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# Anisotropic Charged Stellar Models

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**Abstract.** A new class of exact solutions of the Einstein-Maxwell system is found in closed form for a static spherically symmetric anisotropic star in the presence of an electric field by generalizing earlier approaches. The field equations are integrated by specifying one of the gravitational potentials, the anisotropic factor and electric field which are physically reasonable. We demonstrate that it is possible to obtain a more general class of solutions to the Einstein-Maxwell system in the form of series with anisotropic matter. For specific parameter values it is possible to find new exact models for the Einstein-Maxwell system in terms of elementary functions from the general series solution. Our results contain particular solutions found previously including models of Thirukkanesh and Maharaj (2009) and Komathiraj and Maharaj (2007) charged relativistic models.

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## 1. Introduction

To obtain an understanding of the gravitational dynamics of a general relativistic star it is necessary to solve the Einstein-Maxwell equations. The matter distribution may be anisotropic in the presence of an electromagnetic field. On physical grounds we should include an equation of state relating the radial pressure to the energy density in a barotropic distribution. In this way we can model relativistic compact objects including dark energy stars, quark stars, gravastars, neutron stars and ultradense matter. Since the pioneering paper by Bowers and Liang [3] there have been extensive investigations in the study of anisotropic relativistic matter distributions in general relativity to include the effects of spacetime curvature. The anisotropic interior spacetime matches to the Schwarzschild exterior model. Stellar models consisting of spherically symmetric distribution of matter with presence of anisotropy in the pressure have been widely considered in the frame of general relativity [24]. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [33] or another physical phenomenon by the presence of an electrical field [40]. In such systems, the radial pressure is different from the tangential pressure. This generalization has been very used in the study of the balance and collapse of compact spheres [2,10,11]. Malaver [25] studied the effect of local anisotropy on the bulk properties of spherically symmetric static general relativistic compact objects. Tello-Ortiz et al. [36] found an anisotropic fluid sphere solution of the Einstein-Maxwell field equations with a modified

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version of the Chaplygin equation. Also generalised version of the Chaplygin equation of state was successfully used in the study of charged anisotropic matter [27]. More recently, Malaver and Kasmaei [28] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state specifying particular forms for the gravitational potential and electric field intensity.

Many researchers have used various analytical techniques to try in order to obtain solutions of Einstein field equations for relativistic stars as it has been shown by Thirukkanesh and Ragel [38], Feroze and Siddiqui [8], Pant et al. [32] and Malaver [23,26]. These studies suggest that the Einstein-Maxwell field equations are very important in the description of the stellar structures. In addition, it needs to be considered that Einstein Field Equations lie in the category of Systems of Differential equations and many new analytical and approximate methods can be suggested to solve these types of equations [1,5,13,30,31,34]. For some recent models investigating the properties of charged anisotropic stars see the treatments of Komathiraj et al. [19] and Komathiraj and Sharma [18], Thirukkanesh and Ragel [39], Malaver and Kasmaei [29].

Incorporation of electromagnetic field and anisotropy makes the system of field equations even more difficult to solve unless one adopts some simplifying techniques to make them tractable. In an earlier work, by identifying a conformal Killing vector, Mak and Harko [21] developed a relativistic model of an isotropic quark star. The work was later extended by Komathiraj and Maharaj [15] who provided a more general class of exact solutions by incorporating an electromagnetic field in the system of field equations. In a more recent work, Maharaj et al. [20] and Komathiraj [14] have made a further generalization of [15] model by incorporating anisotropic stress into the system. In a subsequent paper, Sunzu et al. [35] performed a detailed physical analysis of the solution obtained in [20] and discussed its relevance in the context of compact quark stars candidates.

From the above motivation it is clear that both anisotropy and the electromagnetic field are important in astrophysical processes. However previous treatments have largely considered either anisotropy or electromagnetic field separately. The intention of this paper is to provide a general framework that admits the possibility of tangential pressures with a nonvanishing electric field intensity. We believe that this approach will allow for a richer family of solutions to the Einstein-Maxwell field equations and possibly provide a deeper insight into the behaviour of the gravitational field.

