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Hall and Ion-Slip Effects on Magneto-Micropolar Fluid with Combined Forced and Free Convection in Boundary Layer Flow over a Horizontal Plate with Viscous Dissipation

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Abstract. In this paper, we study the effects of Hall and ion-slip currents on the steady magneto-micropolar of a viscous incompressible and electrically conducting fluid over a horizontal plate by taking in to account the viscous dissipation effects. By means of similarity solutions, deviation of fundamental equations on the assumption of small magnetic Reynolds number are solved numerically by using quasilinearised first and finite difference method. The effects of various parameters of the problem, e.g. the magnetic parameter, Hall parameter, ion- slip parameter, buoyancy parameter and material parameter and Eckert number are discussed and shown graphically.

Keywords: Hall effects, Ion-slip, buoyancy parameter, Eckert number, finite difference method.

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1. Introduction

The study of magneto-micropolar fluid flows in a slip-flow regime with Hall and ion- slip currents has important engineering applications, e.g. in power generators, magnetohydrodynamic (MHD) accelerators, refrigeration coils, transmission lines, electric transformers and heating elements. The theory of micropolar fluids that displays the effects of local rotary inertia and coupled stresses was formulated by Eringen [4]. The theory can be used to explain the flow of colloidal fluids, liquid crystals, etc. The present problem finds applications in MHD generators with neutral fluid seeding in the form of rigid microinclusions. Also, many industrial applications involve fluids as a working medium, and in such applications unclean

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fluids (i.e. clean fluid plus interspersed particles) are the rule and clean fluids an exception. Control of convection is also important in many of these non- isothermal applications. Literature on magneto-micropolar fluid and heat transfer is very extensive due to its technical importance in the scientific community. Some literature surveys and reviews of pertinent work in this field are documented by Ahmadi and Shahinpoor [1] who recently analyzed the universal stability of magneto-micropolar fluid motions. Mori [10] and Sparrow and Minkowycz [21] were the first investigators to treat the problem of combined convective heat transfer over a horizontal flat plate with the effects of buoyancy forces. Balaram and Sastry [2] and Maiti [9] studied the convection heat transfer in a micropolar fluid through a vertical channel and through a horizontal parallel plate channel. The free convection in the boundary layer flow of a micropolar fluid past a non- isothermal vertical flat plate has been studied by Jena and Mathur [7] and Hassanien [5] studied.

It should be noted that in the above studies the Hall current as well as the ion-slip effects are ignored. In fact, the Hall effect is important when the Hall parameter, which is the ratio between the electron-cyclotron frequency and the electron-atom-collision frequency, is high. This happens when the magnetic field is high or when the collision frequency is low (Crammer and Pai, [3] Sutton and Sherman [23]). Furthermore, the masses of the ions and the electrons are different and, in turn, their motions will be different. Usually, the diffusion velocity of electrons is larger than that of ions and, as a first approximation , the electric current density is determined mainly by the diffusion velocity of the electrons. However, when the electromagnetic force is very large (such as in the case of strong magnetic field), the diffusion velocity of the ions may not be negligible (Crammer and Pai [3] Sutton and Sherman [23]). If we include the diffusion velocity of ions as well as that of electrons, we have the phenomena of ion slip. In the above mentioned work, the Hall and ion slip terms were ignored in applying Ohms law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magnetohydrodynamic is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable under these conditions , and the Hall current as well as the ion slip are important ;they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic-force term (Crammer and Pai [3] Sutton and Sherman[23]).

In most of the MHD flow problems, the Hall and Ion-slip terms in Ohms law were ignored . However, in the presence of strong magnetic field, the influence of Hall current and Ion-slip are important. Tani[24] studied the Hall effects on the steady motion of electrically conducting viscous fluid in channels. Hall and Ion-slip effects on MHD Couette flow with heat transfer have been considered by Soundelgekar et al [20]. Hazem Ali et al [16] examined the MHD flow and heat transfer in a rectangular duct with temperature dependent viscosity and Hall Effect. Emmanuel Osalusi et al [11] investigated on the effectiveness of viscous dissipation and Joule heating on steady MHD flow and heat transfer of a Bingham fluid over a porous rotating disk in the presence of Hall and Ion-slip currents. Hayat et al [6] studied the Hall effect on the unsteady flow due to non coaxially rotating disk and fluid at infinity. Many of the problems in the literature deal with MHD flow between parallel plates / flow through circular pipes with Hall and Ion-slip effects, but not much attention has been given to the flow through a closed rectangular channel and concentric cylinders.

