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Using Fuzzy Interest Rates for Uncertainty Modelling in Enhanced Annuities Pricing

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Abstract. The modeling of uncertainty resources is very important in insurance pricing. In this paper, fuzzy set theory is implemented to model interest rates as an uncertainty resource for calculating the price of enhanced annuities. In this regard, the single fuzzy premium for a fixed annuity payouts is calculated using adjusted mortality probabilities for an insured with health problems and the results are compared with standard status. As the adjustment multiplier increases, which means that the health problems of the insured are worse, the life expectancy of the person decreases. In addition, as adjustment multiplier increases, the insurance premium decreases, which is due to the adjustment of survival and mortality probabilities based on the individual's health status. Also, to show the validity of the proposed fuzzy method, the random interest rate has been used. The results of the fuzzy and random models are close to each other which indicates the validation of proposed method.

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Index to information contained in this paper

1. Introduction

Traditionally, the same price has been considered for all insureds of the same age. In other words, the health status of all the insured has been assumed standard. This is despite the fact that the insured may have some health problems and, as a result, have a lower life expectancy than the standard health status. Therefore, under the assumption of paying the same annuities by insurance company compared to the

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standard status, lower premium should be paid. This is considered a sales advantage for annuity products, and recently researchers have focused on the pricing problem of these types of products, which are called enhanced or substandard annuities in the literature (see e.g. [7–9, 12, 17]). In order to issue an enhanced annuity policy, a person's health status must be examined by a medical underwriter and the survival and mortality probabilities for this person should be adjusted based on the medical underwriter's opinion (see e.g. [3, 4, 10, 13]).

In addition, uncertainty factors such as interest rate, future lifetime and insured health factors are important issues in the pricing of life insurance products, which are recently modeled using fuzzy random variables. The future lifetime has been considered as a fuzzy random variable in [18]. The present value of life insurance liabilities using the fuzzy random variable of interest rate is calculated in [14]. The life insurance and annuity evaluation framework presented in [11, 16] has been developed using symmetric triangular fuzzy numbers in [15] to price life insurance liabilities. Fuzzy random variables are used in [20] to estimate the discount function related to the generalized interest rate and future lifetime to calculate the life insurance premium. The Fuzzy pricing of substandard annuities that are issued based on medical underwriting is considered in [17]. It has proposed two approaches based on Fuzzy Set Theory tools to address imprecise or vague information about the insured health status and lifestyle. For example, the information such as high blood pressure or overweight is hard to define precisely. Therefore, fuzzy variables can be very suitable to model this kind of information.

Interest rates fluctuate over time due to changes in monetary, fiscal and foreign exchange policies, and this fluctuation can affect the determination of premiums, reserves and liabilities of insurance companies. This is especially important for life insurance products due to their long-term nature and the possibility of higher interest rate fluctuations during the policy period. Therefore, it is necessary to consider the interest rate risk and uncertainty in life insurance products by the regulator and insurance companies. In this regard, using the fuzzy interest rate, the permitted range for the interest rate can be considered that insurance companies, taking into account their risk aversion, to determine the amount of premium in the desired period. The pricing of insurance products with a fuzzy interest rate is considered in [1, 14, 16].

In this article, we implement a fuzzy interest rate for pricing enhanced annuities. According to our latest information from the research works, it is believed that the pricing of this kind of annuity products based on proposed fuzzy variables has been investigated for the first time in this paper. The difference between this article and researches [11, 14–16] is that in these researches, the annuity pricing for people with standard health status have been calculated, while in this article, the pricing of enhanced annuity products for people with health problems is considered and based on the insured health status, the mortality probabilities have also been adjusted.

The remainder of this article is organized as follows.The definitions and basic concepts such as the single premium of annuity, the adjustment multiplier for the survival and mortality probabilities and life expectancy based on health status of insured are described in section 2. Fuzzy pricing approaches for standard and adjusted premiums are proposed in section 3. Numerical test results for pricing enhanced annuities using fuzzy interest rates are given in section 4. Finally, our conclusions are given in section 5.

