. The generated offsprings have features from both parents and thus may be better or worse than either parent according to the objective function. As we assigned pr% of the chromosomes of generation to reproduction, the (1–pr)% remaining chromosomes are generated through crossover operator. In this paper, we employ the One-point crossover.

Local search and Mutation:
Mutation is expressed as a probability. For each solution in the parent population a random number is generated from uniform distribution between 0 and 1 giving this solution a percent chance of being mutated. If this solution is chosen for mutation then a copy of the solution is made and job sequences mutated. In this paper, we employ the Big Swap Mutation. Local search is carried out for a fixed number ($n_{\max}$) of neighborhood or mutation searches for each offspring.

4. Experimental Design

4.1. Instances

Hajiaghaei-Keshteli et al. [11] generated random test problems to verify the efficiency of their solution approach. We extend their plan to step costs in this paper. To cover various problem configurations, several levels of influencing inputs are considered. After determining the size of test problems in a given instance, considering the important effect of the step fixed costs to the solution for each size, four problem types (A–D) are generated. For a given problem size, problem types differ from each other by the range of step fixed costs, which increases upon progressing from problem type A through problem type D. The problem sizes, types, step fixed costs ranges and their detail are shown in Table 1.

Table 1. Test problems characteristics.

Problem size	Total Demand	Problem type	A_{ij}	Range of variable costs		Range of first and second fixed costs	
				Lower limit	Upper limit	Lower limit	Upper limit
10×10	10,000	A	400	3	8	50	200
10×20	15,000	B	400	3	8	100	400
15×15	15,000	C	400	3	8	200	800
10×30	15,000	D	400	3	8	400	1,600
50×50	50,000						
30×100	30,000						
50×200	50,000						

4.2. Experimental Results

We set searching time to be identical for both algorithms which is equal to $1.7 \times (n + m)$ milliseconds. Hence, this criterion is affected by both n and m. The number of suppliers and customers, the more rise of searching time increases. Considering twenty instances for each of the twenty eight problem type, or eighty instances for each of the seven problem sizes, for both algorithms, the instances have been run five times. Due to having different objective functions scale in each instance their relative percentage deviation (RPD) is used. The RPD is obtained by the following formula:

$$\text{RPD} = \frac{\text{Alg}_{\text{sol}} - \text{Min}_{\text{sol}}}{\text{Min}_{\text{sol}}} \times 100$$

where Alg_{sol} is value of algorithm and Min_{sol} is the best value between the algorithms.
In order to verify the statistical validity of the results, we have performed an analysis of variance (ANOVA) to analyze the results. The means plot and LSD intervals (at the 95% confidence level) for GA and MA are shown in Fig. 8. As can be seen from the result figure, the performance of MA is better than GA.

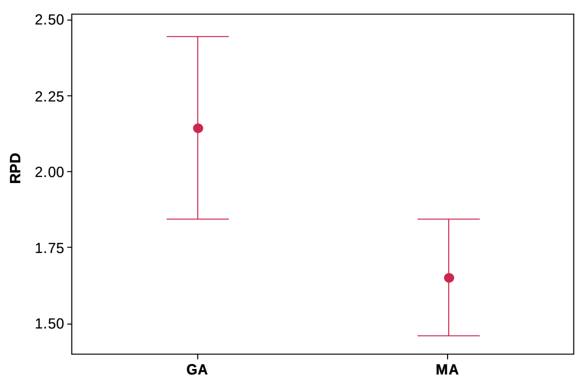

Fig . 8. Means plot and LSD intervals for the MA and GA.

5. Conclusion and Future Works

In this paper, we have developed a GA and a MA to solve the Step Fixed Charge Transportation Problem. In order to evaluate the efficiency of developed algorithm, a new plan is extended based on previous test problems to generate random instances. The comprehensive set of computational experiments for instances with different configuration and problem sizes show that the MA provides good average RPD results and outperforms the GA. As a direction for future research, addressing mentioned problem in the solid transportation [22] is a promising research avenue with significant practical relevance. Also our approach can be extended to the case of inventory cost [17] or fuzzy numbers [23]. Since the model was considered for single objective optimization, in future the multi objective model may be considered. In the present model, only transportation cost of products was concerned. Other possible objective such as delivery earliness tardiness can be added to the model. In addition, the probabilistic demand pattern may also be considered in the future study.

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