International Journal of
Mathematical Modelling & Computations
Vol. 12, No. 02, Spring 2022, 131- 142

Analytical Solution of the Effect of Awareness Program by Media on the Spread of an Infectious Disease by Homotopy Perturbation Method

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Abstract. In this paper, the nonlinear dynamical system modeling the effect of awareness program by media on spread of infectious disease is considered. The model is mathematically formulated by the deterministic compartmental model consisting of susceptible population, infected population, aware population and cumulative density of awareness spread by the media. Homotopy perturbation method is used to obtain the approximate solution of the governing nonlinear differential equation, which consists in determining the series solution convergent to the exact solution or enabling to built the approximate solution of the problem. Numerical solutions are obtained and the results are discussed graphically using Maple. The method allows to determine the solution in form of the continuous function, and shows the significance of awareness program driven by media in spread of an infectious disease, but due to immigration, the disease may remain endemic. The simulation analysis of the model with different parameter values confirms the analytical results.

Received: 09 August 2021, Revised: 25 January 2022, Accepted: 05 February 2022.

Keywords: Nonlinear system of differential equation; Approximate solution; Homotopy Perturbation method; Numerical simulation.

AMS Subject Classification: 34B60, 34D10, 37M05, 74H10.

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1. Introduction

As global population increases, life expectancy rises, and living standards improves, causes of death across the world are changing. Infectious diseases are a leading

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cause of death worldwide, particularly in low income countries, especially in young children [7]. Illness caused by any infectious agents namely microbes or parasites or its toxic products that arises through transmission from an infected person, animal or reservoir to a susceptible host, either directly or indirectly through an intermediate plant or animal host, vector or inanimate environment. Some of the infectious disease are leptospirosis, malaria, measles, mumps, small pox, typhoid etc. To evaluate the control strategies of the spread of infectious diseases in the environment, mathematical modeling and data analysis are significant. The modeling can help in deciding which interventions to avoid and which to trial. The classical models governing the spread of infectious diseases depend mainly on the interactions between susceptible and infective [9]. However, there are other factors, such as media coverage, vaccination, migration of population etc., which also affect the spread of infectious diseases.

Recently, many authors have been contributed towards the understanding of the infectious disease models. S. Edward and N. Nyerere [6], formulated a mathematical model of dynamics of cholera transmission with public health educational campaigns, vaccination, sanitation and treatment as control strategies in limiting the disease. O.J. Peter et. al., [14], studied the dynamics of typhoid fever model and tested for the existence and uniqueness of solution for the model using the Lipchitz condition to determine both the disease free equilibrium and the endemic equilibrium for the system of the equation. Y. Li et.al., [11], designed and analyzed long - term mumps surveillance data in an SVEILR (susceptible - vaccinated - exposed - severely infectious - mildly infectious - recovered) dynamic transmission model with optimized parameter values to describe the dynamics of mumps infections in China and finally proposed to increase the vaccine coverage and make two doses of MMR (Measles, mumps and rubella) vaccines freely available in China in order to control the disease. A.K. Misra and Rajanish Kumar Rai [12], analysed the effect of awareness through radio and tv advertisement with the control of infectious disease and result shown that the augmentation in dissemination rate of awareness among susceptible individuals due to TV and radio ads may cause stability switches through Hopf-bifurcation and also the analytical findings are supported through numerical simulations.

Mathematical modeling of most of the biological problems are inherently nonlinear and difficult to find the exact solutions to understand the biological phenomena. Therefore, numerical methods are necessary to find the exact solution and approximate solution to these nonlinear problems. In the numerical methods, stability and convergence of the solution is inevitable to avoid divergence or inappropriate results. To overcome these drawbacks in finding the approximate solution to the nonlinear dynamical models, Homotopy perturbation method is one of the best technique. The Homotopy Perturbation Method (HPM) is an analytical method for solving linear /nonlinear differential equations. It is a powerful and efficient technique for finding solutions of nonlinear equations without linearizing the problem. The method was first introduced by He [8]. The HPM is a combination of the perturbation and Homotopy methods. When two continuous functions moves from one topological space to another and one deforms into another it is said to be homotopic. It uses the idea of the homotopy from topology to create a convergent series solution - a Maclaurin series which transform the non linearities in the system of differential equation. This method takes the advantages of the conventional perturbation method while eliminating its restrictions.