The effects of Hall and ion-slip currents in steady MHD free convective flow past an infinite vertical porous plate in a rotating frame of reference when the heat flux is maintained as constant at the plate was studied by Ram [14]. Later, Ram and Takhar [15] dealt with MHD free convection from an impulsively moving infinite vertical plate in a rotating fluid with Hall and ion-slip currents. Pop and Watanable [12] studied the effects of Hall current on MHD free convection flow past a semi-infinite vertical flat plate. The effects of Hall and ion-slip currents on free convective heat generating flow in a rotating fluid were analyzed by Ram [13]. Kinyanjui et al. [8] studied the effects of Hall current in the MHD Stokes problem for a vertical infinite plate in a rotating fluid. Seddeek [17] studied the effects of Hall and ion-slip currents on magneto-micropolar fluid and heat transfer over a non- isothermal stretching sheet with suction and blowing. Recently, Seddeek [18] studied the flow of a micropolar fluid past a moving plate by the presence of magnetic field.

In the present work, the aim is to investigate the Hall and ion-slip current effects of neutral seeding of the fluid, in the form of introducing relatively rigid microelements in the fluid which is moving in the presence of a magnetic field. A linear micropolar fluid as presented by Eringen [4] is assumed to represent the model of the fluid under consideration. We have chosen to call the fluid under discussion a magneto-micropolar fluid. The criteria for Hall and ion-slip current effects on magneto-micropolar fluid with boundary layer mixed convection flow over a horizontal plate are then obtained. Numerical results are presented for range of values of Hall parameter, ion-slip parameter, magnetic parameter, material parameter and buoyancy parameter of the fluid.

2. Mathematical Analysis

Consider a steady laminar, incompressible, viscous, electrically conducting fluid flowing past a semi-infinite horizontal plate having a uniform free stream velocity U_∞, denisity ρ_∞ and temperature T_∞. The x-axis is assumed to be taken along the plate and the y-axis normal to the plate. A transverse strong magnetic field B_o with constant intensity is imposed along the y-axis. For an electrically conducting fluid, the Hall and ion-slip currents significantly affect the flow in the presence of large magnetic field. The induced magnetic field is neglected, since the magnetic Reynolds number is assumed to be very small (Shercliff[19]). The effects of Hall current gives rise to a force in the z-direction, which induces a cross flow in that direction, and hence, the flow becomes three-dimensional. We assume that there is no variation of flow or heat transfer quantities in the z-direction. The governing boundary layer equations may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(v+\frac{k}{\rho_\infty}\right)\frac{\partial^2 u}{\partial y^2}-\frac{1}{\rho_\infty}\frac{\partial p}{\partial x}+\frac{k}{\rho_\infty}\frac{\partial N}{\partial y}-\frac{B_o}{\rho_\infty}j_z, \tag{2}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\left(v+\frac{k}{\rho_\infty}\right)\frac{\partial^2 w}{\partial y^2}+\frac{B_o}{\rho_\infty}j_x \qquad \frac{1}{\rho_\infty}\frac{\partial p}{\partial y}=\beta_\infty(T-T_\infty) \tag{3}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma}{\rho_\infty J^*}\frac{\partial^2 N}{\partial y^2}-\frac{K}{\rho_\infty J^*}\left(2N+\frac{\partial u}{\partial y}\right), \tag{4}$$

(5)

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial v} = \frac{k_f}{\rho_\infty c_p}\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2, \quad (6)$$

Where u, v, and w are the velocity components in the x, y, and z directions, respectively; N is the angular velocity; T is the fluid temperature; v, k and γ are the viscosity coefficients, J^* is the micro inertia per unit mass; k_f is the thermal conductivity; C_P is the specific heat at constant pressure, g is the gravitational constant and β_∞ is the coefficient of volumetric expansion.