2. Basic concepts

In this section, we provide the definitions and basic concepts. Annuity contracts offer a regular series of payments. Life annuity is an annuity that depends on the survival of the recipient. The individual that receives annuity is called an annuitant. Here, we consider a deferred m year life annuity due of 1 monetary unit with payments within n years for an insured aged x which is a term life annuity. For the whole life annuities, $n = \varpi - m + 1$ where ϖ is the maximum attainable age in the considered mortality table. The present value of 1 m.u. payments to an annuitant for this policy is ${}_{m|n}\ddot{a}_x$ and it is defined with the following values and their probabilities based on [16], where ${}_{m+r-1|}q_x$ is the probability that the insured aged x dies the rth year and ${}_{m+n-1}p_x$ is the probability that the insured survives $m + n - 1$ years.

${}_{m|n}\ddot{a}_x$

Outcomes	Probabilities	
0	${}_{m}q_x$	
$\sum_{t=m}^{m+r-1} \tilde{d}_t$	${}_{m+r-1\|}q_x$	$r = 1, \cdots, n-1$
$\sum_{t=m}^{m+n-1} \tilde{d}_t$	${}_{m+n-1}p_x$	

Based on [1, 5, 14], the single premium (SP) of life annuity is the expectation of ${}_{m|n}\ddot{a}_x$ and is calculated as follows:

$$SP = E({}_{m|n}\ddot{a}_x) = \sum_{s=m}^{m+n-2} \sum_{t=m}^{s} d_t {}_{s|}q_x + \sum_{t=m}^{m+n-1} d_t {}_{m+n-1}p_x = \sum_{t=m}^{m+n-1} d_t {}_t p_x.$$

In calculating the conventional premium of annuity products, it is assumed that the insured has a standard health status according to his age. If a person has health problems, the mortality and survival probabilities should be calculated using the adjustment multiplier, according to his health condition. The more health problems a person has, the higher the adjustment multiplier. Consider that the adjustment multiplier $\beta = 1$ indicates the standard state.
Calculating the adjustment multiplier is discussed in a few researches. In [13], the adjustment multiplier is calculated as follows:

$$\beta = 1 + \sum_{j=1}^{m} \rho_j,$$

where coefficient ρ_j supposes as a measure of negative factors for the lifestyle of the insured or positive factors based on the medical diagnosis which effected the insured mortality probabilities as explained in [6, 13, 21]. Therefore, the adjusted mortality probability q_x^* which is considered as a linear function of the standard probability q_x is calculated as follows:

$$q_x^* = \beta q_x = (1 + \sum_{j=1}^{m} \rho_j) q_x. \tag{1}$$

Since $0 \leqslant q_x^* \leqslant 1$, the following inequality should be satisfied:

$$-1 < \sum_{j=1}^{m} \rho_j < \frac{1}{q_x} - 1.$$

For this aim, Xu in [21] proposes the following formula:

$$q_{x+t}^* = min\{1, (1 + \sum_{j=1}^{m} \rho_j) q_x\}, t = 1, 2, \cdots, \omega - x.$$

The adjusted probability that the insured of age x survives k years can be calculated as follows using the mortality probabilities q_{x+t}^* , $t = 1, 2, , \omega - x$:

$$_k p_x^* = \prod_{t=0}^{k-1} (1 - q_{x+t}^*), \tag{2}$$

and

$$_{k|} q_x^* =_{k-1} p_x^* \times q_{x+k-1}^*. \tag{3}$$

Therefore, the life expectancy of the insured is:

$$e_x^* = \sum_{k=1}^{\omega - x} {}_k p_x^* = \sum_{k=1}^{\omega - x} \prod_{t=0}^{k-1} (1 - q_{x+t}^*).$$

