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Collocation method for Fredholm-Volterra integral equations with weakly kernels

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Abstract. In this paper it is shown that the use of uniform meshes leads to optimal convergence rates provided that the analytical solutions of a particular class of Fredholm-Volterra integral equations (FVIEs) are smooth.

Keywords: Collocation, Integral equations, Weakly kernels, Generalized Grownwall-Type inequality.

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1. Introduction

The collocation method for Volterra integral equation was introduce and studied in [4–8]. Other concept of integral equation are given and studied in. e.g. [1]. This leads us to the idea of developing method for Fredholm-Volterra integral equation with weakly kernels. In this paper we consider the problem of Fredholm Volterra-Fredhom integral equation with weakly kernels. the structure of this paper is as follows. In Section 2 we present the basic concepts of our work. In Section 3 we show the Gronwall inequality and convergence of collocation methods is shown in Section 4.

2. Basic Concept

This paper will be concerned with high-order collocation methods for the Fredholm-Volterra integral equations (FVIEs)

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$$y(t) = g(t) + \int_a^b p(t,s)k(t,s,y(s))ds + \int_0^t p'(t,s)k'(t,s,y(s))ds, \qquad t \in [0,T] \quad (1)$$

where $y(t)$ is the unknow function whose value is to be determined in the interval $0 \leqslant t \leqslant T < \infty$, the kernels $k(t,s,y(s))$ and $k'(t,s,y(s))$ are lipschits continuous in their variable and $p(t,s)$ and $p'(t,s)$ are unbounded in the region of integration but integrable over $[0,T]$.

The following notation and methods were introduced in [2, 3] and will be used throughout this paper. The collocation methods generate , as approximation to the solution of (1) elements of the polynomial spline space

$$S_{m-1}^{(d)}(Z_N,T) := \{u \in C^{(d)}(I(T)) : u \,|_{\sigma_n} := u_n \in \pi_{m-1}, 0 \leqslant n \leqslant N-1\}, \qquad (2)$$

associated with a given partition

$$\Pi_N : 0 = t_0 < t_1 < \cdots < t_N = T, \qquad N \geqslant 1 \qquad (3)$$

of the interval $[0,T]$. Here, π_{m-1} is the set of real polynomials of degree not exceeding $m-1$ and we have set $\sigma_0 := [t_0,t_1]$ and $\sigma_n := (t_n,t_{n+1}], n = 1,\ldots,N-1$, $Z_N := \{t_n : 1 \leqslant n \leqslant N-1\}$ (the set of interior grid points). The quantity h, $h := \max\{h_n := t_{n+1} - t_n : 0 \leqslant n \leqslant N-1\}$, is often called the diameter of the grid $\prod_N$. If $h_n \equiv h$ all $0 \leqslant n \leqslant N-1$, then the grid $\prod_N$ is called a uniform mesh.

The desired approximation to y is the element $u \in S_{m-1}^{(d)}(Z_N,T)$ satisfying

$$u(t) = g(t) + \int_a^b p(t,s)k(t,s,u(s))ds + \int_0^t p'(t,s)k'(t,s,u(s))ds, \qquad t \in X(N) \quad (4)$$

where $X(N) := \bigcup_{n=0}^{N-1} X_n$ with

$$X_N := \{t_{nj} := t_n + c_j h_n : 0 \leqslant c_1 < \cdots < c_m \leqslant 1\},$$

where $\{c_j\}_{j=1}^m$ are collocation parameters.

3. A generalized Grownwall-Type inequality

Throughout this paper, c_i where i is an integer, will denoted constants which are independent of h .

Definition 3.1. Let $p_1(t,s) := p(t,s), p'_1(t,s) := p'(t,s)$ and set

$$\begin{array}{l} p_n(t,s) := \int_a^b p_1(t,\xi)p_{n-1}(\xi,s)d\xi \\ p'_n(t,s) := \int_0^t p'_1(t,\xi)p'_{n-1}(\xi,s)d\xi \;\; (t,s) \in S, \, n \geqslant 2 \end{array} \qquad (5)$$

where $S := \{(t,s), 0 \leqslant s < t \leqslant T\}$. The functions $\{p_n, p'_n, n = 1,2,\ldots\}$ are called the iterated kernels associated with the given kernels p and p'.

