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A Mathematical Review on Machine Intelligence Quotient (MIQ) Theory in Real-World Applications

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Abstract. Complex systems and information flow such as command, control, communication, computer intelligence, inference, and expert systems exhibit an increasing dependence on the system's nature and information flow. For this reason, it is both the flow exchange of information in the system and the emergent information properties of the system, which are central to the definition, design and analysis of such systems. Besides, the achievement of human-level machine intelligence has long been one of the principal objectives of Artificial Intelligence (AI). Machine intelligence methods have gained many orders of magnitude in computational capability in the last decade, especially in cognitive science and cognitive informatics. This research discussed the difference between Human IQ and Machine Intelligence Quotient (MIQ). Also, we show that MIQ is an abstract intelligence and soft concept of artificial intelligence. Finally, we conclude that the fuzzy sets can better represent MIQ instead of probabilistic and numerical methods.

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1. Introduction

Humans have many remarkable capabilities, including two primary capabilities. First, the capability to converse, reason, and make rational decisions in an uncertain environment and partiality of truth. Second, perform various physical and mental tasks such as driving a car in city traffic without any measurements and any computations [1]. IQ and MIQ's fundamental difference is that IQ is more or less constant. In contrast, MIQ changes with time and is machine-specific. Furthermore, the dimensions of MIQ and IQ are not the same. For example, speech recognition might be an essential dimension of MIQ. However, in the case of IQ, it is taken for granted. Underlying these capabilities is the human brain's capability to process and reason with perception-based information. It should be noted that a natural language is a system for describing perceptions [2]. Cognitive Informatics (CI) is a transdisciplinary inquiry of the internal information processing mechanisms and processes of the brain and abstract intelligence and their cognitive computing and cognitive engineering applications. These systems include computational intelligence, which is an autonomous system that mimicking the brain's mechanisms. CI and CC emerged from transdisciplinary studies in both natural intelligence in cognitive brain sciences. CI recognized that the brain might be explained by a hierarchical structure at the logical, cognitive, and neurological levels from the bottom up. This representation is known as the abstract intelligence or brain informatics system.

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The synergy of multidisciplinary studies at all levels leads to the theory of CI for explaining the brain. The fundamental theories underpinning the framework of the brain and intelligence sciences are abstract intelligence [3]. Informally, Human-level machine intelligence (HLMI) is a machine with a human brain. A machine has human-level machine intelligence if it has human-like capabilities to Understand, Learn, Reason, Recall, Remember, Inference, and Answer questions [4]. A machine may have superhuman intelligence in some respects and subhuman intelligence in other respects like Google or Siri. So, the MIQ of a machine is relative to the MIQ of other machines in the same category. For example, the MIQ of Google should be compared with the MIQ of other search engines.

2. Theoretical background

This section will overview relevant aspects of machine intelligence, fuzzy logic theory, and computing with words. After a short introduction to corporate events' institutionalization, the section is concerned with Machine Intelligence in Subsection 2.1. Second, it will focus on Computing with Words (CWW) in Subsection 2.2. In Subsection 2.3, we introduced the role of fuzzy Logic in MIQ systems.

2.1 *Machine intelligence*

Albus once insisted that a useful de3nition of intelligence should span a wide range of capabilities. We shall not attempt to define the notion of "machine intelligence" explicitly. Instead, we shall confine the target machines of interest to a class of engineering products and consider machine intelligence in a limited sense. Some mobile robot systems can plan their paths autonomously, avoiding obstacles based on information obtained. Simultaneously, some washing machines can discriminate ordinary clothes from dirty clothes to decide the washing time and the amount of detergent. These systems are said to have intelligence. All those systems can process input information to give a proper decision with their controllers, processors, or computers, which correspond to a human's brain. The other component of machine intelligence is interfaced intelligence. Interface intelligence indicates the degree of intelligence of the human-computer interaction (HCI). Recently, HCI has become more and more critical in complex control systems. Such as the control room of nuclear power plants or Aeroplan's, since control systems designed as user-friendly can reduce human errors and effectively utilize human ingenuity. We are going to formulate control intelligence and interface intelligence using the CWW and equations. We face many products and various forms of, called system intelligence. The meaning of intelligence seems to be different from one system to another, implying something in a non-unique way. To make this point clear, we review several definitions of machine intelligence and consider some essential questions.