3. Fuzzy pricing approach

In the real world, uncertainty can arise due to randomness, ambiguity, and inaccuracy. Due to the long-term nature of life annuity contracts, the interest rate is one of the sources of uncertainty in the pricing of these products. Due to the extreme fluctuations of the economic components, it is not possible to determine the interest rate exactly in practice. Using fuzzy numbers for interest rates makes it possible to have a range including all possible values for annuity premium instead of considering a crisp premium. In this paper, we calculate the single net premiums of the enhanced annuities using fuzzy interest rates presented in [1, 14] Assume that the interest rate is a fuzzy number with the α cuts, $r_\alpha = [\underline{r}_\alpha, \overline{r}_\alpha]$. Therefore, the fuzzy discount rate $\tilde{d}_t$ is defined as $\tilde{d}_{t_\alpha} = [\underline{d_t}_\alpha, \overline{d_t}_\alpha]$ for 1 monetary unit payable in t years, with α cuts for each $\alpha \in [0, 1]$. Since, the discount rate is a decreasing function of interest rate, it is obvious that:

$$\tilde{d}_{t_\alpha} = [\underline{d_t}_\alpha, \overline{d_t}_\alpha] = [(1 + \overline{r}_\alpha)^{-t}, (1 + \underline{r}_\alpha)^{-t}]. \tag{4}$$

The fuzzy random variable of present value of 1 m.u. payments to an annuitant for this policy is $_{m|n}\tilde{\ddot{a}}_x^*$ and is defined with the following values and their probabilities: where the mortality and survival probabilities $_{m+r-1|}q_x^*$ and $_{m+r-1}p_x^*$ are calculated using Equations (2) and (3) for insured with health problems[1]. So $\forall \alpha \in [0, 1]$, we have:

[1]The symbol * means the calculations are related to the insured with health problems instead of insured with standard health condition

${}_{m|n}\tilde{\ddot{a}}^*_x$

Outcomes	Probabilities
0	${}_{m}q^*_x$
$\sum_{t=m}^{m+r-1} \tilde{d}_t$	${}_{m+r-1\vert}q^*_x$
$\sum_{t=m}^{m+n-1} \tilde{d}_t$	${}_{m+n-1}p^*_x$

$r = 1, \cdots, n-1$

$\underline{{}_{m|n}\ddot{a}^*_x}_\alpha$

Outcomes	Probabilities
0	${}_{m}q_x$
$\sum_{t=m}^{m+r-1} \underline{d_r}_\alpha$	${}_{m+r-1\vert}q^*_x$
$\sum_{t=m}^{m+n-1} \underline{d_n}_\alpha$	${}_{m+n-1}p^*_x$

$\overline{{}_{m|n}\ddot{a}^*_x}_\alpha$

Outcomes	Probabilities
0	${}_{m}q^*_x$
$\sum_{t=m}^{m+r-1} \overline{d_r}_\alpha$	${}_{m+r-1\vert}q^*_x$
$\sum_{t=m}^{m+n-1} \overline{d_n}_\alpha$	${}_{m+n-1}p^*_x$

The fuzzy single premium of enhanced annuity is:

$$SP^* = E({}_{m|n}\tilde{\ddot{a}}^*_x)_\alpha = [\underline{E({}_{m|n}\tilde{\ddot{a}}^*_x)}_\alpha, \overline{E({}_{m|n}\tilde{\ddot{a}}^*_x)}_\alpha], \tag{5}$$

where the expectation is calculated using the mentioned outcomes and probabilities. Considering that all the components are positive, based on the monotone convergence theorem (see [19]), one can show that the expectation indeed satisfies countable additivity for non-negative random variables. Therefore, we have:

$$\underline{E({}_{m|n}\tilde{\ddot{a}}^*_x)}_\alpha = E(\underline{{}_{m|n}\ddot{a}^*_x}_\alpha) = \sum_{s=m}^{m+n-2} \sum_{t=m}^{s} \underline{d_t}_\alpha {}_{s|}q^*_x + \sum_{t=m}^{m+n-1} \underline{d_t}_\alpha {}_{m+n-1}p^*_x = \sum_{t=m}^{m+n-1} \underline{d_t}_\alpha {}_{t}p^*_x,$$

$$\overline{E({}_{m|n}\tilde{\ddot{a}}^*_x)}_\alpha = E(\overline{{}_{m|n}\ddot{a}^*_x}_\alpha) = \sum_{s=m}^{m+n-2} \sum_{t=m}^{s} \overline{d_t}_\alpha {}_{s|}q^*_x + \sum_{t=m}^{m+n-1} \overline{d_t}_\alpha {}_{m+n-1}p^*_x = \sum_{t=m}^{m+n-1} \overline{d_t}_\alpha {}_{t}p^*_x.$$

