Efficiency of Centralized Structures in Data Envelopment Analysis Ratio Models

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Abstract. This paper investigates the centralized resource allocation with centralized structures by using the data envelopment analysis-ratio (DEA-R) models. To this end, it proposes a method to determine the resource allocation of centralized structures such that the ratio of inputs to outputs are minimized.

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1. Introduction

Data envelopment analysis (DEA) is a linear programming (LP), non-parametric technique which is used to measure the relative performance of a number of similar decision making units (DMUs). The CCR model, developed by Charnes, Cooper, and Rhodes [1], is one of the best known models to evaluate the efficiency of DMUs in DEA. Many extensions in DEA have been proposed based on the CCR model. Despic et al. [2] used ratio analysis to develop DEA-R models and Wei et al. ([7, 8]) applied DEA-R models to evaluate the efficiency in health care.

In many applications there are situations in which all the DMUs are under the umbrella of a centralized Decision Maker (DM). This type of situation occurs whenever all of the units belong to the same organization. Many DEA applications (such as those by bank branches, hospitals, university departments, supermarket chains and police stations) fall into this category (Lozano and Villa, [6]). In this case, the centralized decision-maker aims to minimize the overall input consumption (or to maximize the overall output production) by all DMUs (Korhonen and Syrjänen, [5]; Fang and Zhang, [4]; Du et al., [3]).

This paper investigates the centralized resource allocation with centralized structures by using the data envelopment analysis-ratio (DEA-R) models. To this end, it proposes a method to determine the resource allocation of centralized structures such that the ratio of inputs to outputs are minimized.

The paper unfolds as follows: Section 2 states some background of DEA and DEA-R. Section 3 proposes a centralized DEA-R model. Section 4 concludes.

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2. Background

Assume there are decision making units, where each DMU uses inputs to produce outputs. Charnes et al. [1] introduced the model to evaluate the relative efficiency as follows:

$$
\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_j \leq \theta x_o \\
& \sum_{j=1}^{n} \lambda_j y_j \geq y_o \\
& \lambda_j \geq 0, \quad j = 1,\ldots,n.
\end{aligned}
\tag{1}
$$

Despic et al. [2] used ratio analysis to evaluate the efficiency of the DMUs and proposed the input-oriented DEA-R model for efficiency evaluation of DMU$_o$ as follows:

$$
\begin{aligned}
\max \quad & \Delta \\
\text{s.t.} \quad & \Delta \leq \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} \frac{x_{ij}/y_{rj}}{x_{io}/y_{ro}}, \quad j = 1,\ldots,n \\
& \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1 \\
& w_{ir} \geq 0, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s.
\end{aligned}
\tag{2}
$$

Let λ_j $(j=1,\ldots,n)$ and γ be the dual variables constraints of model (2). Then, the dual of model (2) can be written as:

$$
\begin{aligned}
\min \quad & \gamma \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \frac{x_{ij}}{y_{rj}} \leq \gamma \frac{x_{io}}{y_{ro}}, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \eta \text{ free}, \quad \lambda_j \geq 0, \quad j = 1,\ldots,n.
\end{aligned}
\tag{3}
$$

Definition 2.1 DMU$_o$ in input-oriented DEA-R efficient if and only if, the optimal value of models (2) and (3) equal 1.

3. Centralized resource allocation in DEA-R models

For the DMUs with centralized structure, the centralized decision-maker aims to minimize the overall ratios of input to outputs production (or to maximize the overall ratios of outputs to inputs) by all DMUs. To this end, we propose the following model:

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{n} \Delta_j \\
\text{s.t.} \quad & \Delta_j \leq \sum_{i=1}^{m} \sum_{r=1}^{s} w_{irj} \frac{x_{ij}/y_{rj}}{\sum_{l=1}^{n} \frac{x_{il}}{y_{rl}}}, \quad j = 1,\ldots,n \\
& \sum_{i=1}^{m} \sum_{r=1}^{s} w_{ir} = 1 \\
& w_{ir} \geq 0, \quad i = 1,\ldots,m, \quad r = 1,\ldots,s.
\end{aligned}
\tag{4}
$$

The dual of the above model is as follows:

$$
\begin{aligned}
&\min \quad \gamma \\
&\text{s.t.} \quad \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_j^p \frac{x_{ij}}{y_{rj}} - \gamma \sum_{l=1}^{n}\frac{x_{il}}{y_{rl}} \leq 0, \quad i=1,\ldots,m, \quad r=1,\ldots,s \\
&\qquad \sum_{j=1}^{n}\lambda_j^p = 1, \quad p=1,\ldots,n \\
&\qquad \lambda_j^p \geq 0, \quad p,j=1,\ldots,n, \quad \gamma \text{ free.}
\end{aligned}
\tag{5}
$$

By the optimal solution of model (5) the projection of all DMUs are determined for the centralized structure.

4. Conclusion

This paper proposed a method to determine the resource allocation of the centralized structures by input orientation of the DEA-R models. The proposed method can be stated for the different DEA-R models, too.

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