The boundary conditions are given by

$$\begin{aligned} &y = 0 : u = v = w = 0, N = 0, T = T_w(w) \\ &y \to \infty : u \to U_\infty, N \to 0, w \to 0, T \to T_\infty, p \to p_\infty \end{aligned} \quad (7)$$

Where ρ_∞ is the hydrostatic pressure in the undisturbed fluid. The equation of conservation of electric charge $\nabla.j = 0$ gives j_y = constant, where $j = (j_x, j_y, j_z)$. This constant is assumed to be zero since $j_y = 0$ everywhere in the flow. The expression for the current density components j_x and j_z as obtained from the generalized Ohms law (Sutton and Sherman [23]), are given by

$$\begin{aligned} j_x &= \frac{\sigma}{\alpha_e^2 + \beta_e^2}[\alpha(E_x - wB_o) + \beta(E_z - uB_o)], \\ j_z &= \frac{\sigma}{\alpha_e^2 + \beta_e^2}[\alpha(E_z + uB_o) - \beta(E_x - wB_o)] \end{aligned}$$

Where β_e is the Hall parameter, β_i is the ion-slip parameter, σ is the electrical conductivity and $\alpha_e = 1 + \beta_i\beta_e$. In the absence of electric filed ($E_x = E_z = 0$), we get

$$j_x = \frac{\sigma B_o}{\alpha_e^2 + \beta_e^2}(\beta_e u - \alpha_e w), \quad (8)$$

$$j_z = \frac{\sigma B_o}{\alpha_e^2 + \beta_e^2}(\alpha_e u + \beta_e w) \quad (9)$$

Now we use the dimensionless variable, which takes the form

$$
\begin{aligned}
&x=\frac{x}{L};\ \ y=\sqrt{\frac{R_e}{X}\frac{y}{L}};\\
&U=\frac{U}{U_\infty};\ \ v=\sqrt{\frac{R_e}{X}\frac{v}{U_\infty}};\ \ w=\frac{w}{U_\infty};\\
&N_1=\frac{NL}{U_\infty}\sqrt{\frac{X}{R_e}};\ \ \theta\frac{T-T_\infty}{T^*};\ \ p=\frac{p-p_\infty}{\rho_\infty U_\infty};\\
&\lambda=\frac{\lambda}{\mu J^*};\ \ B=\frac{v^2R_e}{J^*U_\infty^2};\ \ Ar=\frac{gL\beta_\infty T^*}{U_\infty^2};\text{(Archimedes number)}\\
&Re=\frac{U_\infty x}{v}=\frac{U_\infty XL}{v};\text{(Reynolds number)};\\
&\Delta=\frac{k}{\mu};\text{(Material parameter)};\\
&\Omega=Ar\sqrt{\frac{x}{R_e}};\text{(Buoyancy parameter)};\\
&pr=\frac{\rho_\infty v c_p}{k_f};\text{(Prandtl number)};\\
&M=\frac{\sigma_v R_e B_o^2}{\rho_\infty U_\infty^2};\text{(magnetic parameter)};
\end{aligned}
\tag{10}
$$

Wehre T^* represents a characteristic temperature difference between plate and free stream and L is the value of the x-coordinate where $T^*=T_w-T_\infty$. λ and B are the micropolar fluid parameters.

We further define the following similarity variables.

$$
\begin{aligned}
&\eta=YX^{-\frac{1}{2}},\ \ \psi=X^{\frac{1}{2}}f(\eta);\\
&N_1=X^{-\frac{1}{2}}g(\eta);\ \ \theta=X^{-\frac{1}{2}}\theta(\eta);\ \ W=h(\eta).
\end{aligned}
\tag{11}
$$

With $u=\frac{\partial\psi}{\partial Y}$, $V=-\frac{\partial\psi}{\partial X}$ and introducing equations (8), (9), (10) and(11) in to equations (2), (3), (5) and (6) give

$$2(1+\Delta)f'''+ff''+2\Delta g'+\Omega\eta\theta-\frac{2M}{\alpha_e^2+\beta_e^2}(\alpha_e f'+\beta_e h)=0 \tag{12}$$

$$2(1+\Delta)h''+fh'-\frac{2M}{\alpha_e^2+\beta_e^2}(\alpha_e h-\beta_e f')=0 \tag{13}$$

$$\lambda g''+\frac{1}{2}(f'g+fg')-\Delta .B(2g+f'')=0 \tag{14}$$

$$\frac{2}{pr}\theta''+f\theta'+f'\theta+2E_c f''^2=0 \tag{15}$$

In the above equations, a prime denotes differentiation with respect to η only.