4. Numerical test results

In this section, the numerical test results will be analyzed for different insurance policies. Insurance products in Iran insurance industry are planned based on Regulation No. 68 and its amendments. Therefore, based on this regulation, the following specific criteria are set to price enhanced annuity products. we use Iranian life table and the technical interest rate is set on the maximum amount which is 16 percent for the first 2 years of the policy, 13 percent for the next 2 years and 10 percent for the periods more than 4 years. So, for each $\alpha \in [0, 1]$, the fuzzy interest rate is considered as follows:

$$\begin{aligned} i_{\alpha_1} &= [0.15 + 0.01\alpha, 0.17 - 0.01\alpha], \quad t \leqslant 2; \\ i_{\alpha_2} &= [0.115 + 0.015\alpha, 0.145 - 0.015\alpha], \quad 2 < t \leqslant 4; \\ i_{\alpha_3} &= [0.085 + 0.015\alpha, 0.115 - 0.015\alpha], \quad t > 4. \end{aligned}$$

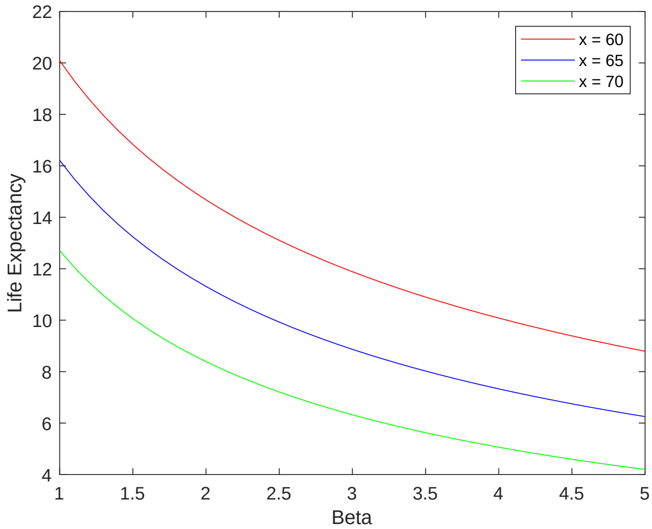

Figure 1. Example of a two-part figure with individual sub-captions showing that all lines of figure captions range left.

Therefore, the fuzzy discount rate specified in Equation (4) is calculated as follows:

$$\underline{d_r}_\alpha = \begin{cases} (1.17 - 0.01\alpha)^{-t}, & \text{if } t \leqslant 2; \\ (1.17 - 0.01\alpha)^{-2}(1.145 - 0.015\alpha)^{-(t-2)}, & \text{if } 2 < t \leqslant 4; \\ (1.17 - 0.01\alpha)^{-2}(1.145 - 0.015\alpha)^{-2}(1.115 - 0.015\alpha)^{-(t-4)}, & \text{if } t > 4, \end{cases}$$

$$\overline{d_r}_\alpha = \begin{cases} (1.15 + 0.01\alpha)^{-t}, & \text{if } t \leqslant 2; \\ (1.15 + 0.01\alpha)^{-2}(1.115 + 0.015\alpha)^{-(t-2)}, & \text{if } 2 < t \leqslant 4; \\ (1.15 + 0.01\alpha)^{-2}(1.115 + 0.015\alpha)^{-2}(1.085 + 0.015\alpha)^{-(t-4)}, & \text{if } t > 4. \end{cases}$$

Furthermore, the results have been analyzed for an insured with standard health conditions and an insured who has cancer. The standard mortality probabilities and the adjusted mortality probabilities are calculated using Iranian life table. Suppose a 65-year-old person is going to buy a deferred 3 years life annuity due of 1 m.u. with payments within 10 years.

It is expected that the life expectancy will decrease with the increase of the β coefficient, which is caused by the worse health condition of the person compared to the standard condition. This issue is shown in Figure 1 for 60, 65 and 70 year olds. It is obvious that $\beta = 1$ means the standard state. Therefore, it is possible to compare standard and adjusted premiums in Figure 1.