In general, this method has been successfully applied to solve many kinds of linear and nonlinear equations in science and engineering by many authors. F. A. Adesuyi et. al., [3], used HPM to solve cholera mathematical model incorporating three control strategy namely vaccination, therapetic treatment and water sanitation to control the spread of cholera epidemic over time. M. A. Khan et. al., [10], studied the analytic solution of leptospirosis model by HPM. Abubakar et.al.,[1], proposed a SIR model for general infectious disease dynamics and the analytic solution is found using HPM. R. Senthamarai and S. Balamuralitharan [15], studied the transmission of malaria disease modelled by SIRS - SI model with treatments given to humans and mosquitoes and using HPM numerical solution is achieved B. Ebenezer et. al., [5], used HPM to obtain the approximation solution of the Ebola mathematical model results shows that the HPM is very effective and accurate as few perturbation and obtained result on HPM is as good reliable as other known standard methods. G. Devipriya [4], considered a mathematical model of dengue virus transmission consists of human and mosquito compartments by incorporating with control strategy of imperfect treatment and delay in vector maturation whose analytical solution is obtained using HPM. G. Adambu et. al., [2], considered mathematical model of Zika virus with two control strategies namely treatment for human and insecticide spray for mosquito and used HPM to get approximate solution and shows that 59% effective administration of infectious spray proved a great reduction in the infected human as well as infected vector population.

Thus we can see application of homotopy perturbation method to obtain the solution for almost all the infectious disease. To best of my knowledge, HPM has not been applied to obtain the solution for effect of awareness programs by media on the spread of infectious disease. Thus an attempt is made in this paper. The Mathematical formulation of the dynamics of effect of awareness program by media on the spread of infectious disease is done in section 2. In section 3, the analytic solution is obtained with aid of HPM. In section 4, the numerical solution is found and results are interpreted graphically using Maple. The variation of susceptible, infected, aware population and the number of awareness program for different parameter are analyzed graphically.

2. Mathematical formulation

Let $N(t)$ be the total population at time t in a region under consideration. The total population is divided into three classes; the susceptible population $X(t)$, the infective population $Y(t)$ and the aware population $Z(t)$. Also, let $M(t)$ be the cumulative density of awareness program driven by the media in that region at time t. We make the following assumptions on the model. The disease spreads due to the direct contact rate between susceptible and infective only. The growth rate of density of awareness programs is proportional to the number of infective individuals. Also, due to the awareness program susceptible individuals form a different class and avoid contact with the infective. Under these assumptions, the governing equation of this non linear dynamics is modeled by the following system of non linear ordinary differential equations [13]:

$$
\begin{aligned}
\frac{dX(t)}{dt} &= A - \beta X(t)Y(t) - \lambda X(t)M(t) - dX(t) + \nu Y(t) + \Omega Z(t) \\
\frac{dY(t)}{dt} &= \beta X(t)Y(t) - \nu Y(t) - \alpha Y(t) - dY(t) \\
\frac{dZ(t)}{dt} &= \lambda X(t)M(t) - dZ(t) - \Omega Z(t) \\
\frac{dM(t)}{dt} &= \mu Y(t) - \sigma M(t) \qquad (1)
\end{aligned}
$$

with initial data
$X(0) \geqslant 0, Y(0) \geqslant 0, Z(0) \geqslant 0, M(0) \geqslant 0$.
where A denote the rate of immigration of susceptible, β represents the contact rate of susceptible with infective. The constant λ denotes the dissemination rate of awareness among susceptible. Let ν be the rate of recovery. Let α be the disease induced death rate and d be natural death rate of the population. The rate of transfer of aware individuals to susceptible class is given by Ω. The rate at which awareness program are being implemented is denoted by μ and σ denotes the depletion rate of these programs due to ineffectiveness, social problems in the population.