The objective of this treatment is to generate exact solutions to the Einstein-Maxwell system, that may be utilised to describe a charged anisotropic relativistic body. In Section 2, we express the Einstein-Maxwell system as a new system of differential equations using a coordinate transformation. We choose particular forms for one of the gravitational potentials, anisotropic factor and the electric field intensity, which enables us to obtain the condition of pressure isotropy in the remaining gravitational potential in Section 3. This is the master equation which determines the integrability of the system. We integrate this equation using the method of Frobenius and the solution is given in terms of series. We demonstrate that it is possible to find two categories of solutions in terms of elementary functions by placing certain restriction on the parameters in Section 4. The advantage of this approach is that one can regain the charged isotropic stellar model simply by setting the anisotropy to zero. It is interesting to note that many previously found explicit solutions of the Einstein-Maxwell system with anisotropic stress e.g., solutions obtained by [4, 7, 9, 12, 22] do not have their corresponding isotropic analogues. In Section 5, we discuss the physical features of the solutions found. Finally, some concluding remarks are made in Section 6.

## 2. Field equations

The gravitational field should be static and spherically symmetric for describing the internal structure of a dense compact relativistic sphere which is charged. For describing such a configuration, we utilise coordinates $(x^a) = (t, r, \theta, \phi)$, such that the generic form of the line element is given by

$$ds^2 = -e^{2\mu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1}$$

where $\mu(r)$ and $\lambda(r)$ are yet to be determined. The Einstein-Maxwell system of field equations corresponding to the line element (1) can be written in the form

$$\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2, \tag{2}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\mu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \tag{3}$$

$$e^{-2\lambda}\left(\mu'' + \mu'^2 + \frac{\mu'}{r} - \mu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \tag{4}$$

$$\frac{1}{r^2}e^{-\lambda}(r^2E)' = \sigma. \tag{5}$$

In the above $\rho$ is the energy density, $p_r$ is the radial pressure, $p_t$ is the tangential pressure, $E$ is the electric field intensity and $\sigma$ is the proper charge density, and a prime $(')$ denotes derivative with respect to the radial coordinate $r$.

A different but equivalent form of the field equations is generated if we introduce new variables

$$A^2y^2(x) = e^{2\mu(r)}, \quad Z(x) = e^{-2\lambda(r)}, \quad x = Cr^2, \tag{6}$$

where A and C are arbitrary constants. Under the transformation (6) due to Durgapal and Bannerji [6], the system (2)-(5) becomes

$$\frac{1 - Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{1}{2}E^2, \tag{7}$$

$$4Z\frac{\dot{y}}{y} + \frac{Z - 1}{x} = \frac{p_r}{C} - \frac{1}{2C}E^2, \tag{8}$$

$$4Zx^2\ddot{y} + 2\dot{Z}x^2\dot{y} + \left(\dot{Z}x - Z + 1 - \frac{\Delta x}{C} - \frac{E^2x}{C}\right)y = 0, \tag{9}$$

$$\sigma^2 = \frac{4CZ}{x}\left(x\dot{E} + E\right)^2, \tag{10}$$

where $\Delta = p_t - p_r$ represents the measure of anisotropy which is required to vanish at the center and dots denote derivative with respect to the new coordinate $x$. The system of equation (7)-(10) governs the gravitational behavior of a charged star with anisotropic pressure.

The mass of a self-gravitating object for a given radius is an important measure for comparison with observational data. In this case, the mass contained within a radius $x$ of the sphere is obtained as

$$m(x) = \frac{1}{4C^{\frac{3}{2}}}\int_0^x \sqrt{x}\rho(x)dx. \tag{11}$$

## 3. Exact models

We have a nonlinear system of four equations (7)-(10) in seven unknowns $Z,\ y,\ \rho,\ p_r,\ \Delta,$ $E$ and $\sigma$. To integrate the system (7)-(10) it is necessary to specify three of the variables. In our approach we choose $Z,\ \Delta$ and $E$ on physical grounds. The remaining quantities are then obtained from the rest of the system.