Definition 3.2. If the functions p and p' satisfies

$$(i)p(t,s) \geqslant 0, \qquad p'(t,s) \geqslant 0, (t,s) \in S \tag{6}$$

$$(ii) \int_a^b p(t,s)dt \leqslant c_1, \qquad \int_0^t p'(t,s)dt \leqslant c'_1 \tag{7}$$

$$(iii)p_v(t,s) \leqslant c_2, \qquad p'_v(t,s) \leqslant c'_2, (t,s) \in S \tag{8}$$

where v is a certain integer, then p and p' are said to satisfy conditions C.

Theorem 3.1. Let $A \geqslant 0$ be a constant, and Let the function x satisfy to condition C. The function $x(t)$ is defined as

$$x(t) = \kappa_n, \quad t \in [t_n, t_{n+1}], \quad 0 \leqslant n \leqslant N-1 \tag{9}$$

where the t_n is given by (3) and $\kappa_n \geqslant 0$, if the function x satisfies the integral inequality

$$x(t) \leqslant \int_a^b p(t,s)x(s)ds + \int_0^t p'(t,s)x(s)ds + A \quad t \in [0,T) \tag{10}$$

then it can be bounded by

$$x(t) \leqslant c_2 \int_a^b x(s)ds + c'_2 \int_0^t x(s)ds + c_3 A \quad t \in [0,T), \tag{11}$$

Furthermore, if $h := \max\{h_n := t_{n+1} - t_n, 0 \leqslant n \leqslant N-1\} \leqslant c_4/N$, then

$$\kappa := \max\{\kappa_n, 0 \leqslant n \leqslant N-1\} \leqslant c_5 A \tag{12}$$

Proof

Consider

$$x(s) \leqslant \int_a^b p(s,\lambda)x(\lambda)d\lambda + A_1 \tag{13}$$

$$x'(s) \leqslant \int_a^b p'(s,\lambda)x(\lambda)d\lambda + A_2 \tag{14}$$

where $A_1, A_2 \geqslant 0$ and $A_1 + A_2 = A$.

Multiplying (13) by $p(t,s)$ and integrate from a to b and multiplying (14) by $p'(t,s)$ and integrate from 0 to t, so we have

$$\int_a^b p(t,s)x(s)ds \leqslant \int_a^b \int_a^b p(t,s)p(s,\lambda)x(\lambda)d\lambda ds + c_1 A_1$$

$$\int_0^t p'(t,s)x(s)ds \leqslant \int_0^t \int_0^s p'(t,s)p'(s,\lambda)x(\lambda)d\lambda ds + c'_1 A_2$$

or

$$\int_a^b p(t,s)x(s)ds \leqslant \int_a^b p_2(t,s)x(s)ds + c_1 A_1 \tag{15}$$

$$\int_0^t p'(t,s)x(s)ds \leqslant \int_0^t p'_2(t,s)x(s)ds + c'_1 A_2 \tag{16}$$

By adding (15) and (16) we obtain

$$\int_a^b p(t,s)x(s)ds + \int_0^t p'(t,s)x(s)ds \leqslant \int_a^b p_2(t,s)x(s)ds + \int_0^t p'_2(t,s)x(s)ds + c_1A_1 + c'_1A_2$$

From (10) we have

$$x(t) \leqslant \int_a^b p_2(t,s)x(s)ds + \int_0^t p'_2(t,s)x(s)ds + [(1+c_1)A_1 + (1+c'_1)A_2]$$

Repeating the above procedure, we have

$$x(t) \leqslant \int_a^b p_\nu(t,s)x(s)ds + \int_0^t p'_\nu(t,s)x(s)ds + \sum_{j=0}^{\nu-1}[(1+c_1)A_1 + (1+c'_1)A_2]^j$$