3. Analytical solution by homotopy perturbation method

Consider the following nonlinear differential equation

$$L(u) + N(u) = f(r), \quad r \in \Omega \tag{2}$$

with the boundary conditions:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma$$

where L is a linear operator, N is a nonlinear operator, B is a boundary operator, Γ is the boundary of the domain Ω and $f(r)$ is a known analytic function.
By the homotopy perturbation technique [8], construct a homotopy:

$$v(r,p) : \Omega \times [0,1] \to R \tag{3}$$

which satisfies:

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(r)] = 0 \tag{4}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{5}$$

where $r \in \Omega, p \in [0,1]$ is an imbedding parameter and u_0 is an initial approximation which satisfies the boundary conditions. Obviously, from (3.3) and (3.4), we have:

$$H(v,0) = L(v) - L(u_0) = 0 \tag{6}$$

$$H(v,1) = L(v) + N(v) - f(r) = 0 \tag{7}$$

The changing process of p from zero to unity is just of $v(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation, $L(v) - L(u_0)$ and $L(v) + N(v) - f(r)$ are called homotopic. The basic assumption is that the solution of equation (3.4) can be expressed as a power series in p :

$$v = v_0 + pv_1 + p^2 v_2 + \cdots \tag{8}$$

The approximate solution of (3.1) is given as,

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \tag{9}$$

which leads to rapid convergence.

Now, we apply the homotopy perturbation techniques to the model (1). Define the operator $\mathcal{L} = d/dt$. According to HPM, we construct a homotopy system as given below:

$$\begin{aligned}
\mathcal{L}X(t) - \mathcal{L}X^0(t) &= p(A - \beta X(t)Y(t) - \lambda X(t)M(t) - dX(t) + \nu Y(t) + \\
&\quad \Omega Z(t) - \mathcal{L}X^0(t)) \\
\mathcal{L}Y(t) - \mathcal{L}Y^0(t) &= p(\beta X(t)Y(t) - \nu Y(t) - \alpha Y(t) - dY(t) - \mathcal{L}Y^0(t)) \\
\mathcal{L}Z(t) - \mathcal{L}Z^0(t) &= p(\lambda X(t)M(t) - dZ(t) - \Omega Z(t) - \mathcal{L}Z^0(t)) \\
\mathcal{L}M(t) - \mathcal{L}M^0(t) &= p(\mu Y(t) - \sigma M(t) - \mathcal{L}M^0(t))
\end{aligned} \tag{10}$$

with initial data,
$X_0(t) = X^0(t) = X(0);$ $Y_0(t) = Y^0(t) = Y(0);$ $Z_0(t) = Z^0(t) = Z(0);$ $M_0(t) = M^0(t) = M(0).$

The approximate solution of the above homotopy (10) is a power series in $p \in [0, 1]$ and is given by

$$\begin{aligned}
X(t) &= X_0^*(t) + pX_1^*(t) + p^2 X_2^*(t) + \cdots \\
Y(t) &= Y_0^*(t) + pY_1^*(t) + p^2 Y_2^*(t) + \cdots \\
Z(t) &= Z_0^*(t) + pZ_1^*(t) + p^2 Z_2^*(t) + \cdots \\
M(t) &= M_0^*(t) + pM_1^*(t) + p^2 M_2^*(t) + \cdots
\end{aligned} \tag{11}$$

Making use of (11) in equation (10) and comparing the same coefficient of same power of p, we get
zeroth order system:

$$\begin{aligned}
\mathcal{L}X_0^*(t) - \mathcal{L}X^0(t) &= 0 \\
\mathcal{L}Y_0^*(t) - \mathcal{L}Y^0(t) &= 0 \\
\mathcal{L}Z_0^*(t) - \mathcal{L}Z^0(t) &= 0 \\
\mathcal{L}M_0^*(t) - \mathcal{L}M^0(t) &= 0
\end{aligned} \tag{12}$$

with initial conditions,
$X_0^*(0) = X(0);$ $Y_0^*(0) = Y(0);$ $Z_0^*(0) = Z(0);$ $M_0^*(0) = M(0);$
and

first order system:

$$\begin{aligned}\mathcal{L}X_1^*(t) &= A - \beta X_0^*(t)Y_0^*(t) - dX_0(t) + \gamma Y_0^*(t) - \lambda X_0^*(t)M_0^*(t)\\ &\quad +\Omega Z_0^*(t) - \mathcal{L}X^0(t) \qquad (13)\\ \mathcal{L}Y_1^*(t) &= \beta X_0^*(t)Y_0^*(t) - \gamma Y_0^*(t) - \alpha Y_0^*(t) - dY_0^*(t) - \mathcal{L}Y^0(t)\\ \mathcal{L}Z_1^*(t) &= \lambda X_0^*(t)M_0^*(t) - dZ_0^*(t) - \Omega Z_0^*(t) - \mathcal{L}Z^0(t)\\ \mathcal{L}M_1^*(t) &= \mu Y_0^*(t) - \sigma M_0^*(t) - \mathcal{L}M^0(t) \qquad (14)\end{aligned}$$