In the integration procedure we make the specific choices:

$$Z(x) = \frac{(1+kx)^2}{1+mx}, \tag{12}$$

$$\frac{\Delta}{C} = \frac{\alpha m(k-m)x}{(1+mx)^2}, \tag{13}$$

$$\frac{E^2}{C} = \frac{\beta k(m-k)x}{(1+mx)^2}, \tag{14}$$

where $k\ ,m,\alpha$ and $\beta$ are constants. The choice (12) ensures that the metric function is regular at the center and is well behaved within the stellar interior. A similar choice has been used by Komathiraj and Maharaj [16] and Thirukkanes and Maharaj [37]. As far as the second choice is concerned, it is a reasonable assumption in the sense that $\Delta$ vanishes at the center (i.e., $p_r = p_t$ at the origin) which is consistent with the physical requirement for a realistic stellar model. The form $E^2$ in (14) is physically palatable because $E$ remains regular and continuous throughout the sphere.

Substitution of (12)-(14) into (9) gives

$$\begin{aligned} 4X^2[mX-(m-k)]\frac{d^2Y}{dX^2} &+ 2X[mX-2(m-k)]\frac{dY}{dX} \\ &+ (m-k)\left[\frac{m(1+\alpha)}{k} - (1+\beta)\right]Y = 0, \end{aligned} \tag{15}$$

which is the second order differential equation in terms of the dependent variable $Y$ and independent variable $X$, where we have set

$$1+kx = X, \qquad y(x) = Y(X) \tag{16}$$

Once (15) is integrated we can directly find the remaining quantities $\rho,\ p_r,\ p_t$ from the system (7)-(9) as $\Delta$ and $E$ are known from (13) and (14) respectively. It is difficult to obtain a closed form solution to the equation (15). However, one can transform it to a differential equation which can be integrated by the method of Frobenius. This can be done in the following way. We introduce a new function $U(X)$ such that

$$Y(X) = X^a U(X), \tag{17}$$

where $a$ is a constant. A similar kind of transformation was utilised earlier by Komathiraj and Sharma [17] for generating charged stellar models. With the help of (17), differential equation (15) can be written as

$$4X^2[mX-(m-k)]\frac{d^2U}{dX^2}+2X[m(4a+1)X-2(2a+1)(m-k)]\frac{dU}{dX}+\left[2ma(2a-1)X-(m-k)\left(\frac{m(1+\alpha)}{k}-1-\beta-4a^2\right)\right]U=0 \quad (18)$$

A substantial simplification of the equation can be achieved if we set

$$\frac{m(1+\alpha)}{k}-\beta-1=4a^2 \quad (19)$$

Equation (18) then reduces to

$$2X\left[X-\frac{(m-k)}{m}\right]\frac{d^2U}{dX^2}+\left[(4a+1)X-2(2a+1)\frac{(m-k)}{m}\right]\frac{dU}{dX}+a(2a-1)U=0 \quad (20)$$

We can utilise the method of Frobenius about $X=\frac{m-k}{m}$, since this is a regular singular point of the differential equation (20). We write the solution of the differential equation (20) in the series form

$$U=\sum_{i=0}^{\infty}b_i\left[X-\frac{(m-k)}{m}\right]^{i+d}, \quad b_0\neq 0, \quad (21)$$

where $b_i$ are the coefficients of the series and $d$ is a constant. For a legitimate solution we need to determine the coefficients $b_i$ as well as the parameter $d$. On substituting (21) in to (20) we obtain the indicial equation as:

$b_0d(2d-3)=0$ which determines the value of the parameter $d=0,\ d=3/2$ as $b_0\neq 0$.