From (8)we have

$$x(t) \leqslant c_2 \int_a^b x(s)ds + c'_2 \int_0^t x(s)ds + c_3 \tag{17}$$

where $c_3 = \sum\limits_{j=0}^{\nu-1}[(1+c_1)A_1 + (1+c'_1)A_2]^j$, nothing that $h \leqslant \frac{c_4}{N}$ from (9) and (17) we obtain

$$\kappa_n \leqslant c'_2 c_4 \sum_{i=0}^{n-1} \kappa'_i \frac{1}{N} + D$$

where $D = c_2 c_4 \sum_{i=a}^{b} \kappa_i \frac{1}{N} + c_3, 0 \leqslant n \leqslant N-1$. The above inequality is the standard discrete Gronwall inequality which yields (12). ∎

4. Convergence of collection methods

Throughout this paper, we write $E = \varepsilon(h)$ as shorthand for the inequality $\mid E \mid \leqslant ch^{\delta}$ that c and δ are positive constants.

Definition 4.1. If the functions p and p' satisfies condition C and

$$(i) \int_{t_n}^{t_{n+1}} p(t_{nj}, s)ds = \varepsilon(h), \int_{t_n}^{t_{nj}} p'(t_{nj}, s)ds = \varepsilon'(h) \tag{18}$$

$$(ii) \int_{0}^{t_n} |p(t_{nj}, s) - p(t, s)|ds = \varepsilon(h), \int_{0}^{t_n} |p'(t_{nj}, s) - p'(t, s)|ds = \varepsilon'(h) \tag{19}$$

$$t \in [t_n, t_{n+1})$$

where $t_{nj} \in X_n$, $0 \leqslant n \leqslant N-1$, then p and p' are said to condition D.

Definition 4.1. Let the function p and p' in (1) satisfy condition D, and $H(t, s, z) := k_z(t, s, z)$, $H'(t, s, z) := k'_z(t, s, z)$ satisfy

$$|H(t, s, z)| \leqslant c_6, \quad |H'(t, s, z)| \leqslant c'_6, \quad (t, s) \in S, \quad -\infty < z < \infty. \tag{20}$$

Theorem 4.1. If the solution y of (1) belongs to $C^m(I(T))$ with $m \geqslant 1$, then for a uniform mesh sequence and for any choice of the collocation parameters $\{c_j\}$ with $0 \leqslant c_1 < \cdots < c_m \leqslant 1$, the error $e(t) := y(t) - u(t)$ satisfies

$$\parallel e \parallel_{\infty} := \max\{|e(t)|, \quad t \in I(T)\} = O(h^m), \quad h \leqslant 1, \tag{21}$$

where u is the solution of the collocation equation (4) and h is the step size of the uniform mesh sequence.

Proof

Set $t = t_{nj}$ in (1) and subtract the collocation equation (4). Denoting by $e_n := y - u_n$ the restriction of the collocation method error to subinterval σ_n, we obtain

