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Applications of Partial Differential Equations in Stability Index and Critical Length in Avalanche Dynamics

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Abstract. In this study, Stability analysis of snow slab which is under detonation has developed in the present model. The model has been studied by using the basic concepts of non-detonation model and concepts of underwater explosions with appropriate modifications to the present studies. The studies have also been extended to account the effect of critical length variations at the time of detonation and its effects on various material parameters through the concepts of fracture mechanics. The results indicate that the stability and critical length values are lower for the detonation (present) values in comparison with the non-detonated values. The importance of the studies in Avalanche forecasting has been highlighted.

Keywords: Stability index, Shear strength, Avalanche dynamics.

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1. Introduction

Avalanches are quite destructive in nature in snow bound hilly region [1-2]. The loss of property and lives during this avalanche are many and cannot be comprehended owing to its unpredictable in nature. Western countries of late adopted technique to release the avalanches of those sites which are highly affected by traffic and habitation by explosive device (detonating a particular avalanche site by high energy explosives). This has reduced the risk of someone getting affected by an avalanche and losses due to the property damage. Though method of artificial release has eased out the mountaineers and skiers to some extent however, it is of interest to note that, not all explosive detonation yields for an avalanche. The cause for such failure needs to be analyzed before going for second sound of firing immediately. Whether particular site is just night for an avalanche to get initiate?

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Or is it a stable slope so that one needs to wait for some more days before going for an explosion? Stability analysis of snow mass which is lying on the slope hill and which get affected by explosions needs to be analyzed with respect to snow materials [density, slab depth, slope angle, internal friction, weak layer thickness, etc] and dynamic parameters [pressure, acceleration, stress, elastic modulus, Poisons ratio, time decay, etc]. Stability index has been earlier studied for the snow lying on slope hill without explosion and derived relation with respect to snow material parameters. In the present investigations, since explosion is involved, stability needs to be checked with respect to of snow mass dynamic parameters. This checking with dynamic parameters need to be linked to equilibrium and momentum equation for getting relation with respect to snow slab length. The study focus onto two aspect :(i) stability index and for the ii) relation for snow slab length.

2. Analysis

Laboratory experiments have shown that the mechanical behavior of snow depends on the rate of loading at low rates, snow shows predominately nonlinear viscous behavior. Considerable strain energy can be dissipated. Crack initiation and development has been modeled from the following fundamental equations:

$$\sigma_{ij} + F_j = 0 \rightarrow \begin{array}{c} \partial\sigma_{xx}/\partial x + \partial\sigma_{yx}/\partial y + F_1 = 0 \\ \\ \partial\sigma_{yy}/\partial y + \partial\sigma_{xy}/\partial x + F_2 = 0 \end{array} \tag{1}$$

The satiability and critical length will obtain in next sections.

3. Stability Index

The relation for the stability index (S) as referred by Bruce Jamieson [4] is reproduced below.

$$S = [c + \sigma_{zz}.\Phi(c, \sigma_{zz})]/\sigma_{xz} \tag{2}$$

where

$$\sigma_{zz} = \rho.g.H.\cos\theta \text{ and } \sigma_{xz} = \rho.g.H.\sin\theta \tag{3}$$

σ_{xz}, σ_{zz}, Φ, and c are shear stress, normal stress, internal friction, and shear strength respectively (see Figure 1).

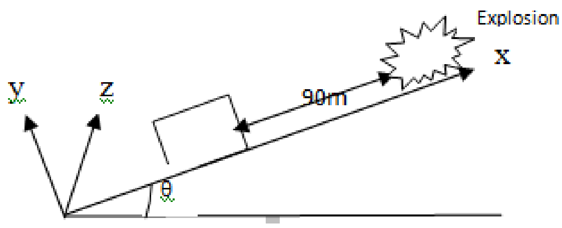

Figure 1. Source point located on the boundary, surrounded by a semicircular region.