It is possible to express the coefficient in terms of the leading coefficient $b_0$ by establishing a general structure for the coefficients by considering the leading terms. These coefficients generate the pattern

$$b_i=\left(\frac{m}{m-k}\right)^i\prod_{p=1}^{i}\frac{(p+d-1)(2p+2d+4a-3)+a(2a-1)}{(p+d)(2p+2d-3)}b_0, \qquad b_0\neq 0 \quad (22)$$

Now it is possible to generate two linearly independent solutions to the differential equation (20) with the help of (21) and (22). For the parameter value $d=0$, the first solution can be written as:

$$U_1(X)=b_0\left[1+\sum_{i=1}^{\infty}\left(\frac{1}{\gamma}\right)^i\prod_{p=1}^{i}\frac{(p-1)(2p+4a-3)+a(2a-1)}{p(2p-3)}\times[X-\gamma]^i\right] \quad (23)$$

For the parameter value $d=3/2$, the second solution can be written as:

$$U_2(X)=b_0[X-\gamma]^{\frac{3}{2}}\times\left[1+\sum_{i=1}^{\infty}\left(\frac{1}{\gamma}\right)^i\prod_{p=1}^{i}\frac{(2p+1)(p+2a)+a(2a-1)}{p(2p+3)}\times[X-\gamma]^i\right], \quad (24)$$

where we let $\gamma = \frac{(m-k)}{m}$ for convenience. With the help of (16) and (17) we obtain the equivalent expressions for $U_1(X)$ and $U_2(X)$ given in (23) and (24) in terms of the original variable $x = Cr^2$ as:

$$y_1(x) = b_0(1+kx)^a \times \left[1 + \sum_{i=1}^{\infty}\left(\frac{1}{\gamma}\right)^i \prod_{p=1}^{i} \frac{(p-1)(2p+4a-3)+a(2a-1)}{p(2p-3)} \; [(1+kx)-\gamma]^i\right] \tag{25}$$

and

$$y_2(x) = b_0(1+kx)^a[(1+kx)-\gamma]^{\frac{3}{2}} \times \left[1 + \sum_{i=1}^{\infty}\left(\frac{1}{\gamma}\right)^i \prod_{p=1}^{i} \frac{(2p+1)(p+2a)+a(2a-1)}{p(2p+3)} \; [(1+kx)-\gamma]^i\right] \tag{26}$$

Thus, the general solution to the differential equation (20), for the choice of the anisotropic factor (13) and the electric field (14), is given by

$$y(x) = A_1 y_1(x) + A_2 y_2(x), \tag{27}$$

where $A_1$ and $A_2$ are arbitrary constants, $a^2 = \frac{m(1+\alpha)}{4k} - \frac{\beta+1}{4}$, $\gamma = \frac{(m-k)}{m}$ and $y_1(x)$ and $y_2(x)$ are given by (25) and (26) respectively. It is clear that the quantities $y_1(x)$ and $y_2(x)$ are linearly independent functions. From equations (7)-(10), the general solution to the Einstein-Maxwell system can be written as

$$e^{2\lambda} = \frac{1+mx}{(1+kx)^2}, \tag{28}$$

$$e^{2\mu} = A^2 y^2, \tag{29}$$

$$\frac{\rho}{C} = \frac{(3+mx)(m-2k)}{(1+mx)^2} - \frac{k^2x(5+3mx)}{(1+mx)^2} - \frac{\beta k(m-k)x}{(1+mx)^2}, \tag{30}$$

$$\frac{p_r}{C} = 4\frac{(1+kx)^2}{1+mx}\frac{\dot{y}}{y} + \frac{k(2+kx)-m}{1+mx} + \frac{\beta k(m-k)x}{(1+mx)^2}, \tag{31}$$

$$p_t = p_r + \Delta, \tag{32}$$

$$\frac{\Delta}{C} = \frac{\alpha m(k-m)x}{(1+mx)^2}, \tag{33}$$

$$\frac{E^2}{C} = \frac{\beta k(m-k)x}{(1+mx)^2}, \tag{34}$$

where $y$ is given in (27). The result in (28)-(34) is a new solution to the Einstein-Maxwell field equations. Note that if we set $\beta = 0$, (28)-(34) reduce to models for charged stars with isotropic matter which may contain new solutions to the Einstein-Maxwell field equations (7)-(10).