A) Define the MIQ Theory: The following sentences show the attributes intelligent machines should have: 1) Machine intelligence is analyzing, organizing, and converting data into knowledge. Machine knowledge is defined as the structured information acquired and applied to remove ignorance and uncertainty about the intelligent machine's specific task. 2) For a human-made intelligent system to act appropriately, it may emulate living creatures' functions and, ultimately, human mental faculties. 3) Intelligent control is the discipline in which control algorithms are developed by emulating intelligent biological systems' specific characteristics. 4) An intelligent control system is a control system with the highest degree of autonomy in self-learning, self reconfigurability, reasoning, planning, and decision-making. 5) Intelligent machines are designed to perform anthropomorphic tasks with minimum interaction with a human operator.

B) Several Questions About Measuring the MIQ: to develop a practical procedure for measuring a MIQ, the following problems must be resolved. *Question 1:* Machine intelligence can be divided into two components—control intelligence and interface intelligence, as illustrated in Figure 1. Control intelligence is required to perform control in response to unanticipated events in uncertain environments. *Question 2:* The method of measuring the MIQ should be designed with a human orientation. The MIQ represents the degree of machine intelligence as closely as users feel. *Question 3:* The concept of question 3 is depicted in Figure 1. In most cases, the entire control system is made up of human supervisors and machines. Therefore, computer controllers cannot complete the overall control job without a human supervisor. The boundary between humans and machines is not clearly defined because they must complement each other. Unanticipated events are exceptional or randomly occurring conditions such as machine faults and plant abnormalities.

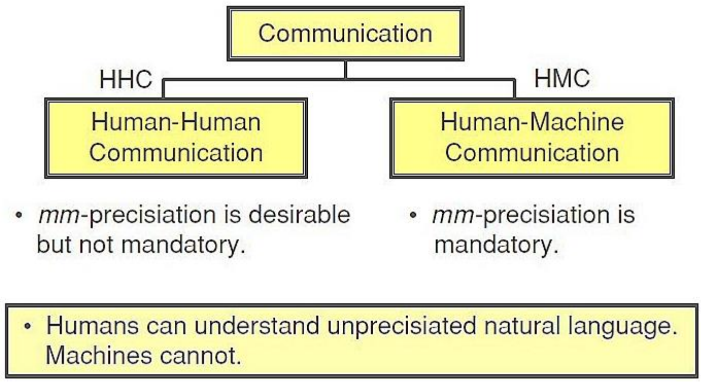

Figure 1. Human-Machine cooperation model.

2.2 *Computing with words (CWW)*

Computing with Words relates to computation with the information described in a natural language [6]. More concretely, in CWW, computation objects, computation words, propositions are drawn from a natural language. The importance of computing with words derives from the fact that most human knowledge is mostly world knowledge described in natural language. What is needed for this purpose is the methodology of Computing with Words (CWW). There is a vast literature on propositions in natural languages. Underlying CWW are two postulates and two concomitant rationales:

A. *Words are less precise than numbers*
B. *Precision carriers a cost*
C. *When numerical information is not available or too costly, CWW becomes necessary*
D. *CWW's advantages can be exploited to reduce cost, simplify design, and enhance robustness when there is a tolerance for imprecision.*

Table 1. Level of uncertainty analysis.

Order	Methodology
Probability events	Probability theory: inaccurate and/or incomplete data or partially true.
First-order uncertainty	Possibility Theory: fuzzy probability approaches
Vague information.	Fuzzy logic: imperfect information and approximate reasoning
High-order Imprecision	Type-2 fuzzy logic: Handling the imprecision and uncertainty about uncertainty
Real-world applications	Hybrid Intelligent with HLMI: Decision analysis under real-world uncertainty

2.3 *Fuzzy logic*

The fuzzy logic theory is based on the notion of relative graded membership, as inspired by the processes of human perception and cognition. Lotfi A. Zadeh published his first famous research paper on fuzzy sets in 1965. Fuzzy logic (1988) deals with computational perception and cognition: uncertain, imprecise, vague, partially true, or without sharp boundaries. Fuzzy logic allows for the inclusion of preliminary human assessments in computing problems. Also, it provides an effective means for conflict resolution of multiple criteria and better assessment of options. In fuzzy logic, everything is or is allowed to be granulated. Granulation involves the partitioning of an object into granules. A fundamental concept in fuzzy logic that plays a crucial role in many applications, especially in fuzzy control and fuzzy expert systems, is a linguistic variable. As its name suggests, a linguistic variable is a variable whose values are words or sentences in a natural or synthetic language. New computing methods based on fuzzy logic can develop intelligent systems for decision making, identification, pattern recognition, optimization, and control [10].