Equation (2) is modified to explosive detonation and given by

$$S = [c + \sigma_{zz}.\Phi(c, \sigma_{zz}) - \rho.H.a.\Phi(c, \sigma_{zz})]/(\sigma_{xz} + \rho.H.a) \tag{4}$$

Where ρ is snow density, H is slab depth and a is acceleration due to detonation force. Roch [7] found that the internal friction term Φ depended on shear strength and normal stress. He [7] determined empirical for mules for Φ several different microstructures:

Fresh snow	$\Phi1 = 0.1 + 0.08c + 0.04\sigma_{zz}$	(5)
Rounded grains	$\Phi2 = 0.4 + 0.08c$	(6)
Facets and depth hoar	$\Phi3 = 0.8 + 0.08c - 0.01\sigma_{zz}$	(7)

In the present study, only rounded and facet have been taken for the analysis purpose. Stability index for rounded grains ($S2$) and for facet grain ($S3$) have been given by

$$S2 = [c + (\rho.g.H.\cos\theta - \rho.a.H).(0.4 + 0.8c)]/(\rho.g.H.\sin\theta + \rho.a.H) \tag{8}$$

$$\begin{aligned} S3 =& [c + \rho gH\cos\theta(0.08c + 0.8 - 0.01\rho gH\cos\theta) \\ & - \rho aH(0.8 + 0.08c - 0.01\rho gH\cos\theta)]/(\rho.g.H.\sin\theta + \rho.a.H) \end{aligned} \tag{9}$$

Taking the data from Table 1, Stability has been computed with respect to material parameters and shown in Figure 2 and Figure 3. Also internal friction Φ has been computed and shown in Figure 4. Roch [7] has analyzed stability index for over 36 avalanches and found that, this index S various from 0.76 to 7.5 and found an average of 2.05 with mean of 1.2. Roch [7] concluded that for $S \geqslant 2$, the slopes are unstable whereas for $S \leqslant 2$, the avalanche may induce.

4. The effect of detonation wave on critical length (L)

Critical length plays an important role in the release of an avalanche. This information helps to ski area manages for giving sufficient guidelines for skiers for taking appropriate precautions. In cause of an artificial release of an avalanche, this initial length estimations and its influence on various material and dynamical parameters helps in planning for the location of an avalanche spot for triggering onto the same. In the proposed study, the critical length has been estimated by the following method.

Equilibrium Equation (6) under the action of detonation is modified as

$$\partial\sigma_{xx}/\partial x + \rho.g.H.sin\theta - \sigma_r/H + P/H = 0 \tag{10}$$

Where σ_r, H, P, θ, and σ_{xx} are residual stress, slab depth, detonation pressure, slope angle and normal stress respectively and are defined as [6,9]

$$\sigma_r = (2/3).\rho.g.H.\sin\theta \ , \ P(t) = P_m.\exp(-t/T) \tag{11}$$

Where t, T, and P_m are arrival time, time decay, and peak pressure respectively.

Integrating Equation (9) with x under initial $0 \leqslant x \leqslant L$, we get

$$\sigma_{xx} = -((1/3).\rho.g.H.\sin\theta + [P_m.\exp(-t/T)]/H).L \tag{12}$$

We know that [3]

$$\sigma_{xx} = E.(1 - v^2).\varepsilon_x \tag{13}$$

where E, v, and ε_x are elastic modulus, poison's ratio, and strain in x-direction respectively. Also we know from basic concept of strain energy that, the deformation energy is related to [8]

$$\int \sigma_{xx} d\varepsilon_x = (\sigma_g - \sigma_r).\delta/H \tag{14}$$

Where $\sigma_g = \rho.g.H.\sin\theta$, $\sigma_g = 1.5\sigma_r$, $\delta =$ weak layer thickness/100 $= d/100$ and σ_{xx} and ε_x are normal stress and normal strain in x-direction.

Substituting Equation (12) and (13) in Equation (14), and integrating we get:

$$L = \{2.E.\delta.\rho.\sin\theta.(1 - v^2)/3\}^{0.5}/(\sigma + [P_m.\exp(-t/T)]/H) \tag{15}$$

where $\sigma = (1/3).\rho.g.H.\sin\theta$.