$$
\begin{aligned}
e_n &= y - u_n \\
&= g(t) + \int_a^b p(t,s)k(t,s,y(s))ds + \int_0^{t_nj} p'(t,s)k'(t,s,y(s))ds \\
&\quad - g(t) - \int_a^b p(t,s)k(t,s,u(s))ds - \int_0^{t_nj} p'(t,s)k'(t,s,u(s))ds \\
e_n(t_{nj}) &= \sum_{i=0}^{n} h \int_0^1 p(t_{nj}, t_i+\nu h)[k(t_{nj}, t_i+\nu h, y(t_i+\nu h)) - k(t_{nj}, t_i+\nu h, u(t_i+\nu h))]d\nu \\
&\quad + h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)[k'(t_{nj}, t_n+\nu h, y(t_n+\nu h)) - k'(t_{nj}, t_n+\nu h, u(t_n+\nu h))]d\nu \\
&\quad + \sum_{i=0}^{n-1} h \int_0^1 p'(t_{nj}, t_i+\nu h)[k'(t_{nj}, t_i+\nu h, y(t_i+\nu h)) - k'(t_{nj}, t_i+\nu h, u(t_i+\nu h))]d\nu \\
&= \sum_{i=0}^{n} h \int_0^1 p(t_{nj}, t_i+\nu h)[y(t_i+\nu h) - u(t_i+\nu h)]k_z(t_{nj}, t_i+\nu h, \xi_i)d\nu \\
&\quad + h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)[y(t_n+\nu h) - u(t_n+\nu h)]k'_z(t_{nj}, t_i+\nu h, \xi_i)d\nu \\
&\quad + \sum_{i=0}^{n-1} h \int_0^1 p'(t_{nj}, t_i+\nu h)[y(t_i+\nu h) - u(t_i+\nu h)]k'_z(t_{nj}, t_i+\nu h, \xi_i)d\nu \\
&= \sum_{i=0}^{n} h \int_0^1 p(t_{nj}, t_i+\nu h)H(t_{nj}, t_i+\nu h, \xi_i)e_i(t_i+\nu h)d\nu \\
&\quad + h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)e_n(t_n+\nu h)d\nu \\
&\quad + \sum_{i=0}^{n-1} h \int_0^1 p'(t_{nj}, t_i+\nu h)H'(t_{nj}, t_i+\nu h, \xi_i)e_i(t_i+\nu h)d\nu \\
&= \sum_{i=0}^{n-1} h \int_0^1 [p(t_{nj}, t_i+\nu h)H(t_{nj}, t_i+\nu h, \xi_i) + p'(t_{nj}, t_i+\nu h)H'(t_{nj}, t_i+\nu h, \xi_i)] \\
&\quad e_i(t_i+\nu h)d\nu + h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)e_n(t_n+\nu h)d\nu \\
&\quad + h\int_0^1 p(t_{nj}, t_n+\nu h)H(t_{nj}, t_n+\nu h, \xi_n)e_n(t_n+\nu h)d\nu
\end{aligned} \tag{22}
$$

where $\xi_i \in (\min(y, u_i), \max(y, u_i))$. Here we have made use of the mean value theorem applied to the third variable of the function κ. For $\nu \in (0, 1]$ we follow Brunner [1] and write

$$
e_n(t_n+\nu h) = \sum_{l=1}^{m} \beta_{nl}\nu^{l-1} + h^m R_n(\nu), \quad 0 \leqslant n \leqslant N-1, \tag{23}
$$

where β_{nl} are constants, and

$$R_n(\nu) = \frac{y^{(m)}(t_n + \theta_n \nu h)\nu^m}{m!} \quad (0 < \theta_n < 1).$$

Combining (22) and (23)

$$\begin{aligned}
\sum_{l=1}^{m} \beta_{nl}\{c_j^{l-1} + h^m R_n(\nu) = \sum_{i=0}^{n-1} h \int_0^1 [p(t_{nj}, t_i + \nu h)H(t_{nj}, t_i + \nu h, \xi_i) \\
+ p'(t_{nj}, t_i + \nu h)H'(t_{nj}, t_i + \nu h, \xi_i)](\sum_{l=1}^{m} \beta_{nl}\nu^{l-1} + h^m R_n(\nu))d\nu \\
+ h \int_0^1 p(t_{nj}, t_n + \nu h)H(t_{nj}, t_n + \nu h, \xi_n)(\sum_{l=1}^{m} \beta_{nl}\nu^{l-1} + h^m R_n(\nu))d\nu \\
+ h \int_0^{c_j} p'(t_{nj}, t_n + \nu h)H'(t_{nj}, t_n + \nu h, \xi_n)(\sum_{l=1}^{m} \beta_{nl}\nu^{l-1} + h^m R_n(\nu))d\nu \\
\Rightarrow \sum_{l=1}^{m} \beta_{nl}[c_j^{l-1} - h \int_0^1 p(t_{nj}, t_n + \nu h)H(t_{nj}, t_n + \nu h, \xi_n)\nu^{l-1} d\nu \\
- h \int_0^{c_j} p'(t_{nj}, t_n + \nu h)H'(t_{nj}, t_n + \nu h, \xi_n)\nu^{l-1} d\nu] \\
= \sum_{i=0}^{n-1} h \sum_{l=1}^{m} \beta_{il}[\int_0^1 (p(t_{nj}, t_i + \nu h)H(t_{nj}, t_i + \nu h, \xi_i) \\
+ p'(t_{nj}, t_i + \nu h)H'(t_{nj}, t_i + \nu h, \xi_n))\nu^{l-1}] + q_{nj}
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
q_{nj} = -h^m R_n(c_j) + h \int_0^1 p(t_{nj}, t_n + \nu h)H(t_{nj}, t_n + \nu h, \xi_n)(h^m R_n(\nu))d\nu \\
+ h \int_0^{c_j} p'(t_{nj}, t_n + \nu h)H'(t_{nj}, t_n + \nu h, \xi_n)(h^m R_n(\nu))d\nu \\
+ \sum_{i=0}^{n-1} h \int_0^1 (p(t_{nj}, t_i + \nu h)H(t_{nj}, t_i + \nu h, \xi_n) \\
+ p'(t_{nj}, t_i + \nu h)H'(t_{nj}, t_i + \nu h, \xi_i))(h^m R_i(\nu))d\nu
\end{aligned} \tag{25}$$