with the initial conditions
$X_1^*(0) = 0;\quad Y_1^*(0) = 0;\quad Z_1^*(0) = 0;\quad M_1^*(0) = 0;$
and
second order system:

$$\begin{aligned}\mathcal{L}X_2^*(t) &= -\beta X_0^*(t)Y_1^*(t) - \lambda X_1^*(t)M_0^*(t) - \beta X_1^*(t)Y_0^*(t) - X_0^*(t)M_1^*(t)\lambda - dX_1^*(t)\\ &\quad +\gamma Y_1^*(t) + \Omega Z_1^*(t)\\ \mathcal{L}Y_2^*(t) &= \beta X_0^*(t)Y_1^*(t) + \beta Y_0^*(t)Y_1^*(t) - \gamma Y_1^*(t) - \alpha Y_1^*(t) - dY_1^*(t)\\ \mathcal{L}Z_2^*(t) &= \lambda X_0^*(t)M_1^*(t) + \lambda M_0^*(t)X_1^*(t) - dZ_1^*(t) - \Omega Z_1^*(t)\\ \mathcal{L}M_2^*(t) &= \mu Y_1^*(t) - \sigma M_1^*(t) \qquad (15)\end{aligned}$$

with $X_2^*(0) = 0;\quad Y_2^*(0) = 0;\quad Z_2^*(0) = 0;\quad M_2^*(0) = 0.$
Solving the system of non - linear differential equation given in (12), (13) and (15) we will obtain the zeroth order solution, first order solution and second order solution and considering $p = 1$ in (11), we get

$$\begin{aligned}X(t) &= X_0^*(t) + X_1^*(t) + X_2^*(t) + \cdots\\ Y(t) &= Y_0^*(t) + Y_1^*(t) + Y_2^*(t) + \cdots\\ Z(t) &= Z_0^*(t) + Z_1^*(t) + Z_2^*(t) + \cdots\\ M(t) &= M_0^*(t) + M_1^*(t) + M_2^*(t) + \cdots \qquad (16)\end{aligned}$$

The convergence of HPM is rapid, for few iteration.

4. Numerical results and discussion

In this section, numerical results of the problem is discussed. The solution of the dynamical model is obtained by homotopy perturbation method and results are graphically interpreted with the aid of Maple. In order to obtain the analytical solution to the problem, let us take the values of the parameter involved in the model as the following [13]:
$A = 400,\quad \beta = 0.00002,\quad \lambda = 0.0002,\quad \Omega = 0.2,\quad \nu = 0.6,\quad \alpha = 0.02,$
$d = 0.01,\quad \mu = 0.0005,\quad \sigma = 0.06.$
Let us assume the initial condition for the problem as
$X(0) = 10000;\quad Y(0) = 1000;\quad Z(0) = 100;\quad M(0) = 5;$

Solving the system of non - linear differential equation in (12), (13) and (15) using Maple, we will obtain
Zeroth Order Solution:

$X_0(t) = 10000; \quad Y_0(t) = 1000; \quad Z_0(t) = 100; \quad M_0(t) = 5;$

First Order Solution:
$X_1(t) = 710t; \quad Y_1(t) = -430t; \quad Z_1(t) = -11t; \quad M_1(t) = \frac{1}{5};$

Second Order Solution:
$X_2(t) = -\frac{19661}{200}t^2; \quad Y_2(t) = \frac{1991}{20}t^2; \quad Z_2(t) = \frac{171}{100}t^2; \quad M_2(t) = -\frac{227}{2000}t^2;$

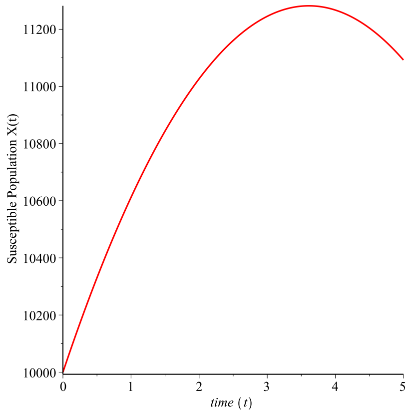

Figure 1. Variation of susceptibles with respect to time (t).