Now the equation is a linear differential equation which is solved by using the implicit finite difference method. With the implementation of the implicit finite difference method the equation (12), (13), (14) and equation (15) are becomes

$$a(i)f(i+1)+b(i)f(i)+c(i)f(i-1)-rf(i-1)-d(i)=0 \tag{16}$$
$$a_1(i)h(i+1)+b_1(i)h(i)+c_1h(i-1)-d_1(i)=0 \tag{17}$$
$$a_2(i)g(i+1)+b_2(i)g(i)+c_2g(i-1)-d_2(i)=0 \tag{18}$$
$$a_3(i)\theta(i+1)+b_3(i)\theta(i)+c_3\theta(i-1)-d_3(i)=0 \tag{19}$$

where

$$r=1+\Delta;\quad r_1=2M/\alpha_e^2+\beta_e^2$$
$$a(i)=r+\Delta_1\ \eta F(i)=(\Delta_1\ \eta)^2 r_1\alpha_e^2/2$$
$$b(i)=-3r-2\ \Delta_1\ \eta F(i)+(\Delta_1\ \eta)^3F2(i)$$
$$c(i)=3r+\Delta_1\ \eta F(i)+(\Delta_1\ \eta)^2 r_1\alpha_e^2/2$$
$$d(i)=\Delta_1\ \eta^3[F2(i)F(i)-2\ \Delta\ g_1(i)-\Omega\eta\theta+r_1\beta_e h(i)$$
$$a_1(i)=r+\frac{\Delta_1\ \eta f(i)}{2}$$
$$b_1(i)=-2r-(\Delta_1\ \eta)^2 r\alpha_e$$
$$c_1(i)=r-\frac{\Delta_1\ \eta f(i)}{2}$$
$$d_1(i)=-r_1(\Delta_1\ \eta)^2\beta_e F_1(i)$$
$$a_3(i)=2+\frac{\Delta_1\ \eta P_r f(i)}{2}$$
$$b_3(i)=-4+(\Delta_1\ \eta)^2 P_r F_1(i)$$
$$c_3(i)=2-\frac{\Delta_1\ \eta P_r f(i)}{2}$$
$$d_3(i)=-2(\Delta_1\ \eta)^2 P_r E_c(F_2(i))^2$$

The transformed boundary conditions are given by:

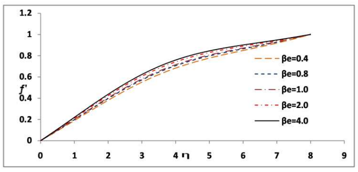
$$f(0)=0,\ f'(0)=0,\ h(0)=0,\ g(0)=0,\ \theta(0)=1$$
$$f'(\infty)=1,\ h(\infty)=0,\ g(\infty)=0,\ \theta(\infty)=1 \tag{20}$$

3. Numerical results and discussion

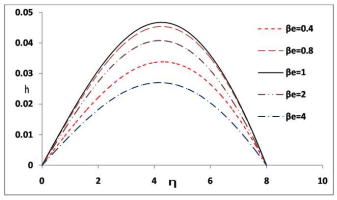
Equations (12)(15) are non-linear coupled ordinary differential equations first, we have linearsied by using the quasilinearisation technique. The Equations (16)-(19) with the boundary conditions (16) have been solved by using the finite difference method. A stem size of $\Delta_1\ \eta = 0.01$ was selected. The results obtained for steady flow are displayed in tables 1-5 and Figures 1(a)-9(b) for $Pr = 0.7$, $\lambda = 0.5$ and $B = 0.01$ at different values of the Hall parameter β_e, the ion-slip parameter β_i, the material parameter Δ, the magnetic field parameter M and the buoyancy parameter Ω, Eckert number E_c.

Results

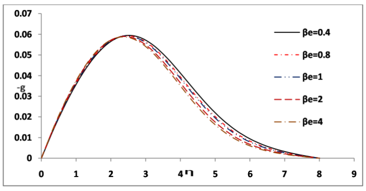
The results of numerical computations are displayed in Figures 1-6 for the velocity profiles f'', h, g and temperature profiles θ for different flow parameters β_e, β_i, Δ, M, Ω and E_c. It is seen from Figure 1(a) that the variation of distribution of velocity profile f' increases with increasing hall parameter β_e. In Figure 1(b) shows that the induced flow in the η-direction begins to develop as β_e increases in the interval $0 \leqslant \beta_e \leqslant 1$ and decreases for $\beta_e > 1$. The angular velocity profiles $g(\eta)$ are shown in Figure 1(c) for different values of β_e. It is observed that the angular velocity $g(\eta)$ decreases with the increase of hall parameter β_e. The effect of β_e on temperature profiles is less. It is seen that with the increase of β_e the temperature profiles increases near the plate and the reverse phenomena is observed far away from the plate.



