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Sensitivity Analysis of Efficient and Inefficient Units in Integer-Valued Data Envelopment Analysis

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Abstract. One of the most important issues in Data Envelopment Analysis (DEA) is sensitivity and stability region of a specific Decision Making Unit (DMU), including efficient and inefficient DMUs. In sensitivity analysis of efficient DMUs, the largest region should be found namely stability region that data variations are only for under evaluation efficient DMU and the data for the remaining DMUs are assumed fixed. Also under evaluation efficient DMU remains efficient with these variations. In sensitivity analysis of an inefficient DMU, it can obtain an efficiency score which is defined by the manager. In traditional DEA, we assume that all inputs and outputs are real amounts and consider continuous inputs and outputs. Although,there are some applications in which one or more inputs and/or outputs can only take integer quantities. In this paper, we obtain a stability region for efficient DMUs with integer data. Thus the inefficient DMU which is under evaluation can satisfy the decision maker and also it can be improved itself to gain a defined efficiency score by management, with integer data. The procedures are illustrated by numerical examples.

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1. Introduction

Data Envelopment Analysis (DEA) is introduced by Charnes et al. [3] (CCR model) and extended by Banker et al. [1] (BCC model). It is one of the best ways for assessing the relative efficiency of group of homogenous Decision Making Units (DMUs)

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that use multiple inputs to produce multiple outputs. In recent years, one of the most important issues in DEA is the sensitivity analysis including efficient and inefficient DMUs which more researchers have great attention. Sensitivity analysis of CCR model for a specific efficient DMU with a single output was initiated by Charnes [4]. Charnes and Neralic [5] considered additive model and they obtained sufficient conditions for remaining efficient. Then, Charnes et al. [2] obtained a specific stability region with L_1 and L_∞. These researchers have studied the methods which simultaneous proportional change is assumed in inputs and outputs for a specific efficient under evaluations DMU. Then Zhu [13] provides a modified DEA model to compute a stability region which under evaluation DMU remains efficient.

Seiford and Zhu [12] developed a procedure to determine an input stability region (ISR) and an output stability region (OSR) for efficient DMU. They stated that an efficient DMU will remain efficient after the input increases or output decreases if and only if such changes occur within the ISR or OSR, and this subject are considering in recent years. Jahanshahloo et al. [7] extended the largest stability region for BCC model and Additive model by supporting hyperplanes for under evaluation DMU which all inputs and outputs of DMUs except under evaluation DMU are assumed fixed. The variations of inputs and outputs are included in four cases: (1) increase of outputs and increase of inputs, (2) decrease of outputs and the increase of inputs, (3) decrease of outputs and decrease of inputs and (4) increase of outputs and decrease of inputs. By variation in case 4 the efficient unit preserve its efficiency because increase of outputs accompanied by decrease of inputs cannot worsen the efficiency of a DMU. They obtained this largest stability region by restricted their attention to the cases 1, 2 and 3. In some these works and other works sensitivity analysis are based on the super efficiency DEA approach in which the efficient under evaluation DMU is not included in the reference set. Sensitivity analysis of an inefficient DMU is studied less than sensitivity of an efficient DMU. Charnes et al. [2] obtained an improvement for inefficient DMU by using Chebychev norm. In the recent years data analysis of inefficient units has been more studied. Jahanshahloo et al. [8] have introduced a new frontier with efficiency score of $\alpha < 1$ for a specific inefficient DMU and α is a constant which is defined by the manager. Sometimes, inefficient DMUs can never reach to the efficient frontier and obtain the score1. Our objective is to achieve ways to improve the inefficient units using another method. In mentioned cases, inputs and outputs are assumed real-valued data. Although, some of the input and/or output data exactly have integer amounts in a lot of application such as the number of employees, cars, customers, lectures and ext. (Lozano and villa [11]). In IDEA, DMUs classify in to efficient and inefficient DMUs.

In this paper, we focus on the sensitivity analysis of efficient and inefficient DMUs in IDEA. In efficient DMUs we obtain stability region for DMUs with integer data.