In Equation (15), t is the arrival time where the velocity of snowpack tends to maximum value need to be estimate from the concept of momentum Equation (8).

$$2.P(t) - \rho.V_s.V + \sigma = mdV/dt \tag{16}$$

where m is mass per unit of area, V_s is sound velocity, V is snow pack velocity and ρ is snow density.

Equation (16) is solved by using initial condition $V(0) = 0$, and the solution $V(t)$ which is the velocity of snow pack is given by

$$\begin{aligned} V(t) =& [2.Pm.T/(\rho.(L - c.T))].\{\exp(-ct/L) - \exp(-t/T)\} \\ &+ [(g.H.\sin\theta)/3].\{1 - \exp(-V_s.t/L)\} \end{aligned} \tag{17}$$

In order to obtain maximum velocity, differentiates (17) with respect to 't' and equate it to zero and solve the equation for 't', we get

$$t = L.T.\ln[\sigma.(T.V_s - L) + 2.P_m.V_s.T]/(2.P_m.L)/(V_s.T - L) \tag{18}$$

Substituting of this 't' (which induces maximum snowpack velocity) into the equation for the critical length (Equation (15)), we get

$$\begin{aligned} L =& 2.E.\delta.\rho.\sin\theta.(1 - v^2)/3^{0.5}/(\sigma + (P_m/H) \\ &\{[\sigma.(T.V_s - L) + 2.P_m.V_s.T]/(2.P_m.L)\}) \end{aligned} \tag{19}$$

It could be seen that, Equation (18) is implicit in nature and cannot be solved by explicitly. Hence use of MATLAB SOFTWARE has been made use of for solving

Equation (19). The data required for solving of (19) has been taken from jamieson [4], schewizer [9], Donald Albert [5] in Table 1.

5. Results

Stability index (S) has been has been computed for rounded grain (S2) [Equation (8)] and for faceted and depth hoar grains (S3) [Equation (9)]. Also variation of S has been observed with density (ρ), slope angle (θ), shear strength (c), slab depth (H), acceleration (a). The results have been shown in Figure 2. The results indicated that, stability increase with the increase in shear strength (c), whereas it (S) decreases with increase in density, angle, slab depth, and acceleration. This observation is found to be for both faceted and depth hoar and round grains. The results have been compared to non-detonation methods (see Figure 3) (natural release) and found that, the observations are of similar trend but varies with magnitude (quantum of values).

Internal friction (Φ) has also been computed for faceted and depth hoar crystal for different parametric variations [ρ, θ, H, and c]. The results are shown in Figure 4. The results indicated that, internal friction increase with increase in slope (θ), and shear strength (c), where it (Φ) decreases with increase in density (ρ), and slab depth. These observations are found in qualitative agreement with physics of flow and deformation.

Critical length (L) has been computed for different slope angle and peak pressure and results are shown in Figure 5. The results indicate that, critical length initially increasing up to the slope angle 38 and afterword's it is observed that L keeps decreasing with increase in slope angle (θ). However, it is observed that, L decrease with the increase in peak pressure (P_m). Similar observation has been observed in case of non-detonation concepts (see Figure 6).

Table 1. Data for computation of flow parameters [Ref. 4, 8 and 9]

	Typical value	Range
Shear strength : c	200pa	$200-2000$
Slope angle :θ	38degree	$30-45$
Slab length : L	4m	$2-8$
Slab depth : H	0.5m	$0.4-2$
Detonation wave acceleration : a	0.4m/s^2	$0.4-10$
Snow density :ρ	200kg/m^3	$100-400$
Elastic modulus : E	1Mpa	$0.5-10$
Poisons ratio :v	0.2	$0.1-0.3$
Weak layer thickness : d	10cm	$0.1-20$
Peak pressure : P_m	300pa	$200-300$
Time decay : T	3s	$2-15$
Sound speed in snow : V_s	300m/s	$100-300$