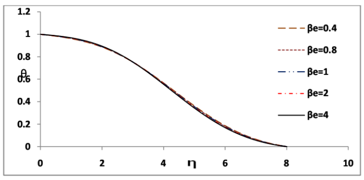
(a) Velocity profiles along the plate



(b) Velocity profiles across the plate



(c) Angular velocity profiles at the plate



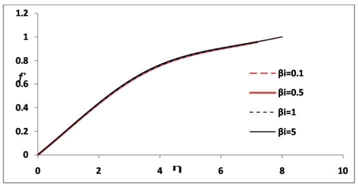
(d) Temperature profiles

Figure 1. $\beta_i = 0.4$, $\Delta = 4.5$, $M = 0.3$, and $\Omega = 0.5$.

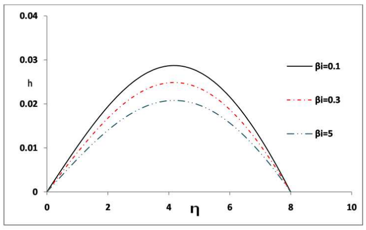
Figure 2 represents the effect of ion slip β_i on f' , h , g and temperature profiles θ. It can be seen that the velocity f' decreases with the increase of ion-slip parameter β_i. The effect of β_i is to reduces the velocity profiles across the plate h is decreases. From the figures it is observed that the effect of β_i is megers on g and temperature θ.

Figure 3 displays the effect of material parameter Δ on f' , h, g and θ. It is noticed from the figures that the magnitude of the velocity of micropolar fluid are smaller than that of Newtonian fluids as the material parameter Δ increases the f' and h decreases. It is also clear from these figures that the angular velocity g and temperature θ increases with the increase of Δ. The magnitude of the maximum values of the angular velocity increase and inflection point for the angular velocity distribution moves further away from the surface.

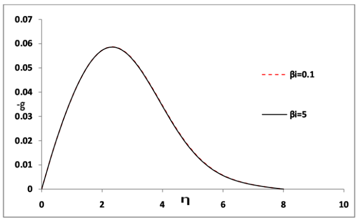
The magnetic field parameter M effect are shown in Figures 4(a)-4(d). It is clear from these figures that effect of magnetic field is to decelerates the f', where as the velocity profiles across the plate h increases . Since, the magnetic field excerts a retarding force on the free convection flow and because of an accelerating force, which acts in a direction parallel to the x-axis when hall an ion-slip current absent with the effect of magnetic field parameter the angular velocity profiles decreases



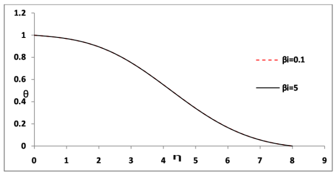
(a) Velocity profiles along the plate



(b) Velocity profiles across the plate

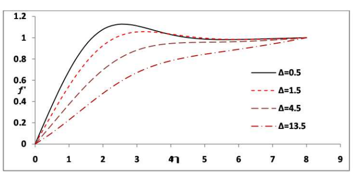


(c) Angular velocity profiles at the plate

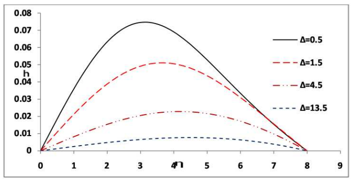


(d) Temperature profiles

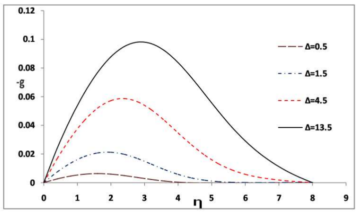
Figure 2. $\beta_i = 5$, $\Delta = 4.5$, $M = 0.3$, and $\Omega = 0.5$.