Define

$$\begin{aligned}
D_{ni} := h \int_0^1 (p(t_{nj}, t_i + \nu h)H(t_{nj}, t_i + \nu h, \xi_i) \\
+ p'(t_{nj}, t_i + \nu h)H'(t_{nj}, t_i + \nu h, \xi_n))\nu^{l-1} \\
0 \leqslant i \leqslant n-1 \quad 1 \leqslant j, l \leqslant m
\end{aligned}$$

and

$$D_{nm} := h\int_0^1 p(t_{nj}, t_n+\nu h)H(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu$$
$$+\, h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu$$

Let $V := (c_j^{l-1})$ denote the vandermonde matrix of order m associated with the collocation parameters $\{c_j\}$. The recurrence relation (24) can thus be written as

$$(V - D_{nn})\beta_n = \sum_{i=0}^{n-1} D_{ni}\beta_i + q_n, \quad 0 \leqslant n \leqslant N-1, \tag{26}$$

where $q_n := (q_{n1}, \ldots, q_{nm})^T$ is the vector whose components are defined by (25). Since p and p' satisfies (18) and H and H' satisfies (20) we have

$$|h\int_0^1 p(t_{nj}, t_n+\nu h)H(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu + h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu|$$
$$\leqslant |h\int_0^1 p(t_{nj}, t_n+\nu h)H(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu| + |h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu|$$
$$\leqslant hc_6\int_0^1 p(t_{nj}, t_n+\nu h)d\nu + hc_6'\int_0^{c_j} p'(t_{nj}, t_n+\nu h)d\nu$$
$$= c_6\int_{t_n}^{t_{n+1}} p(t_{nj}, s)ds + c_6'\int_{t_n}^{t_nj} p'(t_{nj}, s)ds$$
$$\leqslant c_6\varepsilon(h) + c_6'\varepsilon'(h)$$

Let $\varepsilon''(h) = \max\{\varepsilon(h), \varepsilon'(h)\}$, since $h < 1$ and $c_j, c_j' < 1$ we have

$$\Bigg|h\int_0^1 p(t_{nj}, t_n+\nu h)H(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu$$
$$+\, h\int_0^{c_j} p'(t_{nj}, t_n+\nu h)H'(t_{nj}, t_n+\nu h, \xi_n)\nu^{l-1}d\nu\Bigg| \leqslant \varepsilon''(h)$$

Hence the matrix $V - D_{nn}$ possesses a uniformly bounded inverse for sufficiently small h. Thus exists a finite constant c_7 independent of h and N such that

$$\| (V - D_{nn})^{-1} \|_1 \leqslant c_7, \quad 0 \leqslant n \leqslant N-1. \tag{27}$$