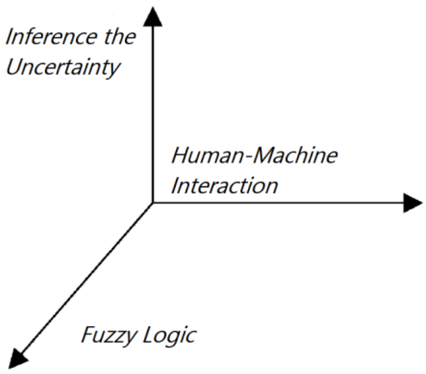

Figure 2. Model space of machine intelligence for a large-scaled system.

The centrality of information in almost everything that we do is that few would care to challenge it. Much less visible, but potentially of equal or even greater importance, might be called the intelligent systems revolution. The artifacts of this revolution are human-made systems that exhibit reason, learn from experience and make rational decisions without human intervention. Professor Zadeh coined the term MIQ (machine intelligence quotient) to describe a measure of intelligence of human-made systems. In this perspective, an intelligent system is a system that has a high MIQ. For example, we may make granulation of MIQ-scale in numbers between 0, and 200 into a fuzzy term set consisting of {Very Low, Low, Medium, High, Very High} as shown in Figure 3. It may be enough to say, for example, that the MIQ of the given robot is High, rather than MIQ = 143 (out of 200).

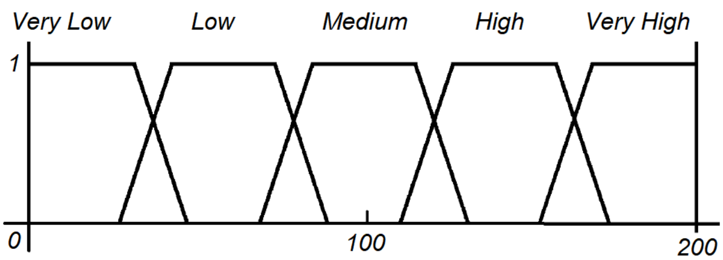

Figure 3. A fuzzy set representation of MIQ.

3. Theory of the Z-numbers

In the real world, uncertainty is a pervasive phenomenon. Much of the information on which decisions are based is uncertain. Humans have a remarkable capability to make rational decisions based on uncertain, imprecise, and/or incomplete information. Formalization of this capability, at least to some degree, is a challenge that is hard to meet. It is this challenge that motivates the concepts and ideas outlined in this note. A Z-number is an ordered pair of fuzzy numbers (A, B). A restriction may be viewed as a generalized constraint [11,14].

$$R(X) = X\ is\ A, \tag{1}$$

is referred to as a possibilistic restriction (constraint), with A playing the possibility distribution of X. More specifically,

$$R(X) = X \text{ is } A \rightarrow Poss(X = u) = \mu_A(u) \tag{2}$$

where μ_A is the membership function of A, and u is a generic value of X. μ_A may be viewed as a constraint that is associated with $R(X)$, meaning that $\mu_A(u)$ is the degree to which u satisfies the constraint. When X is a random variable, the probability distribution of X plays the role of a probabilistic restriction on X. A probabilistic restriction is expressed as:

$$R(X) = X \text{ isp } p, \tag{3}$$

where p is the probability density function of X. In this case,

$$R(X) = X \text{ isp } p \rightarrow Prob(u \leq X \leq u + du) = p(u)du \tag{4}$$

The ordered triple (X, A, B) is referred to as a Z-valuation. In a related way, uncertain computation is a computation system in which computation objects are not values of variables. But restrictions on the values of variables. In this note, unless stated to the contrary, X is assumed to be a random variable. For convenience, A is referred to as X's value, strictly speaking, understanding [13]. Therefore, A is not a value of X but a restriction on the values which X can take. The second component, B, is referred to as certainty. Closely related to certainty are the concepts of sureness, confidence, reliability, the strength of belief, probability, and possibility. When X is a random variable, certainty may be equated to probability. A and B are described in a natural language, the meaning of them is graduated with membership functions, μ_A and μ_B, respectively, Figure 3.