Also, the effect of increasing the beta coefficient on the single premium of the annuity product has been investigated assuming that the annuity payout is fixed 1 monetary unit. As it is shown in Figure 2, with the increase of the beta coefficient, according to the Equation (2), the probability of survival decreases and therefore the single premium calculated using the Equation (5) will decrease.

In order to show validity of the proposed fuzzy method, the numerical results are compared with the random methods. It means that we compared the research findings with the random interest rate with normal distribution used in [5]. In this article, using the interest rates announced in the supplement of Regulation No. 68,

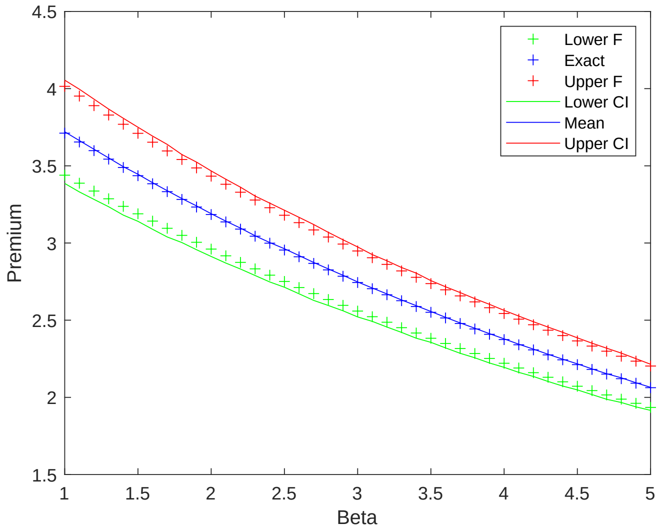

Figure 2. Example of a two-part figure with individual sub-captions showing that all lines of figure captions range left.

we used the normal distribution with the following characteristics:

$$r_1 \sim N(0.16, 0.010), \quad t \leqslant 2;$$
$$r_2 \sim N(0.13, 0.015), \quad 2 < t \leqslant 4;$$
$$r_3 \sim N(0.10, 0.015), \quad t > 4.$$

After simulating the technical interest rate for 5000 samples, we obtained a 95% confidence interval for the single premium and we reflect its results in Figure 2 to compare with the fuzzy method. The comparison of the two methods shows that the fuzzy single premium requires fewer assumptions than the random method, and its calculations are simpler due to the absence of the need to generate random numbers. Also, the comparison of the results of the fuzzy method with the random method in Figure 2 indicates the validity of the findings of the article.

Now, the single premium for a person with standard health condition is compared to a person with health problems. To do this, the standard mortality and survival probabilities are calculated using the standard Iranian life table and the standard fuzzy premiums are shown in Table 1.

Also, we consider for example a person who has regional colon cancer, which, according to the American Cancer Society, has a 5-year relative survival rate of 72%, [2]. It means that the survival probability of this person is 72% of what is calculated based on the standard life table. Our findings can be generalized for other cancers or diseases. Therefore, based on Iranian life table ${}_5P_{65} = 0.9040$, the factor (ρ) is obtained by solving $\prod_{t=0}^{4}[1-(1+\rho)q_{65+t}] = 0.72 \times 0.9040$. So, $\rho = 3.12$.
Therefore, the estimation of multiplier β in Equation (1) will be equal to $3.12+1 = 4.12$. In this way, the multiplier β, adjusted life expectancies and α-cuts of single premium are calculated for localized colon cancer with a 5-year relative survival rate of 90% for ages 60 and 65 and the results are shown in Table 2. The life expectancy for a 60-year-old insured with standard health status is 20.09, while it is 12.58 and 7.65 for insureds with localized and regional colon cancers. Furthermore, all the possible values of the fuzzy premium (0-cuts) for standard status is $[3.65, 4.28]$

Table 1. Standard fuzzy premiums of annuities.