This paper proceeds as follows. In section 2 we briefly present some basic DEA principles. Section 3 develops a proposed method for obtaining stability region of a specific efficiency DMU. Also in section 4 a specific inefficient DMU are improved. Section 5 simple numerical examples, that illustrate the proposed methods are provided. Finally, conclusions are given.

2. Integer-Valued DEA Model

Conventional data envelopment analysis (DEA) models assume that inputs and outputs are continues and have real amount (Cooper et al. (2005). Also, we know that there are many applications in which one or more inputs and outputs are perforce integer amount. First, in these situations, the non-integer targets are rounded off. One of difficulty is that rounding off may easily lead to an infeasible target. Then Lozano and villa [11] consider, $_j(j = 1, \dots, n)$, that each DMU consumes m inputs to produce s outputs. Let $I = \{1, \dots, m\}$, $O = \{1, \dots, s\}$ the sets of input and output indices respectively. Suppose that observed input and output vectors of $_j$ are $X_j = \{x_{1j}, ..., x_{mj}\}\, and\, Y_j = \{y_{1j}, ..., y_{sj}\}$ respectively. Also, let $I' \subset I\, and\, O' \subset O$ the subsets of the corresponding indices that must be integer-valued. Obviously $x_{ij}\, and\, y_{rj}$ must be integer for all $i \in I'\, and\, r \in O'$. An integer-valued Constant Returns to Scale (CRS) Production Possibility Set (PPS) is defined as follows:

$$T'_{CRS} = \left\{ (\widehat{x}, \widehat{y}) | \widehat{x}_i \geq \sum_{j=1}^{n} \lambda_j x_{ij}, \widehat{y}_r \leq \sum_{j=1}^{n} \lambda_j y_{rj}, \lambda_j \geq 0, \right.$$

$$\left. j = 1, ..., n\ \widehat{x}_i \text{ integer } \ i \in I', \widehat{y}_r \ \text{ integer } \ r \in O' \right\}$$

Definition 2.1 *A DMU is CRS integer-efficient if no other integer-valued operating point dominates it.*

Definition 2.2 *The CRS integer-efficiency frontier is the set of non-dominated, integer-valued operating points, i. e. ,*

$$(T'_{CRS})^{eff} = \{(\widehat{x}, \widehat{y}) \in T'_{CRS} : \forall (\overline{x}, \overline{y}) \in T'_{CRS}[\overline{x} \leq \widehat{x}] \cap [\overline{y} \geq \widehat{y}] \leftrightarrow (\overline{x}, \overline{y}) = (\widehat{x}, \widehat{y})\} \subset T'_{CRS}$$

If $_j$ is CRS efficient, then it is CRS integer-efficient.

In this regard, above Production Possibility Set(PPS), Lozano and Villa proposed DEA models by integer data. But, these models form by axiomatic foundation for DEA in the case of integer-valued data.

Kuosmann and Kazemi Matin [10] introduced new axioms consists of “natural disposality” and “natural divisibility” and extended axiomatic foundation for integer DEA under variable, non-decreasing and non-increasing returns to scale. They introduced new axioms consisting of” natural convexity” and “natural augmentability”. Then the efficiency can be computed by solving the following MILP problem which obtained by using above proposed axioms :

$$\begin{array}{lll}
\min \theta - \varepsilon(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- + \sum_{i=1}^{p} s_i^I), & & \\
s.t.\ y_{ro} + s_r^+ = \sum_{j=1}^{n} y_{rj}\lambda_j, & r \in O, & \\
\quad \theta x_{io} - s_i^- = \sum_{j=1}^{n} x_{ij}\lambda_j, & i \in I^{NI}, & \\
\quad \widetilde{x}_i - s_i^- = \sum_{j=1}^{n} x_{ij}\lambda_j, & i \in I^I, & (1) \\
\quad \theta x_{io} - s_i^I = \widetilde{x}_i, & i \in I^I, & \\
\quad \widetilde{x}_i \in Z_+, & i \in I^I, & \\
\quad \lambda_j \geq 0 & j \in J, & \\
\quad s_r^+ \geq 0, \ \ s_i^- \geq 0\ \ s_j^I \geq 0, & r \in O, \ \ i \in I, \ \ j \in I^I. &
\end{array}$$

3. Sensitivity Analysis of Efficient DMUs with Integer Data

In this method, we construct imaginary stability region by assumed supporting hyperplanes for a specific efficient DMU. First,by using model 1, efficient integer frontier is obtained.