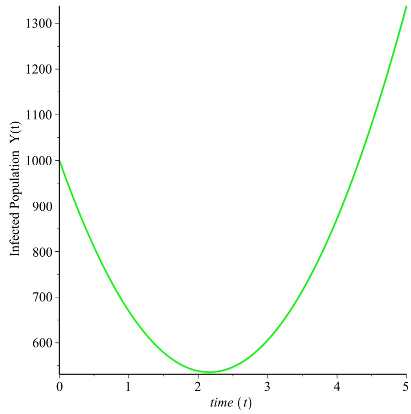

Figure 2. Variation of infected population with respect to time (t).

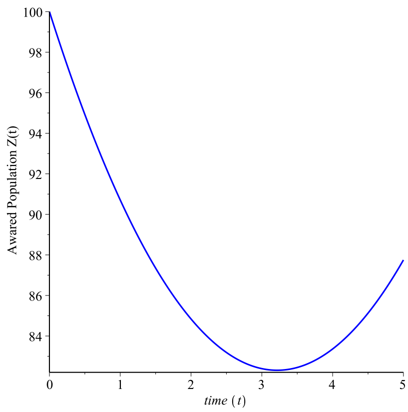

Figure 3. Variation of aware individuals with respect to time (t).

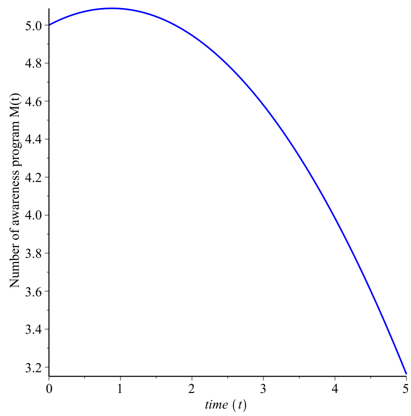

Figure 4. The number awareness program with respect to time (t).

The effect of susceptible population $X(t)$ with respect to time t for different values of rate of dissesmination of awareness among susceptible λ is shown in figure 5. It is noticed that when $\lambda = 0$ the increase in susceptible population is more compared to higher values of λ. As any awareness program for infectious disease are provided for stipulated period of time, the number of susceptible decreases as the preventive measure adopted by them due to awareness program. From the figure, it can be noted that as the rate of dissemination λ increases, susceptible population $X(t)$ decreases.

The effect of the susceptible population $X(t)$ with respect to time t for different values of rate of implementation of μ as shown in figure 6. When there is no implementation of awareness program for infectious disease, the number of susceptible is more compared to that of when awareness program is implemented through media. As the number of awareness programs reached the susceptible population, they undertake the preventive measure which in turn decreases the number of sus-

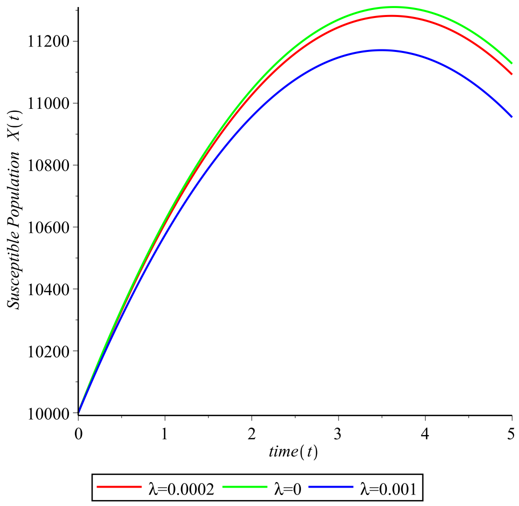

Figure 5. Variation of susceptible population with time for various values of λ.