## 4. Elementary solutions

The general solution (28)-(34) can be expressed in terms of polynomial and algebraic functions. This is possible in general because the series (25) and (26) terminate for restricted values of the parameters $k, m, \alpha$ and $\beta$ so that elementary functions are possible. Consequently, we obtain two sets of general solutions in terms of elementary functions, by determining the specific restriction on the quantity $\frac{m(1+\alpha)}{k} - \beta - 1$ for a terminating series. The elementary functions found using this method, can be written as polynomials and polynomials with algebraic functions. The first category of solution can be written as

$$y_1(x) = -\frac{A_1}{(1+kx)^n} \sum_{i=0}^{n} \left(-\frac{1}{\gamma}\right)^i \frac{(2i-1)}{(2i)!\,(2n-2i+1)!} [(1+kx)-\gamma]^i + \frac{A_2}{(1+kx)^n} \sum_{i=0}^{n-1} \left(-\frac{1}{\gamma}\right)^i \frac{(i+1)}{(2i+3)!\,(2n-2i-2)!} [(1+kx) - \gamma]^{i+\frac{3}{2}} \tag{35}$$

for $\frac{m(1+\alpha)}{k} - \beta - 1 = 4n^2$. The second category of solutions can be written as

$$y_2(x) = -\frac{A_1}{(1+kx)^{n-\frac{1}{2}}} \sum_{i=0}^{n} \left(-\frac{1}{\gamma}\right)^i \frac{(2i-1)}{(2i)!\,(2n-2i)!} [(1+kx)-\gamma]^i + \frac{A_2}{(1+kx)^{n-\frac{1}{2}}} \sum_{i=0}^{n-2} \left(-\frac{1}{\gamma}\right)^i \frac{(i+1)}{(2i+3)!\,(2n-2i-3)!} [(1+kx) - \gamma]^{i+\frac{3}{2}} \tag{36}$$

for $\frac{m(1+\alpha)}{k} - \beta - 1 = 4n(n-1)$.

It is remarkable to observe that the solutions (35) and (36) are expressed completely in terms of elementary functions only. This does not happen often considering the nonlinearity of the gravitational interaction in the presence of charge. We have given our solutions in a simple form which has the advantage of facilitating the analysis of the physical features of the stellar models. Observe that our approach has combined both the charged and uncharged cases for a relativistic star: when $\beta = 0$ we obtain the solutions for the uncharged case directly. It is important to observe that the Einstein-Maxwell solutions (35) and (36) apply to both isotropic and anisotropic relativistic stars. We regain exact solutions with isotropic pressure, which may be possibly new, by setting $\alpha = 0$

It is interesting to observe that we can regain a number of physically reasonable models from the general class of solutions found in this paper. We demonstrate that this is possible in the following cases of physical interest. If we take $k = a, m = b$ and $\alpha = 0$ then it is easy to verify that the equations (35) and (36) correspond to the Thirukkanesh and Maharaj [37] model for a compact sphere in electric fields in the absence of anisotropic matter. When $m = 1$ and $\alpha = 0$ we easily obtain the result of Komathiraj and Maharaj [16] charged models from (35) and (36) after some manipulation.

We, thus, have provided two different class of solutions for $\frac{m(1+\alpha)}{k} - \beta - 1 = 4n^2$ (Case I) and $\frac{m(1+\alpha)}{k} - \beta - 1 = 4n(n-1)$ (Case II). We note that all the solutions are regular. One, however, needs to examine the physical viability of the solutions which can be analyzed by utilizing the junction conditions and systematically fixing the values of the model parameters. An interesting feature of the class of solutions is that they provide a mechanism to examine the impact of anisotropy on the physical properties of a relativistic star simply by using the parameter $\alpha$ as an 'anisotropic switch'.