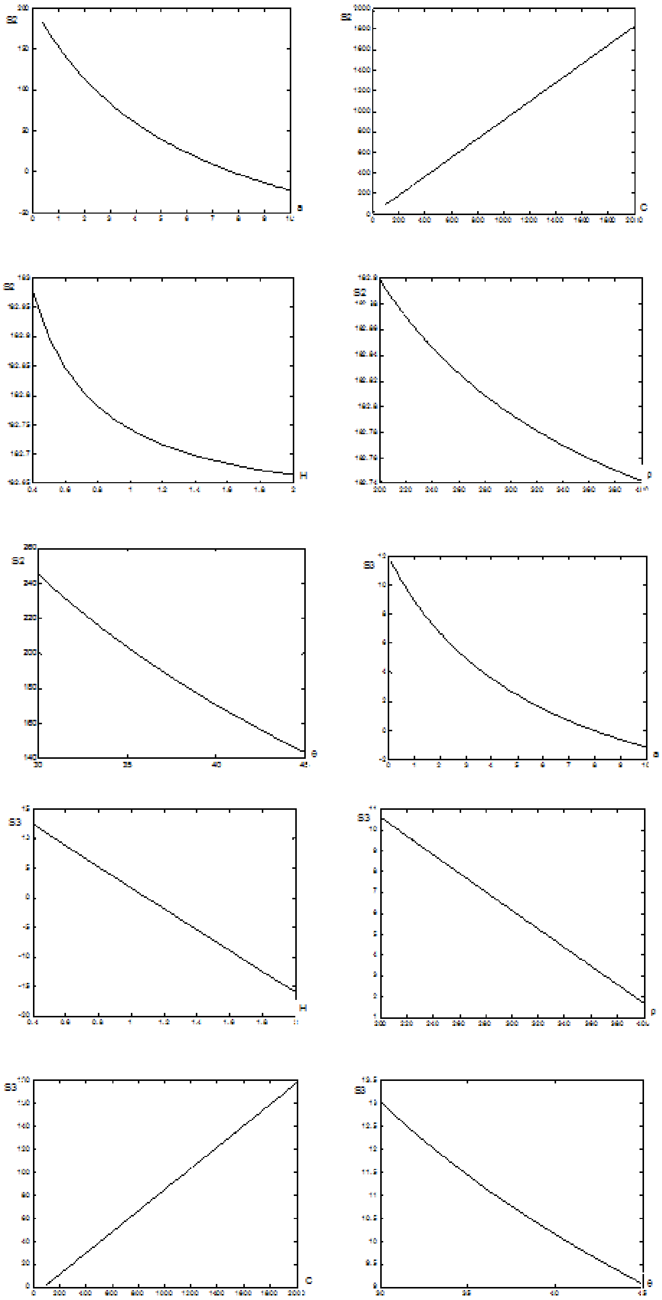

Figure 2. Variation of stability index with density (ρ), slope angle (θ), shear strength (c), slab depth (H) and acceleration (a) [with detonation].

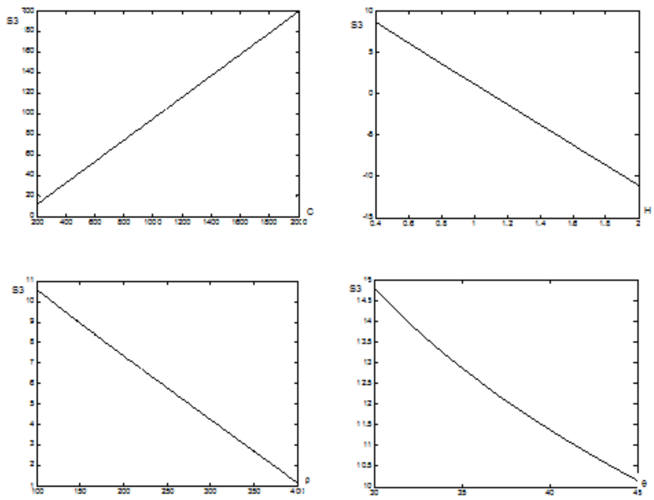

Figure 3. Variation of stability index with density (ρ), slope angle (θ), shear strength (c), slab depth (H) and acceleration (a) [non-detonation].