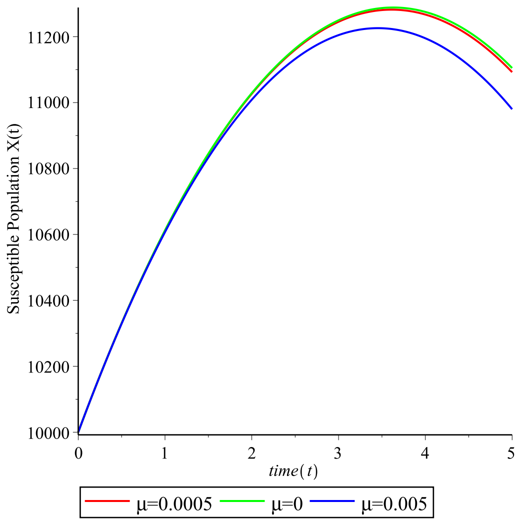

Figure 6. Variation of susceptible population with time for various values of μ.

ceptible individuals. From the figure, it is apparent that as rate of implementation of awareness programs μ increases the susceptible population $X(t)$ decreases

The effect of the infective population $Y(t)$ with respect to time t for different values of rate of dissemination λ is shown in figure 7. From the figure, we can see that change in rate of dissemination λ has no effect on infective population $Y(t)$ since they are infected on spreading awareness among them does not make any significant change.

The Effect of the infective population $Y(t)$ with respect to time t for various values of rate of implementation of awareness programs μ as shown in figure 8. From the figure, it can be noted that the change in rate of implemenation μ' does not change any infective population $Y(t)$ because they are already infected, on implementing awareness program does not make them recover without undergoing treatment.

The effect of the aware population $Z(t)$ with respect to time t for different values of rate of implementation of awareness programs μ as shown in figure 9. When there is no implementation of any awareness program for any infectious disease, it is common tendency that the preventive measure might not be carried out which

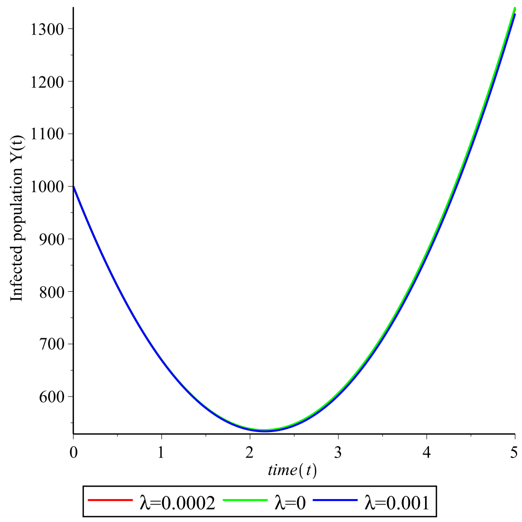

Figure 7. Variation of infective population with time for various values of λ.

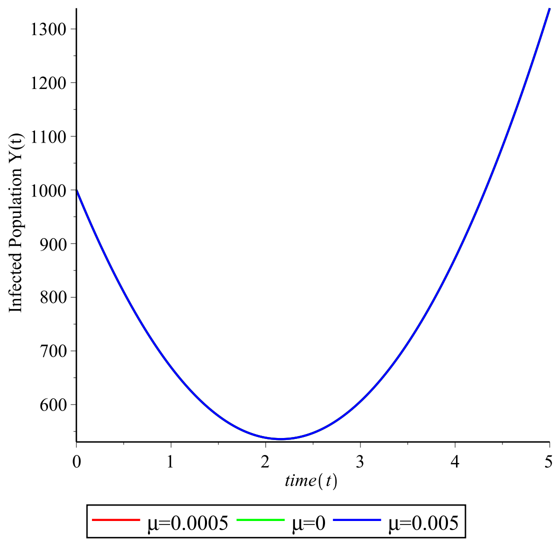

Figure 8. Variation of infective population with (t) for various values of μ.

makes the individual prone to be a susceptible. Thus there is decline in $Z(t)$ when $\mu = 0$. Further, we notice in the figure that the increase in the implementation of awareness program through media increases the aware population. From the figure, we can see that the change in rate of implementation μ increases, the aware population $Z(t)$ also increases.

The effect of the aware population $Z(t)$ with respect to time t for different values of rate of dissesmination of awareness among susceptibles λ is shown in figure 10. When the rate of dissesmination of awareness program is zero we observe a decline in the aware population, this is because they might not taken any preventive measures against the infectious disease. From the figure, it can be noted that the rate of dissemination λ is directly proportional to the aware population $Z(t)$.