We know that the efficient integer frontier is not convex. But, we can approximate the efficient frontier by using this frontier. On the other hand, any integer member of the convex efficient frontier is a member of integer efficient frontier. These integer efficient points are consist on $\alpha_1 = \{_{j1}, ..., _{js}, I_1, ..., I_r\}$

All of imaginary supporting hyperplanes of the Production Possibility Set (PPS)are yielded by using of way finding strong defining hyperplanes, which cross through efficient $_c$. For more details and the method of finding strong defining hyperplanes of Production Possibility Set (PPS), (Jahanshahloo et al. [9]. We named these, $H_1, ..., H_k$. They are defined as follows:

$$H_l : P^t Z_k + \alpha_k = 0 \;\; where\, Z = (x_1, ..., x_m, y_1, ..., y_s) \,\text{and}\, \alpha \text{ is a scalar}$$
$$l = 1, ..., k$$

Corresponding to assumed hyperplane H_k , the halfspace H_k^- is defined as follows:

$H_k^- = P^t Z_k + \alpha_k \leq 0$. Then all efficient DMUs that exist on this hyperplanes ,except $_c$, are found. Let $\alpha_2 = \{_{j_1}, ..., _{j_d}, I_{i1}, ..., I_{ic}\}$ be the set of these DMUs.

Toward this end, inefficient DMUs have been distinguish efficient, which yield by omitting $_c$, are evaluated in following model:

$$\begin{array}{ll}
\max \sum_{r=1}^{s} u_r y_{rk} + u_0 & \\
s.t. \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \leq 0, \; j = 1, ..., n, \;\; j \neq c, & \\
\sum_{i=1}^{m} v_i x_{ik} = 1, & \\
v_i \geq 0, \;\; u_r \geq 0, & i = 1, ..., m, \;\; r = 1, ..., s.
\end{array} \tag{2}$$

Let these DMUs in the set of $\alpha_3 = \{_{j_1}, ..., _{j_t}\}$. By omission of $_c$ in model (1), efficient integer frontier, can evaluate by means of model (3) again.

$$\begin{array}{ll}
\min \theta - \varepsilon(\sum_{r=1}^{s} s_r^+ + \sum_{i=1}^{m} s_i^- + \sum_{i=1}^{p} s_i^I), & \\
s.t. \; y_{ro} + s_r^+ = \sum_{\substack{j=1 \\ j \neq c}}^{n} y_{rj} \lambda_j, & r \in O, \\
\theta x_{io} - s_i^- = \sum_{\substack{j=1 \\ j \neq c}}^{n} x_{ij} \lambda_j, & i \in I^{NI}, \\
\widetilde{x}_i - s_i^- = \sum_{\substack{j=1 \\ j \neq c}}^{n} x_{ij} \lambda_j, & i \in I^I, \\
\theta x_{io} - s_i^I = \widetilde{x}_i, & i \in I^I, \\
\widetilde{x}_i \in Z_+, & i \in I^I, \\
\lambda_j \geq 0, & j \in J, \\
s_r^+ \geq 0, \;\; s_i^- \geq 0 \;\; s_j^I \geq 0, & r \in O, \;\; i \in I, \;\; j \in I^I.
\end{array} \tag{3}$$

Assume that is $\alpha_3 = \{_{j_1}, ..., _{j_f}, E_1, ..., E_m\}$ being the set of these DMUs. Let $H_{i_1}, ..., H_{i_d}$ be the yielded imaginary supporting hyperplanes that are defined by $\beta = \{_{j_1}, ..., _{j_f}, _{j_1}, ..., _{j_t}, I_{k1}, ..., I_{kt}, E_{m_1}, ..., E_{m_h}\}$ as follows [9]:

$H_{i_c} : P^t Z_{i_c} + \alpha = 0 \quad\; c = 1, ..., d$. Corresponding to imaginary hyperplane H_{i_c} , the halfspace $H_{i_c}^+$ is defined as follows:

$$H_{i_c}^+ : P^t Z_{i_c} + \alpha \geq 0 \qquad c = 1, ..., d$$

Now we consider these set for $_c$

$$S_1 = \bigcap_{j=1}^{g} H_{k_j}^-, \quad S_2 = \bigcup_{c=1}^{d} H_{i_c}^+, \quad S = S_1 \cap S_2$$

In this imaginary integer stability region, we should evaluate all integer-valued. In this regard, we should find all integer data in this region. So, the algorithm is introduced as follows:

The algorithm of finding all integer data in integer stability region's algorithm:

step 1. Integer stability region portrait on the coordinates pivots.
step 2. The integer numbers are designated on any pivot.
step 3. By using these points, multi regular are obtained.
step 4. If multi regular is belong to this region stability region go to 5, else choose another multi regular.
step5. These integer points are integer points of stability region.

THEOREM 3.1 *$S = S_1 \cap S_2$ is the imaginary integer stability region of* $_c$

4. Sensitivity Analysis of Inefficient DMUs with Integer Data

In this method, efficiency score of a specific inefficient $_B$ alter to an efficiency score α which is defined by the manager (Jahanshahloo et al. [8]) . We know that a specific inefficient DMU cannot reach itself (θ_B^*) to the integer efficient frontier easily but it can obtain an efficiency score α that $\theta_B^* < \alpha < 1$. What we want is to obtain integer points on efficient integer frontier for $_B$ which efficiency score becomes α after these changes. For this purpose,by applying model (1), distinguish points of integer efficient. Let the set of these points be E. For finding the new frontier, a set of E' was defined as follows:

$$E' = \{(X_j{}', Y_j{}')|\ (X_j', Y_j{}') = (\frac{1}{\alpha}X_j, Y_j),\ \ j \in E\}$$

And the new production possibility set is T_v' :

$$T_v' = \left\{ \left(X', Y'\right) |X' \geq \frac{1}{\alpha}\sum\nolimits_{j\in E}\lambda_j X_j, Y' \leq \sum\nolimits_{j\in E}\lambda_j Y_j, \sum\nolimits_{j\in E}\lambda_j = 1, \lambda_j \geq 0, j \in E \right\}$$

THEOREM 4.1 *The efficiency score of each point of E' in T_v is α.*

Attention 4.2 There is one- to- one correspondence between E and E'.

Attention 4.3 There is one-to-one correspondence between T_v and T_v' frontier points.

We know that most of the point of E' aren't integer. Therefore, by using model (1) for points of E', the integer points are obtained. These points are put in E'' set. Then, we obtain imaginary hyperplanes which pass from points of E''. For improving inefficient DMUs to α and integer point,these DMUs can use different ways for arriving themselves to integer points such as input decreasing or output increasing or combination of them.

5. Numerical Example

We illustrate the proposed methods for sensitivity analysis of efficient and inefficient DMUs with integer data by using two examples.

5.1 *Numerical Example for Sensitivity Analysis of Efficient Units*

Consider a system of nine DMUs with two inputs and single output as shown in Fig. 1. Data is given in Table 1.

Table 1. Nine DMUs with two inputs and single output

	A	B	C	D	E	F	G	H	I
X1	1	2	4	10	17	5	7	6	11
X2	15	10	4	2	1	6	4	10	7
Y	1	1	1	1	1	1	1	1	1

By using model 1, efficient integer frontier is obtained. these integer points are consist on $\alpha_1 = \{I_1,_A,_D,_C,_E,_B, I_2\}$ $\{(1,15),(2,10),(3,9),(4,4),(10,2),(7,3),(17,1)\}$. For finding the imaginary integer stability region of $_C$, we first find all imaginary supporting hyperplanes of PPS which are binding at $_C$ *and* $\alpha_2 = \{I_1,_D, I_2\}$ by using F_j ,that F_j is a subset of F that its member are coplanar . These imaginary supporting hyperplanes are as follows:

$$H_1 = (x_1,x_2)|\, x_2 + 5x_1 = 24\}, \qquad H_2 = \{(x_1,x_2)|\, x_2 + 1/3x_1 = 16/3\}$$

$$S_1 = H_1^- \cap H_2^- = \{(x_1,x_2)|x_2 + 5x_1 \le 24, x_2 + 1/3x_1 \le 16/3\}$$