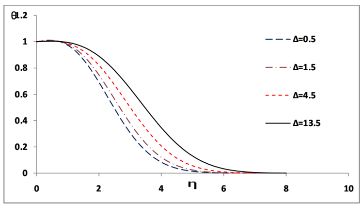
(a) Velocity profiles along the plate



(b) Velocity profiles across the plate



(c) Angular velocity profiles at the plate

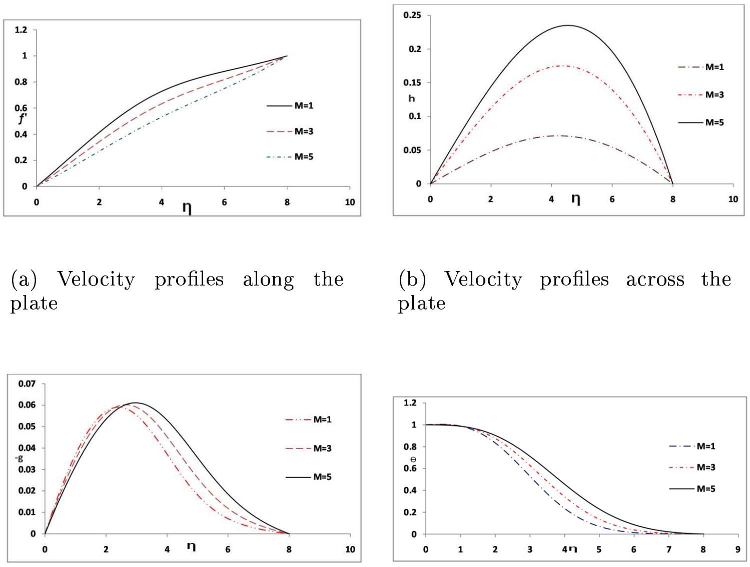


(d) Temperature profiles

Figure 3. $\beta_e = 5$, $\beta_i = 0.4$, $M = 0.3$, and $\Omega = 0.5$.

near the boundary layer and the reverse phenomena is observed far away from the plate .The temperature profiles increases with the increase of magnetic parameter M is observed from the Figure 4(d).

Figure 5 depicts the effect of buyoncy parameter Ω on f', h, g and θ. The velocity distribution f' is increases with the increase of Ω is observed from the Figure 5(a).



(a) Velocity profiles along the plate

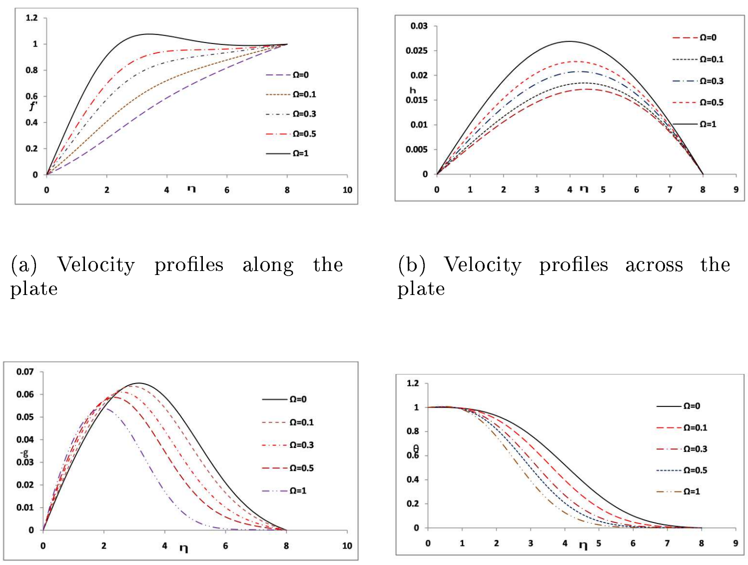
(b) Velocity profiles across the plate

(c) Angular velocity profiles at the plate

(d) Temperature profiles

Figure 4. $\beta_e = 5$, $\beta_e = 5$, $\beta_i = 0.4$, $\Delta = 4.5$, and $\Omega = 0.5$.

It is noticed that the Figure 5(b) the velocity profile across the plate h is increases with the increase of Ω. The effect of buoyancy parameter Ω is to increases the g profiles near the plate where as it decreases far away from the plate. It is noticed from Figure 5(d) the temperature profiles decreases with the increase of Ω.