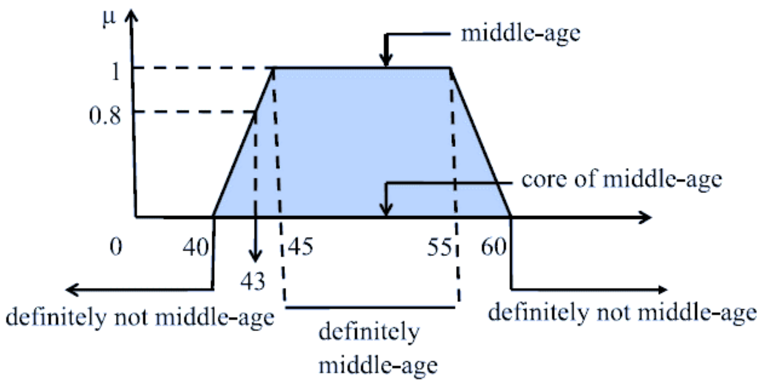

Figure 4. Precipitation of middle-age through graduation with fuzzy logic [15].

The fuzzy set, A, may be interpreted as the possibility distribution of X. The concept of a Z-number may be generalized in various ways. In particular, X can be assumed to take values in R^n, in which case A is a Cartesian product of fuzzy numbers. Simple examples of Z-valuations are [12]:

(anticipated budget deficit, close to 8 million dollars, very likely)
(population of UK, about 40 million, quite sure)

It is important to note that many propositions in a natural language are expressible as Z-valuations. Example: The proposition, p, p: Usually, it takes Robert about one hour to get home from work, is expressible as a Z-valuation:

(David's travel time from office to home, about 1 h, usually)

If X is a random variable, then X is A represents a fuzzy event in R, the real line. The probability of this event, p, may be expressed as [11]

$$p = \int_R \mu_A(u) p_X(u) du \tag{5}$$

where p_X is the underlying (hidden) probability density of X. In effect, the Z-valuation (X, A, B) may be viewed as a restriction (generalized constraint) on X defined by:

$$Prob(X \text{ is } A) \text{ is } B. \tag{6}$$

What should be underscored is that in a Z-number, (A, B), the underlying probability distribution, p_X, is not known. What is known is a restriction on p_X which may be expressed as:

$$p = \int_R \mu_A(u) p_X(u) du \text{ is } B \tag{7}$$

The subtle point is that B restricts the probability measure of $''A''$ rather than on A's probability. Conversely, if B is a restriction on A's probability rather than on the probability measure of A, then (A, B) is not a Z-number. An immediate consequence of the relation between p_X Moreover, B is the following:

If $Z = (A, B)$ then $Z' = (A', 1 - B)$, wherever A' is the complement of A, and $Z'$is the complement of Z. $1 - B$ is the antonym of B. For example, the complement of Z is:

$$Z = (A, likely) \text{ is } Z' \text{ equals to } (not\ A, unlikely)$$

Therefore, according to the mentioned descriptions, an essential qualitative attribute of a Z-number is informativeness. Generally, but not always, a Z-number is informative if its value has high specificity. It is tightly constrained, and certainty is high.

3.1 *Type-2 fuzzy Z-numbers for MIQS*

Type-2 fuzzy sets are known as fuzzy-fuzzy sets [14]. Membership function (MF) of a Type-2 fuzzy set (T2FS) of a given element is itself a type-1 fuzzy set (T1FS). A T2FS represented as $\tilde{A}$, is characterized through a type-2 MF $\mu_{\tilde{A}(x,u)}$ where $x \in X$ and $u \in J_x \subseteq [0,1]$ as follows:

$$\tilde{A} = \left\{ \left((x, u), \mu_{\tilde{A}(x,u)} \right) | \forall x \in X, \forall \in J_x \subseteq [0,1] \right\} \tag{8}$$

In which $0 \leq \mu_{\tilde{A}(x,u)} \leq 1$, X is the domain of fuzzy set and J_x is the domain of the secondary MF at x. $\tilde{A}$ is as follows:

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)}{x, uJ_x} \subseteq [0,1] \tag{9}$$

where $\iint$ represents union over all admissible x and u.

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} 1}{x, u} = \frac{\int_{x\in X} \left[\frac{\int_{u\in J_x} 1}{u}\right]}{x} \tag{10}$$

where x is the primary variable, J_x, an interval in $[0,1]$, is the primary MF of x, u is the secondary variable, and $\int_{u\in J_x}$ is the secondary MF at x.