Consider that I_1, I_2 are two of efficient integer frontier. Now, $_C$ is eliminated and inefficient DMUs which have been distinguished efficient by means of model (5), are obtained. We find out that the DMUs F and G are efficient. Then $\alpha_3 = \{_F,_G\}$. By using model 6, efficient integer frontier after omitting C, is obtained. these integer points are consist on $\alpha_4 = \{E_1,_F,_G, E_2\} = \{(4,8),(9,3),(5,6),(7,4)\}$. Also, The imaginary supporting hyperplanes of the new PPS which are binding at the members of set $\beta = \{I_1, E_1, E_2,_F,_G,_D\}$ are as follows:

$$\begin{aligned} H_{i_1} &= \{(x_1,x_2)|x_2 + x_1 = 12\} \\ H_{i_2} &= \{(x_1,x_2)|x_2 + 2x_1 = 16\} \\ H_{i_3} &= \{(x_1,x_2)|x_2 + x_1 = 11\} \\ H_{i_4} &= \{(x_1,x_2)|x_2 + 1/2x_1 = 15/2\} \\ H_{i_5} &= \{(x_1,x_2)|x_2 + x_1 = 12\} \end{aligned}$$

So,

$$\begin{aligned} S_2 &= H_{i_1}^+ \cup H_{i_2}^+ \cup H_{i_3}^+ \cup H_{i_4}^+ \cup H_{i_5}^+ \\ &= \{(x_1,x_2)|x_2 + x_1 \ge 12\} \;\; or \;\; x_2 + 2x_1 \ge 16 \; or \;\; x_2 + x_1 \ge 11 \;\; or \\ &\quad\; x_2 + 1/2x_1 \ge 15/2 \;\; or \;\; x_2 + x_1 \ge 12\} \end{aligned}$$

Consequently, the imaginary integer stability region of $_C$ is as follows:

$$\begin{aligned} S &= S_1 \cap S_2 \\ &= \{(x_1,x_2)|x_2 + 5x_1 \le 24, x_2 + 1/3x_1 \le 16/3\} \cap \{(x_1,x_2)|x_2 + x_1 \ge 12\} \\ &\quad or \;\; x_2 + 2x_1 \ge 16 \; or \;\; x_2 + x_1 \ge 11 \;\; or x_2 + 1/2x_1 \ge 15/2 \;\; or \;\; x_2 + x_1 \ge 12\}. \end{aligned}$$

Also, integer points of region are obtained according to finding all integer data in integer stability region's algorithm. This points are $A = \{(3,9),(4,9),(4,7),(4,8),(5,4),(5,5),(5,6),(6,4),(7,4),(8,4),(9,3),(10,2)\}$. Namely, can arrives itself in these points without its efficiency become zero. Figure

1 shows DMUs and integer efficient. Also, Figure 2 illustrates the imaginary stability region of the efficient unit C in the case of a single output and two inputs.