(a) Velocity profiles along the plate

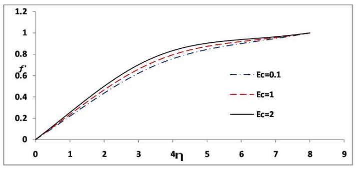
(b) Velocity profiles across the plate

(c) Angular velocity profiles at the plate

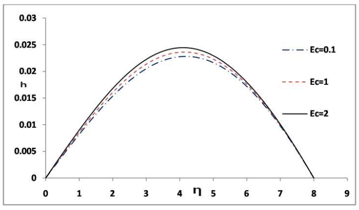
(d) Temperature profiles

Figure 5. $\beta_e = 5$, $\beta_i = 0.4$, $\Delta = 4.5$, and $M = 0.3$.

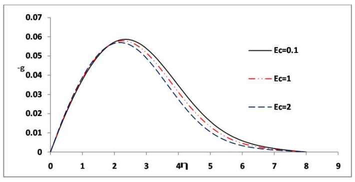
The effect of viscous dissipation on f', h, g and θ are shown in Figure 6. The



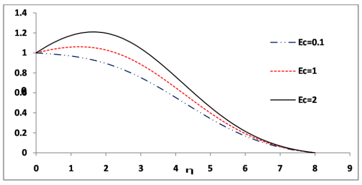
(a) Velocity profiles along the plate



(b) Velocity profiles across the plate



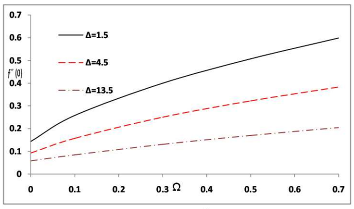
(c) Angular velocity profiles at the plate



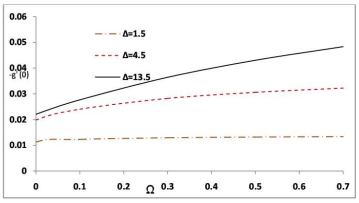
(d) Temperature profiles

Figure 6. $\beta_e = 5$, $\beta_i = 0.4$, $\triangle = 4.5$, $M = 0.3$, and $\Omega = 0.5$

velocity profiles f' , h increases with the increase of Ec is observed. The angular velocity profiles g decreases with the increase of Ec far away from the boundary layer is observed, where as the effect of Ec near the boundary layer is very small. It can be seen that from Figure 6(d) the temperature profiles increases with the increase of Ec.



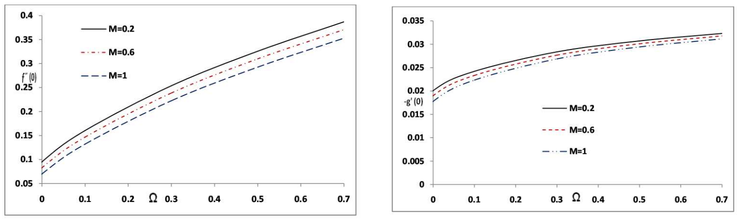
(a) Dimensionless wall shear stress $f''(0)$ as a function of the buoyancy parameter Ω for various material parameter $\triangle$



(b) Dimensionless wall couple stress $g(0)$ as a function of the buoyancy parameter Ω for various material parameter $\triangle$

Figure 7. $\beta_e = 5, \beta_i = 0.4$ and $M = 0.3$.

Results for the dimensional less local wall shear stress $f''(0)$, dimensional less wall coupled stress $g(0)$, the $h(0)$, and $\theta'(0)$ values are presented in table 1-5. For different values of β_e, β_i, Ω, M, and Ec. From the table we observe that the presence of micro elements is found to be an important application in the control of flow separation. With the increase of $f''(0)$, $g(0)$ increases. The $h(0)$ value increases for $0 \leqslant \beta_e \leqslant 1$ where as it decreases $\beta > 1$. The $\theta(0)$ value decreases with the increase of β_e. The $f''(0)$, $g(0)$ increases with the increase of β_i for $\beta_i < 1$ where as it increases for $\beta_i > 1$. The $h(0)$, $\theta(0)$ values decreases with the increase of β_i. the increasing in the value of $\triangle$ the $f''(0)$, $h(0)$ values decreases where as $g(0)$ and $\theta(0)$ values are increasing . From table 4 it is observed that due to the effect of M the

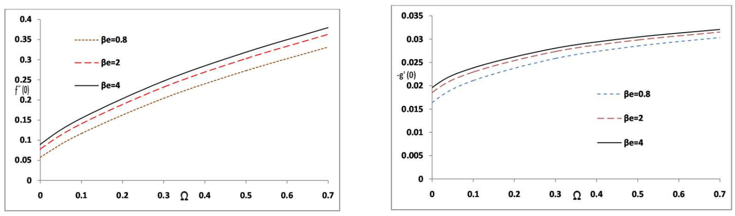


(a) Dimensionless wall shear stress $f''(0)$ as a function of the buoyancy parameter Ω for various magnetic parameter M

(b) Dimensionless wall couple stress $g(0)$ as a function of the buoyancy parameter Ω for various magnetic parameter M

Figure 8. $\beta_e = 5, \beta_i = 0.4$ and $\Delta = 0.3$.