A Gaussian IT2 fuzzy set. the UMF is defined as follows [16]:

$$\mu_{\bar{X}}(x) = \begin{cases} e^{-\frac{(x-m_1)^2}{2\sigma^2}}, & x < m_1 \\ 1, & m_1 \le x \le m_2 \\ e^{-\frac{(x-m_2)^2}{2\sigma^2}}, & x > m_2 \end{cases} \tag{11}$$

Moreover, LMF defines as follows:

$$\mu_{\underline{X}}(x) = min\left(e^{-\frac{(x-m_1)^2}{2\sigma^2}}, e^{-\frac{(x-m_2)^2}{2\sigma^2}}\right) \tag{12}$$

When the standard deviation of the Gaussian T1 fuzzy set is blurred to be an interval $[\sigma_1, \sigma_2]$, the UMF is:

$$\mu_{\bar{X}}(x) = e^{-\frac{(x-m)^2}{2\sigma_2^2}} \tag{13}$$

Moreover, the LMF is:

$$\mu_{\underline{X}}(x) = e^{-\frac{(x-m)^2}{2\sigma_1^2}} \tag{14}$$

In order to compute more precise computation, we applied the concept of Z^+-number, which is closely related to the concept of a Type-2 Fuzzy. A Z^+-number, is a combination of a fuzzy number, A, and a random number, R, written as an ordered pair $Z^+ = (A, R)$. Informally, these distributions are compatible if the centroids of μ_A and p_X are coincident as follows [5]:

$$\int_R u p_X(u) du = \frac{\int_R u\mu_A(u) du}{\int_R \mu_A(u) du} \tag{15}$$

The scalar product of μ_A and p_{X}, μ_A . p_X , is the probability measure P_A, of A, as follows:

$$\mu_A \cdot p_X = P_A = \int_R \mu_A(u) p_X(u) du \tag{16}$$

It is this relation that links the concept of a Z-number to that of a Z^+-number as follows:

$$Z(A, B) = Z^+(\text{A}, \mu_A \cdot p_X \text{ is } B) \tag{17}$$

Computation with Z^+-numbers are much simpler than computation with Z-numbers. Turning to computation with Z-numbers, assume for simplicity that * = sum. Assume that $Z_X = (A_X, B_X)$ and $Z_Y = (A_Y, B_Y)$. The main objective is to compute the sum $Z = X + Y$. Assume that the associated Z-valuations are (X, A_X, B_X), (Y, A_Y, B_Y), and (Z, A_Z, B_Z). The first step involves the computation of p_Z. To begin with, let us assume that p_X and p_Y are known as follows:

$$p_Z(v) = \int_R p_X(u)p_Y(v-u)du \tag{18}$$

In the case of Z-numbers, what we know are not p_X and p_Y but restrictions on p_X and p_Y.

$$\int_R \mu_{A_X}(u)p_X(u)du \text{ is } B_X \tag{19}$$

$$\int_R \mu_{A_Y}(u)p_Y(u)du \text{ is } B_Y \tag{20}$$

In terms of the membership functions of B_X and B_Y, these restrictions may be expressed as:

$$\mu_{B_X}\left(\int_R \mu_{A_X}(u)p_X(u)du\right) \tag{21}$$

$$\mu_{B_Y}\left(\int_R \mu_{A_Y}(u)p_Y(u)du\right) \tag{22}$$

Additional restrictions on p_X and p_Y are:

$$\int_R p_X(u)du = 1 \tag{23}$$

$$\int_R p_Y(u)du = 1 \tag{24}$$

$$\int_R up_X(u)du = \frac{\int_R u\mu_{A_X}(u)du}{\int_R \mu_{AX}(u)du} \text{ (compatibility)} \tag{25}$$

$$\int_R up_Y(u)du = \frac{\int_R u\mu_{A_Y}(u)du}{\int_R \mu_{AY}(u)du} \text{ (compatibility)} \tag{26}$$

Applying the extension principle, the membership function of p_Z expressed as [9]:

$$\mu_{p_Z}(p_Z) = \sup_{p_X,p_Y}\left(\mu_{B_X}\left(\int_R \mu_{A_X}(u)p_X(u)du\right) \wedge \mu_{B_Y}\left(\int_R \mu_{A_Y}(u)p_Y(u)du\right)\right) \tag{27}$$

subject to

$$p_Z = p_X \circ p_Y \tag{28}$$

$$\int_R p_X(u)du = 1 \tag{29}$$

$$\int_R p_Y(u)du = 1 \tag{30}$$

$$\int_R up_X(u)du = \frac{\int_R u\mu_{A_X}(u)du}{\int_R \mu_{A_X}(u)du} \tag{31}$$

$$\int_R up_Y(u)du = \frac{\int_R u\mu_{AY}(u)du}{\int_R \mu_{A_Y}(u)du} \tag{32}$$

The combined restriction on the arguments is expressed as a conjunction of their restrictions, with ^ interpreted as min. In effect, the application of the extension principle reduces the computation of p_Z to a problem in functional optimization.