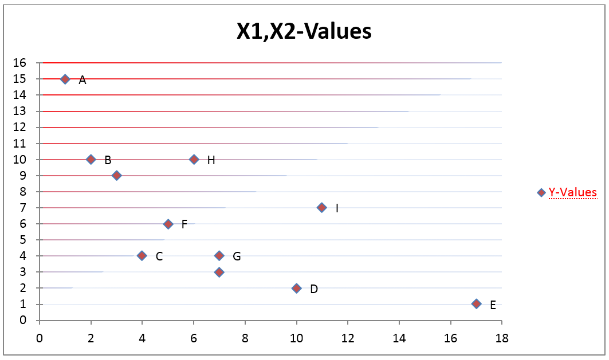

Figure 1. Nine DMUs with two inputs and single output

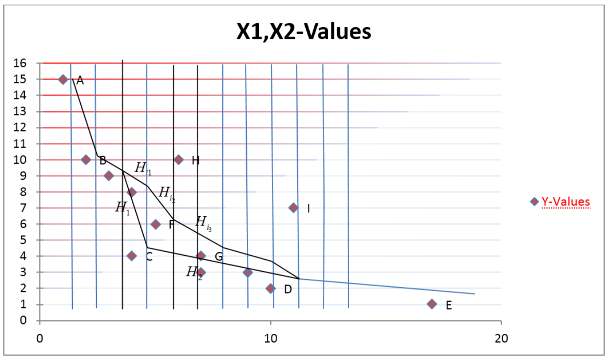

Figure 2. The imaginary stability region of C

5.2 *Numerical Example for Sensitivity Analysis of Inefficient Units*

Consider the seven DMUs with one input and one output as defined in Table 2, again.

Table 2. Seven DMUs with one input and one output

	A	B	C	D	E	F	G
x	1	2	4	5	6	7	10
y	1	3	6	2	5	8	8

By using model 1, efficient integer frontier is obtained. These integer points are consist on $E = \{(1,1),(2,3),(3,4),(4,6),(6,7),(7,8),(10,8)\}$. Suppose $\alpha = 0.8$. Then, by definition of $E', E' = \{(1.25,1),(2.5,3),(3.75,4),(5,6),(7.5,7),(8.75,8),(12.5,8)\}$ will be made. These points aren't integer. For finding integer frontier, model 1 solved again. These integer points are shown with E''. $E'' = \{(2,1),(3,3),(4,3),(5,6),(8,7),(9,8)\}$. Find all imaginary supporting hyperplanes of that pass these points. These hyperplanes are as follows:

$$H_1 = \{(x,y)|y - x = -1\}$$
$$H_2 = \{(x,y)|y - 3x = -9\}$$
$$H_3 = \{(x,y)|y - 1/3x = 13/3\}$$
$$H_4 = \{(x,y)|y - x = -1\}$$

Also, we know that $_D$ and $_E$ are inefficient. $\theta_D^* = 0.3 < \alpha = 0.8$ and $\theta_E^* = 0.56 < \alpha = 0.8$. Furthermore, these DMUs are integer. They want to arrive themselves to efficiency score which management designate (α). For this manner, they can use different ways for arriving themselves to integer. They can decrease inputs or increase outputs or can use combination oriented. In this example, inefficient DMUs utilize combination oriented. Then,they can improve itself to any integer points on imaginary hyperplanes. Here, $_D$ and $_E$ improve themselves to $(4, 4)$ and $(5, 6)$, respectively. Only $_D$ can improved itself to other point namely, $(3, 3)$ because it is in the region which is dominated by D. But $_E$ can improve itself only to $(5, 6)$.

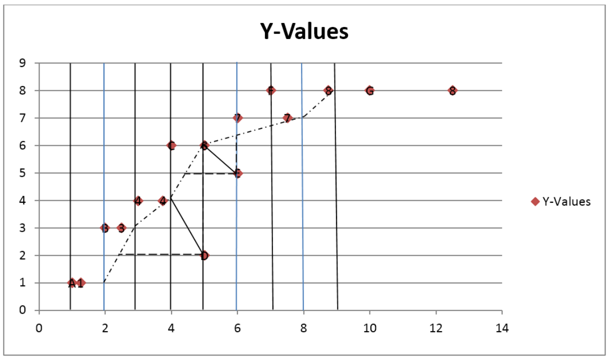

Figure 3. Sensitivity analysis of inefficient $_E$ and $_D$

6. Conclusions

In this paper we developed an approach for the sensitivity analysis of each efficient unit with integer data by using imaginary supporting hyperplanes. In this method we obtain the largest stability region for efficient DMU. First, the integer of region should be found which is obtained by the mentioned algorithm. Then the efficient DMU can reach to the integer of stability region with the same efficiency score. Also, efficiency score of an inefficient DMU can change to an efficiency score which is defined by the manager. Namely, inefficient DMU can improve itself to these integer points that are defined by the management.

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