The number of the awareness program M(t) with respect to time t for different values of implementation of awareness program μ as shown in figure 11. From the figure, we can see that as the rate of implementation of awareness population μ increases, the awareness program $M(t)$ also increases. As awareness program cannot be implemented forever, and hence over period of time number of awareness

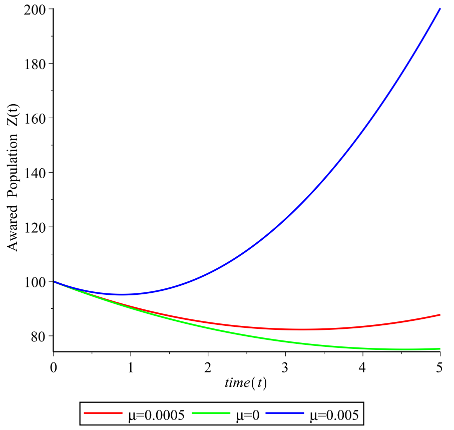

Figure 9. Variation of aware population with time for different values of μ.

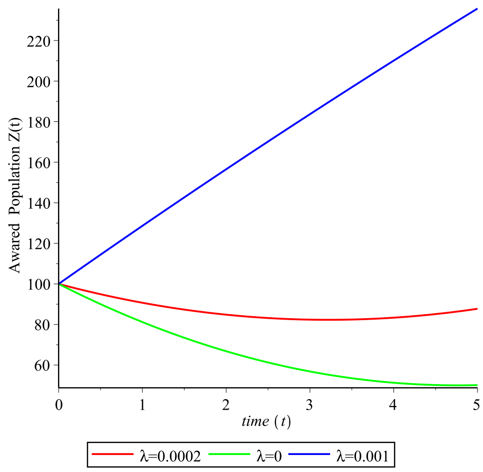

Figure 10. Variation of aware population with time for different values of λ.

program is reduced.

The number of the awareness program $M(t)$ with respect to time t for different values of dissemination of awareness λ is shown in figure 12. From the figure, it can be noted that change in rate of dissemination of awareness λ increases, there is a steady decrease in the awareness program $M(t)$. As the awareness program cannot be spread forever, and hence over a period of time number of awareness program is reduced.

5. Conclusion

In this paper, a non linear mathematical model has been proposed and analysed to study the effects of awareness programs driven by media on the spread of infectious diseases in a variable population with immigration. The analytic solution of the non linear model is obtained with the aid of homotopy perturbation method. The model is solved upto second order using homotopy perturbation technique. The results are then interpreted graphically using Maple 17. The HPM is more reliable and successful method to solve nonlinear problem directly without linearizing the problem and gives convergent approximations that leads to exact solution. From the graphs, we understood that model exhibits a rich dynamics.

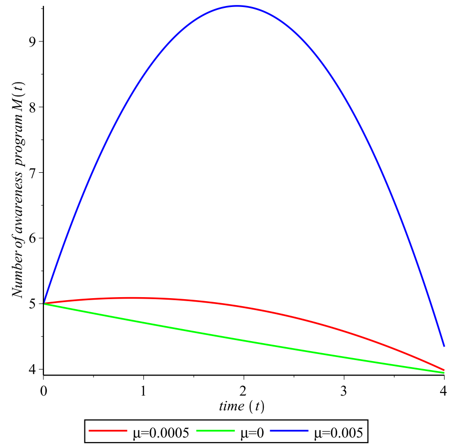

Figure 11. Variation of the number of awareness program with time for various values of μ.

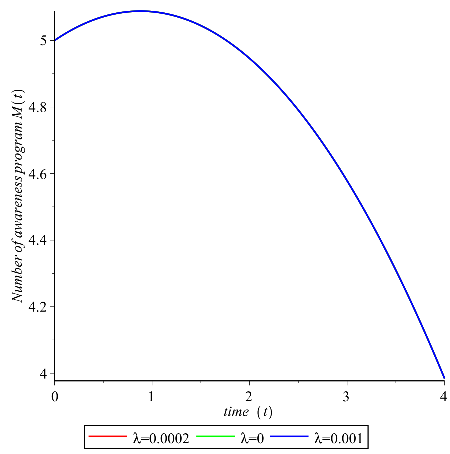

Figure 12. Variation of the number of awareness program with time for various values of λ.

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