4. A real-world application scenario: humanoid intelligence

Humanoid robots are hybrid systems with symbolic knowledge representation and behavior-based action that consider sensory-motor physicality and environmental situation changes. Hybrid systems are controlled in a hierarchically layered approach: Complex behaviors at a higher-level control one or more behaviors at an underlying level. In nature, this hierarchy often corresponds to the development of a living being in phylogeny. The robot control architecture (Figure 3) is given to the modules' arrangement with their connections through the robot's actions. In a symbolically oriented architecture, the hardware's details are abstracted, and cognition is represented as symbol processing in the model. Humanoid cognition is divided into many sub-modules. Sensory data are thus interpreted, evaluated for linguistic representation, conceptual and situational knowledge to realize actions with sensor-motor skills. In this case, symbolic and sequential action planning is possible and quick reaction mainly without symbolic-cognitive instances. However, humanoid intelligence and adaptation will only develop if the artifacts have a body adapted and adaptable to their tasks and can also react to the situation autonomously. Since intelligence in living organisms such as humans develops and changes body-dependently in a person's life, a growing body with highly flexible actuators will also become necessary. This requires cooperation with scientific disciplines until recently seemed utterly meaningless to the engineering sciences: cognitive science and brain research, Systems biology and synthetic biology, nano and materials sciences [8].

On the other hand, behavior-based architectures are based on an action-centered understanding of cognition. All physical details, situatedness by the environment and high adaptability play an essential role. Behavior-based controls ensure that the robot responds quickly to environmental changes by processing stimuli perceived by sensors. With symbolic processing, sensor inputs are first interpreted in an environmental model. A plan for the action to be performed by actuators (e.g., wheels, feet, legs, arms, hands, grippers) is defined. This plan compares different goals as optimally as possible. The behavior-based approach does without sequential programming. Instead, as in a living organism, parallel processes must be coordinated. Behavioral architectures are found more in simple mobile robots, while symbolically oriented architectures are realized in cognitive systems with symbolic knowledge representation. Like humans, humanoid robots should also have both properties [7].

4.1 *Perception with convolutional neural nets*

An ordinary ANN does not take raw data as input; instead, it takes features as input and classifies it based on their features. The features are computed through a separated feature extraction process (handcrafted) and are given as an n-dimensional feature vector. The reason behind this separation of feature extraction and classification is that the data dimension is usually significantly more extensive, typically from tens of thousands to millions. Direct connection of raw data to an ANN would make the network too complicated and too expensive to compute with traditional computing power. Besides, the various data dimension is also a design issue for such a combined ANN. With the rapid increase of computation power, it is possible to combine both the feature extraction and classification processes into a single neural network. The idea is to integrate a feature extraction network in front of an ordinary ANN. The architecture of a CNNs can be demonstrated using the LeNet in Figure 5. A CNN consists of a convolution network in front and a fully connected MLP at the backend. Because each hidden unit in the convolutional network is only connected to a local neighborhood in the input image instead of every pixel, it is also called a locally connected network. In contrast, in an ordinary ANN, each input data element is connected to each network's hidden unit, called a fully connected network. The convolutional network is a repeat process of convolution and pooling, as shown in Figure 5. Depending on the dimension of the input data, the repetitions can occur for several rounds.