Also

$$\begin{aligned}\| D_{ni} \|_1 &\leqslant \sum_{j=1}^{m} hc_6 \int_0^1 (p(t_{nj}, t_i + \nu h))d\nu + \sum_{j=1}^{m} hc'_6 \int_0^1 (p'(t_{nj}, t_i + \nu h))d\nu \\ &= \sum_{j=1}^{m} hc_6 \int_0^1 (p(t_{nj}, t_i + \nu h)c_6 + p'(t_{nj}, t_i + \nu h)c'_6)d\nu \\ &= \sum_{j=1}^{m} \int_{t_i}^{t_{i+1}} (p(t_{nj}, s)c_6 + p'(t_{nj}, s)c'_6)ds\end{aligned} \tag{28}$$

From (26), (27)and (28) we have

$$\begin{aligned}\| \beta_{ni} \|_1 \leqslant c_8 \sum_{i=0}^{n-1} \sum_{j=1}^{m} \int_{t_i}^{t_{i+1}} p(t_{nj}, s)ds \| \beta_i \|_1 \\ + c'_8 \sum_{i=0}^{n-1} \sum_{j=1}^{m} \int_{t_i}^{t_{i+1}} p'(t_{nj}, s)ds \| \beta_i \|_1 + c_7 \| q_n \|_1\end{aligned} \tag{29}$$

where $c_8 = c_6 c_7$, $c'_8 = c'_6 c_7$. Let $x(t) = \| \beta_{ni} \|_1$, so, we have

$$\begin{aligned}x(t) &\leqslant c_8 \sum_{i=0}^{n-1} \sum_{j=1}^{m} \int_{t_i}^{t_{i+1}} p(t_{nj}, s)x(s)ds + c'_8 \sum_{i=0}^{n-1} \sum_{j=1}^{m} \int_{t_i}^{t_{i+1}} p'(t_{nj}, s)x(s)ds + c_7 \| q_n \|_1 \\ &= c_8 \sum_{j=1}^{m} \int_0^1 p(t, s)x(s)ds + c'_8 \sum_{j=1}^{m} \int_0^1 p'(t, s)x(s)ds + c_7 \| q_n \|_1 \\ &\quad + c_8 \sum_{j=1}^{m} \int_0^1 [p(t_{nj}, s) - p(t, s)]x(s)ds + c'_8 \sum_{j=1}^{m} \int_0^1 [p'(t_{nj}, s) - p'(t, s)]x(s)ds \\ &\leqslant mc_8 \int_0^1 p(t, s)x(s)ds + mc'_8 \int_0^1 p'(t, s)x(s)ds + c_7 \| q_n \|_1 \\ &\quad c_8\beta \sum_{j=1}^{m} \int_0^1 \mid p(t_{nj}, s) - p(t, s) \mid ds + c'_8\beta \sum_{j=1}^{m} \int_0^1 \mid p'(t_{nj}, s) - p'(t, s) \mid ds \\ &\leqslant mc_8 \int_0^1 p(t, s)x(s)ds + mc'_8 \int_0^1 p'(t, s)x(s)ds + c_7 \| q_n \|_1 \\ &\quad + mc_8\beta\varepsilon(h) + mc'_8\beta\varepsilon'(h) \\ &= mc_8 \int_0^1 p(t, s)x(s)ds + mc'_8 \int_0^1 p'(t, s)x(s)ds + c_7 \| q_n \|_1 \\ &\quad + m\beta(c_8\varepsilon(h) + c'_8\varepsilon'(h))\end{aligned} \tag{30}$$

where $\beta := \max\{\| \beta_n \|_1, 0 \leqslant n \leqslant N-1\}$ and $\varepsilon''(h) := \max\{\varepsilon(h), \varepsilon'(h)\}$.
Since $t \geqslant t_n$ we obtain

$$x(t) \leqslant mc_8 \int_0^1 p(t,s)x(s)ds + mc_8' \int_0^1 p'(t,s)x(s)ds + c_7 \| q_n \|_1 + m\beta\varepsilon''(h) \quad (31)$$

Since $y \in C^m(I(T))$, we have shown the relation (21).

■

By Theorem 4.1, we have prooved that the analytical solutions of this class of Fredholm-Volterra integral equations (FVIEs) are smooth.

5. Conclusion

In this work we showed that the use of uniform meshes leads to optimal convergence rates provided that the analytical solutions of a particular class of Fredholm-Volterra integral equations (FVIEs) are smooth.

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