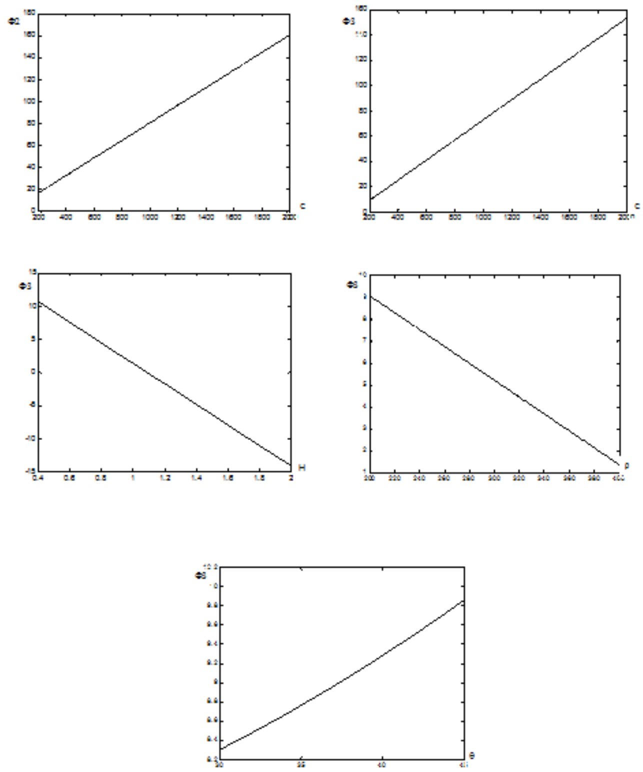

Figure 4. Variation of internal friction (Φ) with density (ρ), slab depth (H), slope angle (θ), shear strength (c).

6. Conclusion

Stability analysis of an avalanche prone slope has been investigated in the present studies. The slopes have been detonated by an explosive and the parametric effects on the stability have been carried out. In addition to the stability, Critical length of snow slab for the failure aspect has also been studied. The results have been compared with those of non-detonation aspect and found that, the stability index

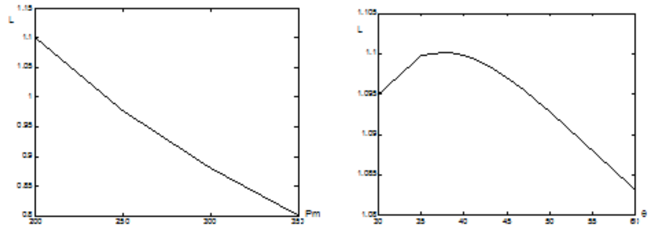

Figure 5. Variation of critical length with slope angle and peak pressure [with detonation].

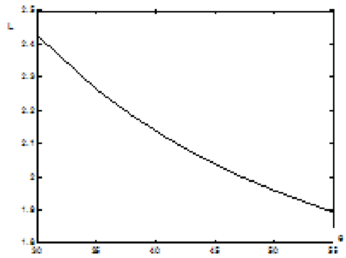

Figure 6. : Variation of critical length with slope angle [non-detonation].

(S) and critical length (L) values reduces considerably in comparison to that of non-detonation aspect. The results have been found in agreement with general physical and science of deformation principles.

One of the primary aspect of the studies is to establish relation between stability index and critical length with respect to material and dynamic parameters and observe its variations so that, the results could be used for forecasting of an avalanche, the by knowing metamorphic stage of the snowpack. This observation has been achieved partially since for forecasting one need actual values for the metamorphic stage of the snow and vis--vis the detonation results which could not be comprehended together due to lack of experimentations. It is intended to take up this aspect in our future scope of studies.

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