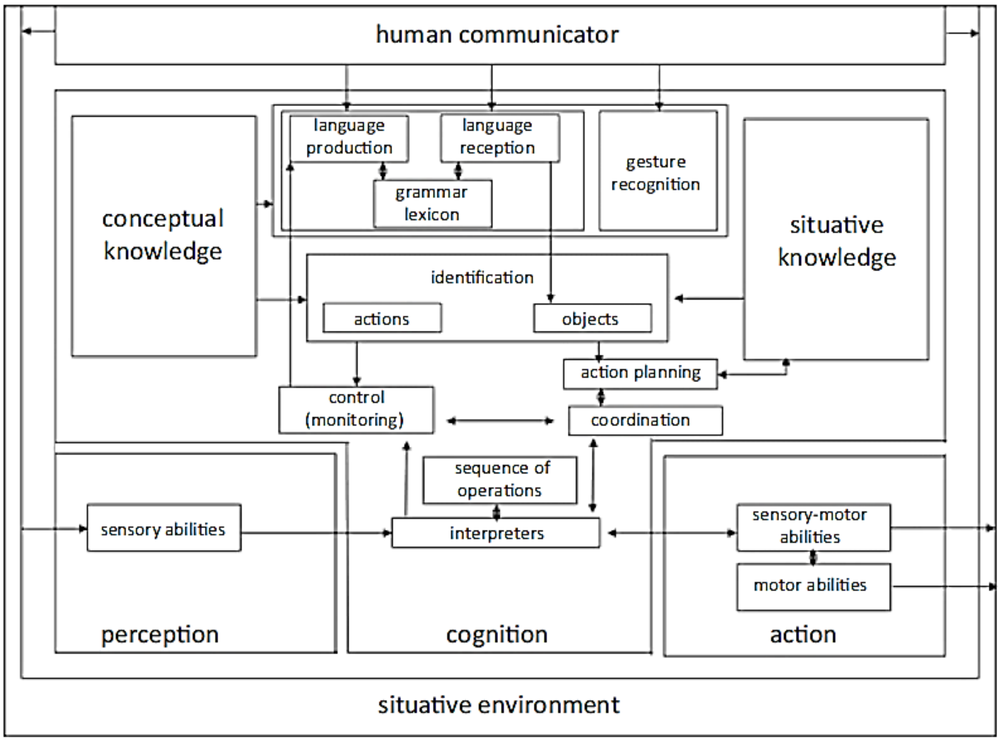

Figure 5. Architecture of a humanoid robot with behavior-based and symbolic cognitive modules.

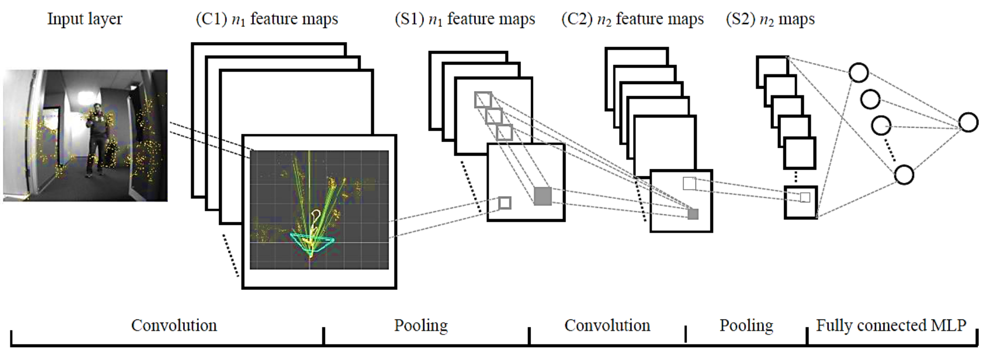

Figure 6. Architecture of a CNN for humanoid robot perception [7].

5. Conclusion

In this research we conclude that there are three main reasons why MIQ is important in real world applications. First, until recently the principal tools in AI's armamentarium were centered on symbol manipulation and predicate logic, while the use of numerical techniques was looked upon with disfavor. Almost all decisions are made in an environment of partial uncertainty, partial knowledge, and partial truth. As we move further into the age of machine intelligence and mechanized decision-making, the need for a better understanding of how to deal with uncertainty is growing in urgency and importance. The achievement of human-level machine intelligence has profound implications for our info-centric society. It has a vital role in enhancing the quality of life, but it is a challenge that is hard to meet. Our presentation articulated that human-level machine intelligence cannot be achieved by using theories based on classical, Aristotelian, bivalent logic. It is argued that to achieve human-level machine intelligence, what is needed is a paradigm shift, a shift from computing with numbers to computing with words. In particular, a critical problem that has to be addressed is that of precision of meaning. Resolution of this problem requires the use of concepts and techniques drawn from fuzzy logic and deep learnings. By adopting the clear definition and the logical estimating method that this paper proposed, the MIQ can be used as a theoretical measure and as a practical index that suggests the design goal and can be cited for comparing a product's intelligence superiority.

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