$f''(0)$, $g(0)$ are decreases where as $h(0)$ and $\theta(0)$ values increases. As the increase the value of Ω from 0 to 1 the value of $f''(0)$, $h(0)$, $g(0)$ are increases . where as $\theta(0)$ value decreases.



(a) Dimensionless wall shear stress $f''(0)$ as a function of the buoyancy parameter Ω for various Hall parameter $\beta_e(\beta_i = 0.4, \Delta = 4.5$ and $M = 0.3)$.

(b) Dimensionless wall couple stress $g(0)$ as a function of the buoyancy parameter Ω for various magnetic parameter $M(\beta_e = 5, \beta_i = 0.4$ and $\Delta = 4.5)$.

Figure 9. Dimensionless wall stress

Table 1. Results of $f''(0)$, $h'(0)$, $-g'(0)$, and $-\theta'(0)$ with β_e ($\beta_i = 0.4$, $\Delta = 4.5$, $M = 0.3$, $\Omega = 0.5$).

β_e	$f''(0)$	$h'(0)$	$-g'(0)$	$-\theta'(0)$
0.4	0.255638	0.025561	0.055836	0.003458
0.8	0.272581	0.036175	0.057566	0.002685
1	0.27977	0.038032	0.058267	0.002411
2	0.302616	0.035302	0.060373	0.001709
4	0.318539	0.024302	0.061744	0.001345

Table 2. Results of $f''(0)$, $h'(0)$, $-g'(0)$, and $-\theta'(0)$ with β_i ($\beta_e = 5$, Δ= 4.5, $M = 0.3$, $\Omega = 0.5$).

β_i	$f''(0)$	$h'(0)$	$-g'(0)$	$-\theta'(0)$
0.1	0.325048	0.026204	0.062293	0.001318
0.5	0.321274	0.018765	0.061971	0.001304
1	0.321219	0.01144	0.061962	0.001307
3	0.326151	0.002544	0.062372	0.001203
5	0.328441	0.001031	0.062561	0.001161

Table 3. Results of $f''(0)$, $h'(0)$, $-g'(0)$, and $-\theta'(0)$ with Δ ($\beta_e = 5$, $\beta_i = 0.4$, $M = 0.3$, $\Omega = 0.5$).

Δ	$f''(0)$	$h'(0)$	$-g'(0)$	$-\theta'(0)$
0.5	0.659937	0.064326	0.009287	0.000180
1.5	0.506901	0.042869	0.026408	0.000201
4.5	0.321785	0.02065	0.062015	0.001283
13.5	0.169781	0.007097	0.092381	0.003510

Table 4. Results of $f''(0)$, $h'(0)$, $-g'(0)$, and $-\theta'(0)$ with M ($\beta_e = 5$, $\beta_i = 0.4$, Δ= 4.5, $\Omega = 0.5$).

M	$f''(0)$	$h'(0)$	$-g'(0)$	$-\theta'(0)$
1	0.292597	0.060383	0.05952	0.001050
3	0.204431	0.118104	0.050565	0.005504
5	0.136128	0.0125393	0.041334	0.016097

Table 5. Results of $f''(0)$, $h'(0)$, $-g'(0)$, and $-\theta'(0)$ with Ω ($\beta_e = 5$, $\beta_i = 0.4$, Δ= 4.5, $M = 0.3$).

Ω	$f''(0)$	$h'(0)$	$-g'(0)$	$-\theta'(0)$
0	0.092296	0.011802	0.043044	0.011869
0.1	0.157264	0.014695	0.050241	0.006987
0.3	0.250101	0.018025	0.057657	0.001676
0.5	0.321785	0.02015	0.062015	0.001283
1	0.463862	0.023631	0.068427	0.000293

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