## 5. Physical analysis

Let us now analyse the physical viability of the class of solutions (28)-(34) obtained in this paper. We need to consider only those values of $k$ and $m$ for which the energy density $\rho$, the radial pressure $p_r$, the tangential pressure $p_t$ and the electric field intensity $E$ remain finite and positive. The choices of $k$ and $m$ must ensure that the gravitational potential $e^{2\lambda}$ remains positive; the other potential $e^{2\mu}$ is necessarily positive. In (28) and (29), we note that

$$e^{2\lambda}(r=0)=1,\ \left(e^{2\lambda}\right)'(r=0)=0,$$

$$e^{2\mu}(r=0)=A^2y^2(r=0),\ (e^{2\mu})'(r=0)=(A^2y^2)'(r=0),$$

where $y$ is given by (25)-(27). Obviously, the gravitational potentials are regular at the origin. Using eq. (30), we obtain the central density $\rho_0=\rho(r=0)=3C(m-2k)$, which implies that we must have $m>2k$. Using eq. (31) at the center of the star $(r=0)$, we must have

$$p_r(r=0)=p_t(r=0)=4C\left(\frac{\dot{y}}{y}\right)(r=0)+C(2k-m)>0 \tag{37}$$

The radial pressure and the tangential pressure will be positive if we choose our model parameters in such a manner that the condition (37) is satisfied.

For a charged and anisotropic system, the anisotropic factor $\Delta$ and the electric field $E$ should be finite and positive. Hence, using (33) and (34) we obtain $\alpha Cm(k-m)>0$ and $\beta Ck(m-k)>0$.

For a realistic star of finite radius, the radial pressure should also vanish at some finite radial distance $r=R$ which yields

$$4(1+kCR^2)^2\left(\frac{\dot{y}}{y}\right)(r=R)+k(2+kCR^2)-m+\frac{\beta k(m-k)CR^2}{(1+mCR^2)}=0$$

This will constrain the values of $k, m, \alpha$ and $\beta$. The solution of the Einstein-Maxwell system for $r>R$ is given by the Reissner-Nordstrom metric

$$ds^2=-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{38}$$

where $M=m(R)$ and $Q=E(R)R^2$ are the total mass and charge of the star. Matching the line element (1) with Equation (38) across the boundary $R$, we have

$$A^2[A_1y_1(CR^2)+A_2y_2(CR^2)\ ]^2=\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right) \tag{39}$$

$$\frac{1+mCR^2}{(1+kCR^2)^2}=\left(1-\frac{2M}{R}+\frac{Q^2}{R^2}\right)^{-1} \tag{40}$$

The matching conditions (39) and (40) place restrictions on the metric coefficients, however there are sufficient free parameters to satisfy the necessary conditions that arise for the model under study. Since these conditions are satisfied by the constants in the solution a relativistic star of radius $R$ is realisable. By providing the necessary bounds on the model parameters, we can examine the physical viability of the solution.

## 6. Conclusion

In this paper, a new class of solutions to the Einstein-Maxwell system is presented in terms of an infinite series by making use of known transformation. This is achieved with the particular choices for one of the gravitational potentials, the anisotropic factor and electric field intensity. Moreover, we have demonstrated that for the specific set of model parameters, it is possible to obtain closed-form solutions from the general series solution. The solutions are expressed in terms of elementary functions which facilitate its physical study. Two class of solutions obtained previously have been shown to be contained in the general class of solutions. The anisotropic factor/electromagnetic field may vanish in the solutions and we can regain isotropic/uncharged solutions. A different choice, would perhaps enable us to regain other previously known stellar solutions. A paper in this direction is under preparation.

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