x	e_x	α	$\underline{SP}(\alpha)$	$\overline{SP}(\alpha)$
60	20.09	1	3.95	3.95
		0.9	3.92	3.98
		0.8	3.89	4.01
		0.7	3.86	4.04
		0.6	3.83	4.07
		0.5	3.80	4.11
		0.4	3.77	4.14
		0.3	3.74	4.17
		0.2	3.71	4.21
		0.1	3.68	4.24
		0	3.65	4.28
65	16.21	1	3.71	3.71
		0.9	3.68	3.74
		0.8	3.65	3.77
		0.7	3.63	3.80
		0.6	3.60	3.83
		0.5	3.57	3.86
		0.4	3.54	3.89
		0.3	3.52	3.92
		0.2	3.49	3.95
		0.1	3.47	3.98
		0	3.44	4.01

and for adjusted status with $\beta = 2.70$ for localized and regional colon cancer is $[3.12, 3.63]$ and with $\beta = 6.19$ for regional colon cancer is $[2.30, 2.64]$. Also, the fair annuity price in the most feasible scenario (1-cut) is 3.36 for localized cancer and 2.46 for regional cancer, while it is 3.95 for standard health status. The results indicates that as β increases, th premium decreases.

5. Conclusions

Considering the importance of annuity insurances as a supplement for financing the elderly and retirees and considering the change of the country's demographic pyramid towards aging, the need to provide new products in the field of annuity insurance is felt more than in the past. Enhanced annuity products that are provided for people with health problems, require a lower premium than the standard state for fixed payouts, which is a positive aspect and a competitive advantage for insurance companies providing these products.

For pricing of these products, the adjustment multiplier of survival and mortality probabilities should be determined based on the individual's health status. In addition, due to the long-term nature of these products, it is better to consider sources of uncertainty such as interest rates in the modeling. In this article, the fuzzy interest rate is used for the pricing of increased annuities and the results are compared in the standard status and in the adjusted status which indicates adjusted premiums are lower than standard premiums. Due to the fact that the fuzzy interest rate is considered, the fuzzy insurance premium is a range of possible states that the actuary can determine the appropriate premium amount from the target range with his deep understanding of the market. In this article, it was assumed that the insured has some type of cancer and the adjustment factor was obtained using the 5-year survival probability based on the reports published by the American Cancer Society in [2]. Meanwhile, no comprehensive information was found for the survival probability of insureds with cancer or other diseases in Iran. Also, considering that the lifestyle like being a smoker also affects the health status of a person, investigating the impact of this health risk factor on enhanced annuity

Table 2. Fuzzy premiums of enhanced annuities.

x	Cancer Stage	$_5P_x$	β	e_x^*	α	$\underline{SP}(\alpha)$	$\overline{SP}(\alpha)$
					1	3.36	3.36
					0.9	3.34	3.39
					0.8	3.31	3.42
					0.7	3.29	3.44
					0.6	3.26	3.47
60	Localized	0.90	2.70	12.58	0.5	3.24	3.49
					0.4	3.22	3.52
					0.3	3.19	3.55
					0.2	3.17	3.58
					0.1	3.14	3.60
					0	3.12	3.63
					1	2.46	2.46
					0.9	2.44	2.48
					0.8	2.43	2.50
					0.7	2.41	2.51
					0.6	2.39	2.53
60	Regional	0.72	6.19	7.65	0.5	2.38	2.55
					0.4	2.36	2.57
					0.3	2.35	2.58
					0.2	2.33	2.60
					0.1	2.32	2.62
					0	2.30	2.64
					1	3.17	3.17
					0.9	3.15	3.20
					0.8	3.13	3.22
					0.7	3.10	3.25
					0.6	3.08	3.27
65	Localized	0.90	2.02	11.24	0.5	3.06	3.29
					0.4	3.04	3.32
					0.3	3.02	3.34
					0.2	2.99	3.37
					0.1	2.97	3.40
					0	2.95	3.42
					1	2.34	2.34
					0.9	2.32	2.35
					0.8	2.30	2.37
					0.7	2.29	2.38
					0.6	2.27	2.40
65	Regional	0.72	4.12	7.19	0.5	2.26	2.42
					0.4	2.24	2.43
					0.3	2.23	2.45
					0.2	2.21	2.47
					0.1	2.20	2.48
					0	2.19	2.50

premium is one of the fields of future research which can be modelled using Fuzzy Set Theory tools.

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